NOTES

Surface Fluxes and the Nocturnal Boundary-Layer Height

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ABSTRACT

The Zilitenkevitch relation for nocturnal boundary-layer (NBL) depth h in terms of scales u_*/f and L is necessarily a poor predictor of h when single-point values of surface fluxes are used. This is because the latter are poor estimates of ensemble-mean fluxes which satisfy this relation. The argument is equally valid both in an evolving and equilibrium NBL.

1. Introduction

The evolving nature of the nocturnal boundary layer (NBL) requires a suitable rate equation to define the depth h, the solution of which gives the time-dependent height $h(t)$ and equilibrium height h_e. Approximate forms of the rate equation have been discussed by Zeman (1979), Mahrt (1981) and Nieuwstadt and Tennekes (1981)—abbreviated henceforth as NT81. They generally take the form

$$\partial h/\partial t \approx (h - h_e)/T, \tag{1}$$

where T is an effective time scale of the NBL evolution. For midlatitude conditions, outside of the initial decay period ($t \gtrsim 2$ h), one finds

$$T\partial h/\partial t \ll h \text{ or } h_e\,, \tag{2}$$

i.e., $h \approx h_e$. Model calculations (e.g., Garratt 1981—abbreviated as G81) confirm this, with h deviating from h_e by no more than 10–15% of h_e for latitudes >30°.

This suggests that relations defining h_e in terms of mean-field (e.g., Mahrt, 1981; NT81) or turbulent quantities (e.g., the scaling law of Zilitinkevitch, 1972) can be used, to a first approximation, as diagnostic relations for h. The Zilitinkevitch relation, with which the present note is mainly concerned, has been studied extensively using NBL observations, with the inequality above taken as an implicit assumption. The major limitation with this approach is not its use in non-equilibrium conditions, but rather the nature of the turbulent quantities. The observations we shall refer to have been used in recent analyses of G81, NT81 and Arya (1981)—abbreviated as A81.

2. The equilibrium height

a. Relations involving mean-field quantities

These have recently been reviewed by A81, and relations developed by Mahrt (1981) and NT81 in their discussions of the rate equation. Mahrt (1981) showed that an equilibrium NBL of height, $h_e^{(1)}$, may exist when the NBL bulk Richardson number becomes critical ($\mathrm{Ri_{BC}}$). Defining V_h and θ_h as wind speed and potential temperature at h_e, θ_0 the surface temperature, $\beta = g/T$ the buoyancy parameter, and f the Coriolis parameter yields

$$h_e^{(1)} = \frac{\mathrm{Ri_{BC}} V_h^2}{\beta(\theta_h - \theta_0)}\,. \tag{3}$$

In contrast NT81 found an equilibrium relation for h_e [$=h_e^{(2)}$] in terms of surface cooling rate, with

$$h_e^{(2)} = \frac{C|f|V_h^2}{\beta|\partial\theta_0/\partial t|}\,, \tag{4}$$

where C is a constant. Use of Eqs. (3) and (4) as diagnostic relations for h (taking $h \approx h_e$) requires values of $\mathrm{Ri_{BC}}$ and C, either from observations or model calculations. In addition iterative procedures must be used since the mean-field quantities themselves depend upon h.

b. The Zilitinkevitch relation

Let s_i be the single-point, time-average estimate of the ensemble-mean turbulent quantity s. Then for

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an equilibrium NBL above a horizontal, homogeneous surface we write (Zilitinkevitch, 1972), with $h_e = h_e^{(3)}$,

$$h_e^{(3)} = \gamma_e(u_* L/f)^{1/2}, \tag{5}$$

where u_* is the surface friction velocity, L the Monin-Obukhov length and γ_e a constant. It can be shown that the heights given by Eqs. (3)–(5) are approximately equivalent (e.g., NT81). Use of Eq. (5) as a diagnostic relation for h (where $h \approx h_e$) requires determination of γ_e from observations, for which we usually have available only estimates s_i of surface fluxes, and heights h in an evolving NBL. Alternatively, model calculations can be used but γ_e then seems to be sensitive to the closure assumptions used (e.g., A81).

