Detecting Climate Change

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ABSTRACT

The likelihood ratio of the data for a hypothesis of some change, relative to the hypothesis of no change, is a suitable statistical measure for the detection of climate change. Likelihood ratios calculated on the basis of Angell and Korshover's (1977) global mean temperature, updated through 1980, do not show convincing evidence of recent climate change. It is possible to calculate probabilities of obtaining future values of likelihood ratios, depending on the postulated future climate change. A modest but significant climate change, such as that expected to occur from an increase of atmospheric carbon dioxide, is likely to be detected from global mean surface temperatures within ten years. The joint behavior of the troposphere and stratosphere is more likely to discriminate between climate change and no change than are surface temperatures. In this case, a climate change that can be attributed to carbon dioxide increase should be detectable by 1986.

1. Introduction

Climate change is difficult to define, let alone to detect or measure. Yet, there is great interest, heightened by concern over the continuing increase in atmospheric CO_2, and its potential climatic effects, in the early detection of subtle changes which may be harbingers of larger changes to come.[1]

Because climate is inherently variable on all time scales, it is not possible to ascribe uniquely any observed variation to some extraordinary cause, or to classify it, with complete confidence, as an aspect of natural variability. This difficulty is exacerbated by the shortness of many of the most reliable global climate records, and the relatively frequent occurrence of events, like volcanoes, that may distort the statistical characteristics of the climatic record. Thus we cannot know with much certainty the nature of the climate record against the background of which we are trying to detect a change that would be otherwise anomalous.

Wigley and Jones (1981) have recently examined the question of detecting a CO_2-induced climate change by comparing the climatic "noise" estimated from gridded monthly mean surface temperature data with a "signal" to be expected from CO_2 effects based on the model results of Manabe and Stouffer (1980). They found that the signals to be expected as a result of CO_2 are similar to climate variations which occurred earlier in the century and which could not be attributed to CO_2. Furthermore, there are reasons to doubt that the early transient signals due to increasing CO_2 would resemble scaled down effects predicted by equilibrium models of the effects of doubling the CO_2 (Schneider and Thompson, 1981).

The approach we describe does not attempt to deal directly with attribution, but rather examines only the question of whether the climate record provides evidence for recent anomalous behavior. We also deal more specifically with the question of how long it will be necessary to wait to detect or reject hypotheses of particular modes of anomalous behavior. For our initial applications of the method, we have used a uniquely uniform set of estimates of global mean temperatures, a much shorter series than that employed by Wigley and Jones, but accompanied by data on the free atmosphere which they did not examine.

We will make grossly simplistic assumptions about the statistical nature of the recent (decades) climate record and seek evidence for some changes from that simplistic behavior. Assuming that the evidence to date is equivocal, we will then ask how long we will have to observe to accumulate convincing evidence and how likely it is that we will detect a change of the type postulated, or be fooled into believing a change that is not real. Although our answers will not be definitive responses to the question of climate

[1] Our interest in this subject was stimulated by a Workshop on the Early Detection of Climate Change from CO_2, Harper's Ferry, WV, 8–10 June 1981, sponsored by the Department of Energy, and by persistent questions at that Workshop from Dr. Hugh Ellsaesser, Lawrence Livermore Laboratory, on the questions of whether any change has yet been observed and how change is to be detected.

change, they will provide quantitative measures of how credible certain hypotheses are, relative to one another. If we suggest that evidence exists for climate change, it means only a change from the assumed simplistic model of how climate behaves. We may be "detecting" a relatively low frequency modulation that is a basic characteristic of climatic behavior. Whether such a variation should be ascribed to the intrinsic behavior of climate, or to some extrinsic cause, like CO_2, cannot be answered entirely by statistics. On the other hand statistics can go a long way to clarifying the issue if one examines, in a multidimensional sense, a set of parameters that should behave in a specific coherent manner to be consistent with some hypothesis of extrinsic cause. We will examine one such two-dimensional example.