For our homogeneous surface, n observations of s_i give estimates γ_i with

$$\begin{aligned}\bar{\gamma} &= n^{-1} \sum \gamma_i\,, \\ &= \gamma_e \text{ for an equilibrium NBL only.}\end{aligned} \tag{6}$$

The non-zero variance

$$\sigma_\gamma{}^2 = n^{-1} \sum (\gamma_i - \bar{\gamma})^2 \tag{7}$$

will depend, *inter alia*, upon the rms deviations of measured s_i about the ensemble mean s.

The use of observations to determine γ_e with a given accuracy depends upon the influence of NBL non-stationarity upon $\bar{\gamma}$ and the magnitude of $\sigma_\gamma{}^2$ related to use of s_i.

In addition, surface nonhomogeneities may exist on a horizontal scale of order the height h (but not greater, since these are generally avoided in the experimental situation). Under these conditions Eq. (2, 5) may still be valid, but only in terms of mesoscale or area averages of NBL height and surface fluxes. In particular, long-term time averages of single-point fluxes may be quite different to the analogous mesoscale fluxes, whence use of observations of s_i will give $\bar{\gamma} \neq \gamma_e$ even if an equilibrium NBL exists.

3. Observations

The analyses of A81 and NT81 used observations from the Cabauw tower site in the Netherlands, with surface fluxes inferred from near-surface mean profiles. In contrast G81 considered observations from four sites, including Cabauw, but with fluxes determined predominantly by eddy correlation techniques. In none of these analyses is the Cabauw data set the same.

a. *Estimates of h*

Numerical models of the NBL define h either in terms of a flux-ratio criterion, e.g., the height at which a covariance $\overline{s'w'_i}$ has decreased to some small fraction of its surface value or in terms of a critical gradient Richardson number (Ri_c) criterion. Both are essentially equivalent, since we assume for Ri $> \mathrm{Ri}_c$ that $\overline{s'w'_i} = 0$ (for practical purposes we may take $\mathrm{Ri}_c = 0.25$). G81 applied these two criteria to observations to estimate h; consequently zero surface fluxes, which relate to $\mathrm{Ri} > \mathrm{Ri}_c$, give $h = 0$ consistent with Eq. (5).

In contrast "direct" estimates of h using acoustic sounder data are used in the Cabauw observations. In our view this technique admits of an ambiguity in the context of Eq. (5). This occurs when surface fluxes are zero, or near-zero, for in this case "observed" h is many times greater than that given by Eq. (5), if a value for γ_e is assumed. This may be seen in the analysis of A81, for the "very stable" and "extremely stable" runs found in his Fig. 4. For relatively large surface fluxes, when h is relatively large, all three methods giving h are generally consistent with Eq. (5) for γ_e = constant (e.g., G81). We believe the ambiguity for near-zero surface flux arises as follows:

(i) Under these conditions Eq. (5) implies $h < 50$ m, which is the lower limit of the acoustic-sounder techniques. At the Cabauw site Eq. (5), with $\gamma_e = 0.4$ (G81), implies $h < 50$ m when $u_* \leqslant 0.1$ m s^{-1}.

(ii) Elevated layers of weak turbulence, isolated from the surface by a layer where fluxes are near zero, may exist giving sounder echoes and an apparent h ($\geqslant$50 m) = h_{app}. Here h_{app} is unrelated to surface fluxes through Eq. (5).

In such circumstances acoustic sounder observations of h are probably reliable only when $h > 50$ m, *viz* when $u_* \geqslant 0.1$ m s^{-1} at Cabauw.

b. *Cabauw observations*

References to site details can be found in Nieuwstadt and Driedonks (1979) and Nieuwstadt (1980):

1) Garratt (1981). He used observations from four sites including Cabauw. The latter were made available by F. T. M. Nieuwstadt, three nights being selected (February 1975), one of which was discussed by Nieuwstadt and Driedonks (1979). All three nights have direct, eddy correlation fluxes, with lowest level of measurement at 20 m. Using a log-linear wind profile, these fluxes and the wind at 10 m imply an aerodynamic roughness length $z_{01} \approx 0.2$ m. For all runs $u_* > 0.1$ m s^{-1}.

2) Nieuwstadt and Tennekes (1981). They used observations from several nights in 1977, with indirect, profile-based fluxes. Apparently these were based on the method of Nieuwstadt (1978), but with profiles restricted to 20 m. These profile data were found to be consistent with eddy fluxes at 20 m

(Nieuwstadt, private communication) and hence their use of a roughness length $z_{02} \approx 0.2$ m, which is apparently representative of the surrounding countryside and not of the local grassy area.

Reference to their individual data made available by the senior author (and contained in their Fig. 5) shows that most observations have $u_* > 0.1$ m s^{-1}.

3) Arya (1981). He used observations from several nights in 1975, with indirect, profile-based fluxes using a different method from NT81. This method is essentially a drag-coefficient technique at a reference level 2 m, and is not constrained to give zero fluxes when local $\mathrm{Ri} > \mathrm{Ri}_c$. Information in the paper suggests only 30% of all these data have $\mathrm{Ri} < \mathrm{Ri}_c$, probably related to his "moderately stable" runs with relatively large fluxes. We consider these the only reliable sub-data set. The remainder of the data set, related to the "very and extremely stable" runs have small inferred surface fluxes, which could be zero on a Richardson number criterion. The associated values of h could be interpreted as h_{app}, described in Section 3a.

In contrast to NT81, A81 uses a local roughness length $z_{03} \approx 0.02$ m, based on near-neutral profiles (height range 2–40 m), and giving fluxes representative only of the local, grassy terrain influencing the surface layer.

4. Data analysis

The observations are used to deduce values of $\bar{\gamma}$ and to illustrate the relatively large uncertainty in its determination. We assume the Cabauw site is characterized by a mesoscale (area) roughness length given by the average of z_{01} and z_{02} (Section 3), i.e. $z_{0A} = 0.2$ m, and by a local value given by z_{03} (Section 3) i.e., $z_{0l} = 0.02$ m. Using the "moderately stable" runs of A81, his Fig. 4 gives 39 data points with

$$\bar{\gamma}_a = 0.74, \quad \sigma_{\gamma a} = 0.26.$$

The fluxes used to derive $\bar{\gamma}_a$ will be systematically smaller than the mesoscale (area) fluxes based on differences in z_{0l} and z_{0A} upon which these indirect fluxes depend. Reference to Eq. (14) of A81 shows that fluxes depend upon z_0, wind at z_1 (2 m) and differences Δu, $\Delta\theta$ between 2 and 9 m. The quantity $(u_*L)^{1/2}$ upon which γ depends will be mainly dependent upon z_0 through $(\ln z_1/z_0)^{-1}$; differences in z_0 will also affect u_1, Δu and $\Delta\theta$ values. If we replace z_{0l} by z_{0A}, and neglect (unknown) changes in u, etc., the corrected $\bar{\gamma}_a$ becomes (with $z_1 = 2$ m)

$$\bar{\gamma}_{a,\mathrm{corr}} = 0.37, \quad \sigma_{\gamma a} = 0.13,$$

while including changes in u_1, etc., by assuming the effective z_1 is the geometric mean of 2 and 9 m (the two levels related to Δu, $\Delta\theta$) gives

$$\bar{\gamma}_{a,\mathrm{corr}} = 0.43, \quad \sigma_{\gamma a} = 0.15.$$

TABLE 1. Data of NT81 used in their Fig. 5, analyzed as hourly averages. Here ddd is wind direction and n is number of hourly values per night.

Date (1977)	n	$\bar{\gamma}_b$	σ_γ	ϵ_γ^2	ddd (20 m)	Comments
29/3	5	0.71	0.18	0.06	60–90	
30/3	8	0.31	0.05	0.03	75–120	
9/4	4	0.36	0.06	0.03	20–30	
18/5	7	0.56	0.25	0.20	25–50	includes one anamolous point
22/5	5	0.61	0.08	0.02	30–35	
23/5	5	0.47	0.05	0.01	40–65	
24/5	6	0.83	0.32	0.15	30–55	low $u_* \approx 0.1$ m s^{-1}
25/5	6	0.44	0.07	0.025	60–75	
4/7	4	0.70	0.20	0.08	40–50	
3/12	14	0.21	0.03	0.02	80–90	
4/12	14	0.47	0.13	0.08	95–120	
8/12	13	0.71	0.30	0.18	125–155	low γ in first half of night; high γ in second half
Total	91	0.51	0.19	0.13		

We now turn to the analysis of NT81. They discussed observations in the context of Eq. (5), mainly to argue its nonvalidity in a non-equilibrium situation. We have available the individual data used in their Fig. 5, and show in Table 1 values of $\bar{\gamma}_b$, σ_γ and the normalized variance, with $\bar{\gamma} = \bar{\gamma}_b$,

$$\epsilon_\gamma^2 = \sigma_\gamma^2/\bar{\gamma}^2, \tag{8}$$

for individual nights. Data have been rejected for $t \leqslant 1$ h (i.e., in the immediate transition period) and where $u_* < 0.1$ m s^{-1}—this involves less than 10% of total data set.