2. Data and postulated modes of climate change

All examples to be presented here are based on Angell and Korshover's (1977) global mean temperatures, based on a set of radiosonde stations carefully selected to give reasonable global coverage. They have been assembled since 1958, and updated to 1980 (J. K. Angell, personal communication, 1981). Three series of global mean temperatures will be involved in the calculations and analysis below, and these are presented as departures from the 1958–78 mean in Fig. 1. Note that both the mean surface temperatures and the mean tropospheric temperatures are somewhat elevated in the last few years, while the stratospheric temperatures are low. This behavior is consistent with the postulated effects on global mean temperatures of increasing atmospheric CO_2 (National Academy of Sciences, 1979). The question to be addressed is how to evaluate this evidence. Is it sufficient to conclude that real change is occurring, or is it reasonable to conclude that such observations could have occurred by chance with no real climate change being involved?

In order to address this question, and further questions about the conclusions we are likely to come to in future years, it is necessary to propose some model for the statistical behavior of the series. The basic model we will use is that T_i (the annual mean temperature for the ith year) is given by

$$T_i = \mu_i + \delta_i + \epsilon_i ,$$

where μ_i is the "natural" climatic mean for the year in question, δ_i is a possible extrinsic "climate change," and ϵ_i is a random variable with zero mean, uncorrelated with μ_i, δ_i or ϵ_j, $j \neq i$.

The crudest model for μ_i is $\mu_i = \mu_0$, a constant. The examples considered below will all involve this model. One alternative is to consider that the annual mean temperatures are autocorrelated:

$$\mu_i = a(\mu_{i-1} + \epsilon_{i-1}) + \mu_0(1 - a),$$

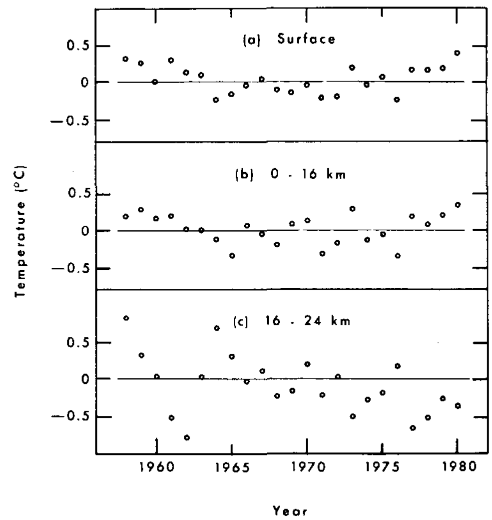

FIG. 1. Global mean temperature, expressed as departures from 1958–76 means, after Angell and Korshover (1977 and private communication).

where $|a| < 1$. Both a and μ_0 must be estimated on the basis of the available data. This and more complex models can be considered, but will not be attempted here. Clearly the more complex the model, the more data are needed to produce credible estimates of the relevant parameters. In the present example, simple autoregressive models do not appear to be particularly helpful, but have not been thoroughly explored.

Another alternative is to allow the μ_i to vary according to the calculated or estimated effects of "known" extrinsic climate processes, like volcanism or variations in the solar constant (Hansen *et al.*, 1981). This would in principle allow one to isolate better the suspected CO_2 effect, but depends very heavily on the specific quantitative attribution of these other effects. We have left this alternative to another exercise, since we are not prepared, here, to argue the merits of the ways in which those effects have been quantified.

The values used for δ_i, and the dependence of the δ_i on time, will be hypothesized. We will assess the credibility of particular hypotheses that $\delta_i \neq 0$. In addition to quantifying the implications of the recent climate record on the credibility of particular hypotheses about δ_i, we will also assess the probability of being led by future observations to various conclusions about δ_i, assuming the "true" values are δ_i^*.