One notable feature of this data set is that significant differences exist in $\bar{\gamma}_b$ and ϵ_γ^2 between nights. This is most evident on the three nights in December where ~50% of the total data occur. We believe there are two probable sources of this behavior:

(i) Systematic errors in the estimated fluxes.

(ii) Effects of sloping terrain, which depend upon wind direction (e.g., Brost and Wyngaard, 1978). Overall the evidence is not compelling, particularly since local surveys give "negligible" slope, although these three December nights show a strong correlation of $\bar{\gamma}$ with wind direction.

There is also the possibility of thermal effects influencing the boundary-layer flow, due to the proximity of the sea (F. T. M. Nieuwstadt, private communication).

From Table 1, we find

$$\bar{\gamma}_b = 0.50, \quad \sigma_{\gamma b} = 0.18,$$

which compares more favorably with the A81 corrected $\bar{\gamma}_a$ (~0.4) than with the value $\bar{\gamma}_a = 0.74$ related to z_{0l}.

The above values of $\bar{\gamma}$ may be compared with the G81 value for Cabauw based on direct fluxes, *viz*

$$\bar{\gamma}_c = 0.42, \quad \sigma_{\gamma c} = 0.14.$$

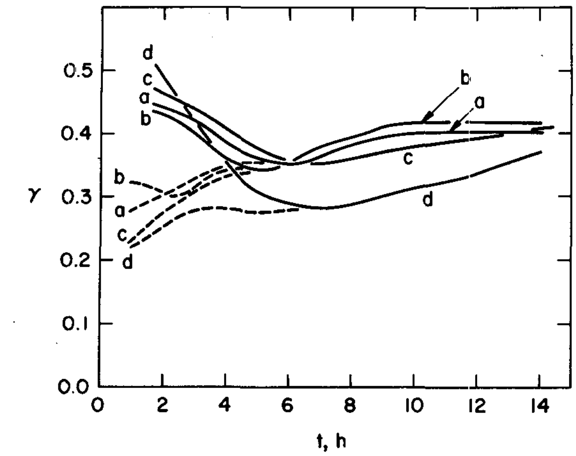

FIG. 1. Model calculations of time variation of parameter γ, taken from G81. Continuous curves use Ri for height criterion; pecked curves use heat flux criterion. Curve a: latitude (ϕ) = 45°, z_0 = 0.01 m; b: latitude (ϕ) = 45°, z_0 = 0.1 m; c: latitude (ϕ) = −30°, z_0 = 0.01 m; d: latitude (ϕ) = −15°, z_0 = 0.1 m.

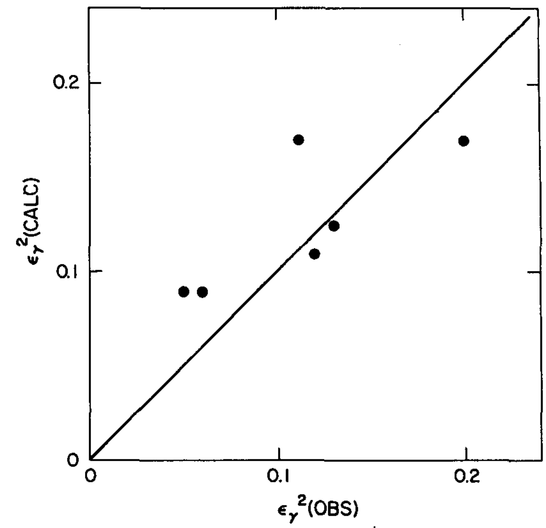

FIG. 2. Comparison of calculated and observed values of ϵ_γ^2.

This value compared well with those from other midlatitude sites, all of which were consistent with a modified rate equation of NT81, and model calculations of Brost and Wyngaard (1978). The latter are shown in Fig. 1, and may be used to estimate a probable contribution of $\partial\gamma/\partial t$ in the nonequilibrium NBL to ϵ_γ^2.

We now consider the curves in Fig. 1 in the range t = 2–12 h where most observations in G81, A81 and NT81 occur. According to these, values of $\bar{\gamma}$ will be less than γ_e in Eq. (5), since h is less than the equilibrium height throughout most of the period. In addition, $\gamma(t)$ varies about $\bar{\gamma}$ giving variance $\sigma_{\gamma 0}^2$, and if we assume γ is available at 1 h intervals then we find from Fig. 1 that

$$\left.\begin{array}{lll}\phi = 45°, & z_0 = 0.01 \text{ m}: & \sigma_{\gamma 0}^2 = 0.026 \\ \phi = -30°, & z_0 = 0.001 \text{ m}: & \sigma_{\gamma 0}^2 = 0.035\end{array}\right\},$$

giving normalized variance $\epsilon_{\gamma 0}^2 \approx 0.006$. Here $\bar{\gamma} \approx 0.39$ with $\gamma_e \approx 0.4$–0.45 *viz* $(\gamma_e - \bar{\gamma})^2/\bar{\gamma}^2 \approx 0.008$, which applies for the homogeneous case only.

We are interested in observed variances ϵ_γ^2, how these compare to differences between $\bar{\gamma}$ and γ_e, and what the major sources are which determine ϵ_γ^2. Table 2 shows a summary of all relevant quantities found in the analyses of G81, A81 and NT81 discussed earlier.

TABLE 2. Results for data analyses at four sites. Sites M (Minnesota) and W (Wangara) are midlatitude; site K (Koorin) at latitude 16°S has low $\bar{\gamma}$ because of katabatic effects (G81). Ec refers to eddy correlation; numbers in parentheses are corrected values.

Source	Site	Flux level	Method	$\bar{\gamma}$	σ_γ	ϵ_γ^2	τ (s)
A81	Cabauw	2–9	profile	0.74	0.26	0.12	4
				(0.37)	(0.13)	0.12	
NT81	Cabauw	10	profile	0.51	0.19	0.13	5
G81	Cabauw	20	Ec	0.42	0.14	0.11	8
	K	20	Ec	0.13	0.06	0.20	8
	M	4	Ec	0.37	0.09	0.06	2.5
	W	4	profile	0.39	0.09	0.05	2.5

All sites, with one exception, have values of $\epsilon_\gamma^2 \gg \epsilon_{\gamma 0}^2$ and $\sqrt{\epsilon_\gamma^2} \gg (\gamma_e - \bar{\gamma})/\bar{\gamma}$; the analysis of A81, for Cabauw, uses the "local" z_0 and shows a large bias in $\bar{\gamma}$. Such biases can be expected for heterogeneous surfaces, unless the local parameters are replaced by appropriate mesoscale values (see discussion in Section 2b). NT81 have argued that relatively large values of ϵ_γ^2 exist because Eq. (5) is not valid in an evolving NBL. However, we show that ϵ_γ^2 depends mainly upon uncertainties in surface fluxes and, to a first approximation, Eq. (5) is valid at all times outside of the immediate transition period, at least for latitudes $\phi > 30°$.

Based on Eq. (5), ϵ_γ^2 will arise, in part, from statistical and experimental errors in determining covariances $\overline{w's'}$ where, since $\gamma \approx h|\overline{w'\theta'}|^{1/2}/(-\overline{u'w'})$, such contributions will be

$$3/2((\epsilon_{ws}^2)_{\text{exp}} + (\epsilon_{ws}^2)_{\text{stat}}).$$

Additional contributions arise from errors in h and time variations in γ. The total variance is given by

$$\epsilon_\gamma^2 = \epsilon_{\gamma 0}^2 + (\epsilon_h^2)_{\text{exp}} + 3/2[(\epsilon_{ws}^2)_{\text{exp}} + (\epsilon_{ws}^2)_{\text{stat}}], \quad (9a)$$

where $(\epsilon_h^2)_{\text{exp}}$ relates to experimental errors in measuring h, $(\epsilon_{ws})^2_{\text{exp}}$ relates to experimental errors in estimating ensemble mean covariance $\overline{w's'}$, and $(\epsilon_{ws}^2)_{\text{stat}}$ relates to rms deviation of single-point, time-averaged values of $\overline{w's'}_i$ about $\overline{w's'}$.