There is literally no limit to the possible time-dependent changes that one might examine and test. Yet only a small number warrant study. In fact, in this study, we have examined three functional forms for the variation of δ with time. One is a step function change occurring between 1976 and 1977. Such a change appears unjustifiable on any physical basis,

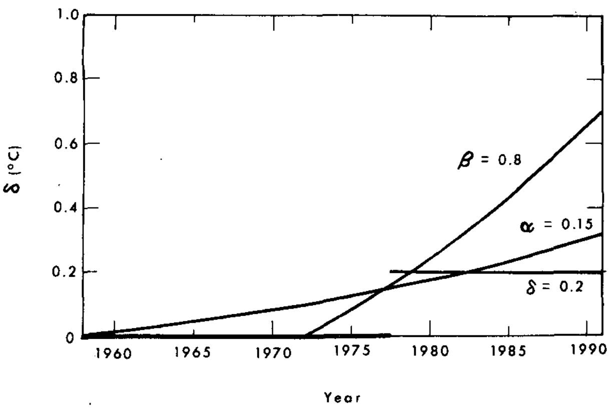

FIG. 2. Illustrative examples of the three climate change scenarios. The curves labeled $\delta = 0.2$, $\alpha = 0.15$ and $\beta = 0.8$ represent examples of Cases 1, 2 and 3, respectively.

but is suggested somewhat by the surface temperature data themselves. Because of this *post facto* selection, the results must be interpreted very cautiously. We find it useful, nevertheless, to compare the results in this case with the results obtained using other functional forms.

The two other functional forms examined are both exponential, with a time constant of 30 years. The 30 years is approximately that suggested by Cess and Goldenberg (1981) as appropriate to the growth of the radiative effects of CO_2. In one case the initial effect of CO_2 is postulated to occur in 1958, coinciding with the beginning of Angell and Korshover's record of global mean temperatures. In the final case, the effect begins 15 years later, or in 1973. This is a crude representation of a delayed CO_2 effect (Cess and Goldenberg, 1981; Schneider and Thompson, 1981). The three cases are

CASE 1 (step function)

$$\delta_i = \begin{cases} 0 & \text{for} \quad i \leqslant 19 \text{ (up to 1976)} \\ \delta_0 & \text{for} \quad i \geqslant 19 \text{ (since 1977)}, \end{cases}$$

CASE 2 (exponential since 1958)

$$\delta_i = \alpha(\exp(i/30) - 1),$$

CASE 3 (exponential since 1973)

$$\delta_i = \begin{cases} 0 & \text{for} \quad i < 15 \\ \beta(\exp(i - 15)/30 - 1) & \text{for} \quad i > 15. \end{cases}$$

Fig. 2 illustrates the three cases. Case 1 is shown for $\delta_0 = 0.2$, Case 2 for $\alpha = 0.15$, and Case 3 for $\beta = 0.8$. The coefficients were chosen, for this illustration only, so that they would be crudely consistent with a temperature increase of ~0.2°C in 1980. Note that the temperatures implied by the three cases differ most profoundly for the future years. Case 3 implies the most substantial increases over the next decade, while Case 1 implies no further changes. This will be important when we examine the likely interpretations of observations to be made in future years.

3. Evaluating the credibility of climate change hypotheses

We will assume that the ϵ_i are normally distributed with constant variance σ^2. It is conceivable that a "climate change" could be manifested by a change in variance, rather than only a change in mean, but we will not examine that possibility in this analysis.

The basic measure we will use to evaluate the credibility of various hypotheses about δ is the likelihood ratio.[2] The likelihood of obtaining a particular value of T_i in the ith year is

$$L(\delta_i; T_i, \mu_i, \sigma) = (2\pi)^{-1/2}\sigma^{-1} \exp(-\epsilon^2/2\sigma^2),$$
$$= (2\pi)^{-1/2}\sigma^{-1} \exp[-(T_i - \mu_i - \delta_i)^2/2\sigma^2].$$

The likelihood of a set of n values of T_i, $i = 1, \ldots, n$, since the ϵ_i are uncorrelated, is

$$L(\delta_i; T_i, \mu_i, \sigma; i = 1, \ldots, n)$$
$$= (2\pi)^{-n/2}\sigma^{-n} \exp[-\sum_{i=1}^{n} (T_i - \mu_i - \delta_i)^2/2\sigma^2].$$