For our sets of observations, we have $\epsilon_{\gamma 0}^2 \simeq 0.006$, $(\epsilon_{ws}^2)_{\text{exp}} \approx 0.01$ and $(\epsilon_h^2)_{\text{exp}} \approx 0.03$ (e.g., G81).

Thus, in Eq. (9a)

$$\epsilon_\gamma^2 \approx 0.05 + 3/2(\epsilon_{ws}^2)_{\text{stat}}. \quad (9b)$$

The quantity $(\epsilon_{ws}^2)_{stat}$ depends mainly upon the spectral and correlation properties of turbulence at the effective height of measurement of $\overline{w's_i'}$ (given in Table 2 for each data set). It is given by the expression (e.g. Wyngaard, 1973),

$$(\epsilon_{ws}^2)_{stat} = 4\tau T_R^{-1}(r_{ws}^{-2} + 1), \qquad (10)$$

with $T_R = 3600$ s for hourly-averaged quantities used here. Here τ, the integral time scale of turbulence, and r_{ws}, the point cross-correlation coefficient, are determined from the results of Kaimal *et al.* (1972) for the stable surface layer for the relevant range of z/L between 0.1 and 0.5. We find $r_{ws} \simeq -0.35$ with values of τ given in Table 2. Calculated values of ϵ_γ^2 from Eqs. (9a) and (10) are shown plotted against observed values (Table 2) in Fig. 2. The quite good agreement emphasizes the important contribution of flux variance $(\epsilon_{ws}^2)_{stat}$, which is generally the greatest contribution to observed ϵ_γ^2.

5. Discussion

The uncertainty in experimentally-determined values of γ_e has direct implications for the prediction of h from Eq. (5). Even for an equilibrium NBL, and assuming γ_e is precisely known, the equation is a poor predictor of h_e if single-point measurements of surface fluxes are used. This is because the latter are poor estimates of the ensemble-mean fluxes satisfying Eq. (5). The true h_e will be given by estimate $h_e^{(3)}$, with normalized variance $\epsilon_3^2(h_e)$, where

$$\epsilon_3^2(h_e) \approx {}^3\!/_2\epsilon_{ws}^2 \approx 0.09 \qquad (11a)$$

for fluxes measured at 10 m height.

In contrast, use of either Eqs. (3) or (4) would generally give estimates $h_e^{(1)}$, $h_e^{(2)}$ with smaller normalized variance, where, assuming Ri_{BC} and C are precisely known and that measured velocity and temperature quantities are accurate to $\pm10\%$,

$$\epsilon_{1,2}^2(h_e) \approx 0.03. \qquad (11b)$$

In the real situation of an evolving NBL, but where Eq. (2) is satisfied so that $h \simeq h_e$, variances will be larger because of uncertainties in the "constants" found in Eqs. (3)–(5). Thus estimates $h^{(3)}$ using the Zilitinkevitch relation will be associated with normalized variance $\epsilon_3^2(h)$, where

$$\epsilon_3^2(h) \approx \epsilon_\gamma^2 + {}^3\!/_2\epsilon_{ws}^2 \approx 0.2, \qquad (12a)$$

while estimates $h^{(1)}$, $h^{(2)}$ based on Eqs. (3) and (4) will have normalized variance $\epsilon_1^2(h)$, taking Eq. (3) to illustrate,

$$\epsilon_1^2(h) \approx \epsilon_{Ri_{BC}}^2 + 3\epsilon_s^2$$
$$\approx \epsilon_{Ri_{BC}}^2 + 0.03. \qquad (12b)$$

Model and analytic calculations (e.g., Mahrt, 1981; NT81; Garratt and Brost, 1981) suggest, for example, that Ri_{BC} lies in the range 0.3–0.55, *viz* $\overline{Ri}_{BC} \approx 0.425$ with $\epsilon_{Ri_{BC}}^2 \approx .09$, whence

$$\epsilon_1^2(h) \approx 0.12. \qquad (12b)$$

The results of Garratt and Brost (1981) do show, however, that Ri_B varies significantly in the evolving NBL, so that an additional contribution can be expected to the variance given by (12b).

6. Summary

Generally speaking, careful investigation of Eq. (5), either for prediction of h or determination of γ, is severely limited by the use of single-point measurements of surface fluxes, even in the case of an equilibrium NBL above a horizontal, homogeneous surface. With such measurements it is not possible to distinguish, to any significant degree, between this hypothetical situation and the real one of non-equilibrium NBL.

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