Assuming $\mu_i = \mu_0$ for all i, this likelihood is maximized if

$$\hat{\mu}_0 = (\sum_{i=1}^{n} T_i - \sum_{i=1}^{n} \delta_i)/n = \bar{T} - \bar{\delta}$$

and

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (T_i - \delta_i - \hat{\mu}_0)^2$$

are used as estimates of the (unknown) parameters μ_0 and σ^2. The maximum likelihood for the observations, under a particular hypothesis concerning the values of δ_i, is then

$$L(\delta_i; T_1, \ldots, T_n)$$
$$= (2\pi)^{-n/2}\hat{\sigma}^{-n} \exp[-\sum_{i=1}^{n} (T_i - \bar{T} - \delta_i + \bar{\delta})^2/2\hat{\sigma}^2]$$
$$= (2\pi)^{-n/2}\hat{\sigma}^{-n}e^{-n/2},$$

by substituting, in the exponential, the maximum likelihood solution for σ^2 and μ. The *relative* likelihood of obtaining the data, given a particular hypothesis concerning δ_0, compared to that for the hypothesis $\delta_i \equiv 0$ is

[2] The likelihood ratio is discussed in most basic statistics texts in connection with maximum likelihood estimators. See also discussions of likelihood-ratio tests as in Mood and Graybill (1963).

$$\lambda = \left[\frac{\sum_{i=1}^{n} (T_i - \bar{T} - \delta_i + \bar{\delta})^2}{\sum_{i=1}^{n} (T_i - \bar{T})^2} \right]^{-n/2}$$

$$= 1 + \frac{\overline{\delta^2} - \bar{\delta}^2 - 2(\overline{\delta T} - \bar{\delta}\,\bar{T})^{-n/2}}{\overline{T^2} - \bar{T}^2}.$$

Given any particular hypothesis concerning the δ_i, one can calculate the corresponding value of λ. If the calculated value of λ is large (say >100), one concludes that this is a much more credible hypothesis about δ than $\delta \equiv 0$. On the other hand, if λ is small (say <0.01), one would be inclined to reject the notion of the particular behavior of δ in favor of the hypothesis that $\delta \equiv 0$.

It is not possible to establish a single value of λ that should be a universal criteria for accepting or rejecting hypotheses. In the context of Bayesian statistics, the likelihood is the ratio of the probability assigned to a hypothesis *after* the data are examined (the posterior probability) to that assigned to the hypothesis *before* the data are examined (the prior probability). To the extent that there are no strong prior preferences for one hypothesis over another, λ is a measure of the relative posterior credibility of the hypotheses. In such a case, one should consider the use of criteria such as $\lambda = 20$ or $\lambda = 100$, which would be equivalent to classical statistical tests of significance at 5% or 1% levels, respectively. On the other hand, if a strong prior preference exists, a different criterion should be adopted. For example, the author would assign prior probability ratios in Case 1, for $\delta > 0.1$ relative to $\delta = 0$, of <0.10. Therefore, for Case 1, he would place a more stringent criteria on δ before he would be prone to accept the validity of a hypothesis that $\delta > 0.1$.

Calculations of λ can be made on the basis of data presently available, and several such calculations are shown below. Calculations will also be possible in future years after more data are accumulated. It is possible to predict, in a probabilistic sense, what the results of these future calculations will be, if one assumes some particular "true" behavior of δ. Such calculations yield estimates of the probability that some real effect will or will not be recognized a few years hence, and, on the other hand, the probability of being falsely led to conclude that an effect is real when it is really not so.

Predictions of the T_i are more uncertain than is implied simply by the estimate σ^2 of the variance of the process. If the variance of the process were known to be given by σ^2 and the mean was known to be exactly equal to μ_0, then indeed one would use a normal distribution to generate predictions of future values of T_i. However, uncertainty in the value of σ^2, and in the value of μ_0 must also be factored into probability statements about future T_i. It is shown by Raiffa and Schlaifer (1968, p. 303) that under these conditions, with all the information about μ_0 and σ^2 contained in the n previous years' observations, the quantities $(T_i - \mu_0 - \delta_i)/V$, $i > n$, have independent Student-t distributions with $n - 1$ degrees of freedom, where

$$V = \frac{n+1}{n-1}\frac{1}{n}\sum_{i=1}^{n} (T_i - \delta_i - \bar{T} + \bar{\delta})^2.$$

By means of Monte Carlo calculations a large number (1000) of possible future realizations of the climate record for $n^* = 1, 2, 5$ and 10 years into the future have been calculated for each model of δ_i, and for various parameter values.

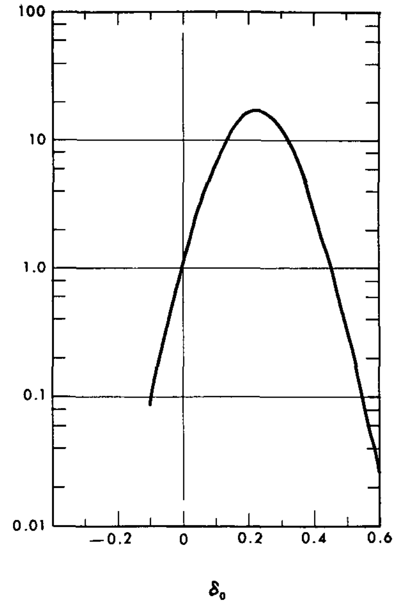

FIG. 3. Likelihood ratios of the global mean surface temperatures in Fig. 1a as a function of the parameter δ_0 of Case 1.

4. Results of calculations involving mean surface temperatures

a. Current likelihood ratios

Likelihood ratios for the data of Fig. 1a are plotted in Figs. 3, 4 and 5 for the three cases, compared always to null alternative hypotheses that α, β, $\delta_0 \equiv 0$. For none of the cases is the maximum likelihood ratio sufficiently large that one would conclude the null hypothesis should be rejected. The largest likelihood ratio, ~ 17, corresponds to $\delta_0 = 0.22$ in Case 1. In other words, it is ~ 17 times more likely to have recorded the 1958–80 surface mean temperatures if

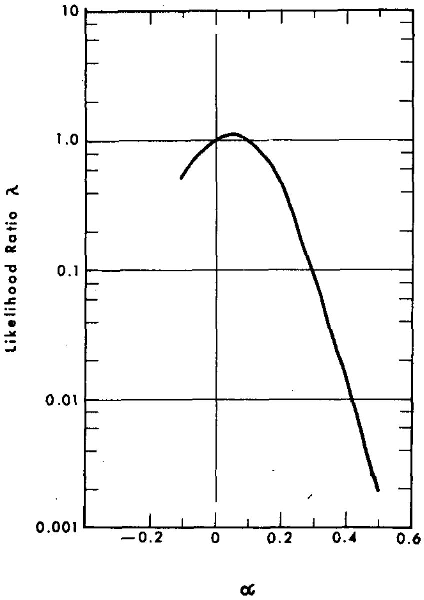

FIG. 4. As in Fig. 3, but for parameter α of Case 2.

a step function increase in temperature had occurred in 1976–77, than if there had been no climate change. But this is not convincing evidence that such a step function change did indeed occur. For some purposes one might use a likelihood ratio of 20 as a criterion (corresponding to a 5% significance level) but this criterion is not met. More importantly, this would be a very weak criterion in a situation where the model itself had been developed only after examining the data.

We conclude that there is not yet any convincing evidence for climate change in the recent record of global mean surface temperatures. This is not to say, however, that the data allow us to exclude the possibility that geophysically significant climate changes are occurring. Values of β near 1.0 are credible, on the basis of observations to date (likelihood ratios > 5.0), and therefore cannot be excluded. From Fig. 2 we can infer that such parameter values would be of considerable geophysical significance. It will be necessary to wait until more data have accumulated before more definitive conclusions can be drawn.

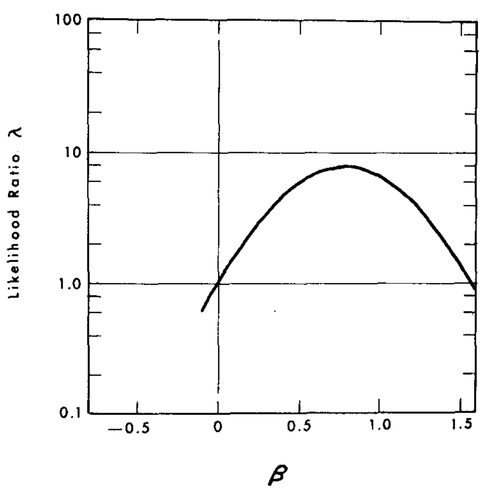

FIG. 5. As in Fig. 3, but for parameter β of Case 3.

b. Future expectations of likelihood ratios

For each of the climate change models and for several values of postulated "true" parameter values (designated δ_0^*, α^*, and β^*), Monte Carlo calculations have produced probability distributions of likelihood ratios that will be encountered 1, 2, 5 and 10 years in the future. From these calculations we estimate how soon more definitive conclusions about climate change can be drawn, and how likely it is that such conclusions will be correct.

Let us first consider the results for Case 1 for $\delta_0^* = 0$, if there is no change. Figs. 6, 7 and 8 present the probabilities that values of λ will be exceeded after 2, 5 and 10 additional years of observations.

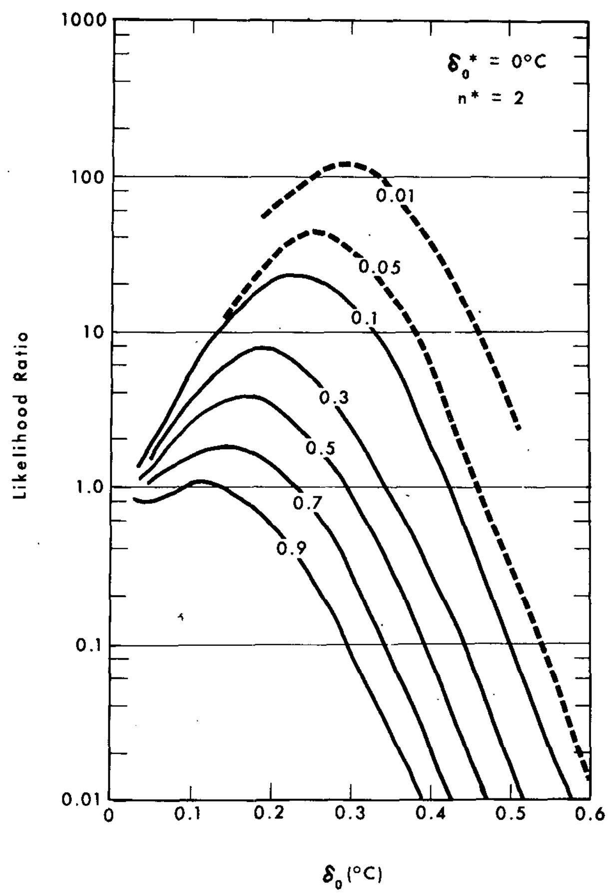

FIG. 6. Probabilities of determining likelihood ratios for Case 1 after including data from two additional years, if there is indeed no climate change. The labels on the curves indicate the probabilities that the likelihood ratios will be exceeded.

For example, from Fig. 6, the probability of calculating, in 1983, a likelihood ratio > 20, corresponding to a hypothetical value of $\delta_0 = 0.2$, when the true value has been 0.0, is ~0.1. The median value of λ for $\delta_0 = 0.2$ is about 3.5. This should be compared with a current calculation of $\lambda = 16.6$ for $\delta_0 = 0.2$. There is about one chance in eight that λ, two years from now, will still be as large as its present value, even though the "true" value δ_0^* is 0.0. Five years in the future (including data through 1985, Fig. 7) the median value of λ for $\delta_0 = 0.2$ is reduced to 1.1, given $\delta_0^* = 0$, and the probability that λ will still be as large as its present value is about 0.09 (or about one chance in 11). By 1990 (Fig. 8) there will be less than one chance in 20 that λ, for $\delta_0 = 0.2$, will be equal to or greater than 16.6.

Examined from another perspective, if $\lambda = 20$ is chosen as a criterion for accepting a climate change hypothesis, then there is somewhat more than one chance in 10 that a hypothesis in the range $0.2 < \delta_0 < 0.25$ will be accepted two years from now, even though it is false. In five years there is about one chance in 12 that such an error would be made, and even after ten years the chance of accepting an incorrect climate change hypothesis is about one in 25. On the other hand, if the criterion used is $\lambda = 100$, then the probability of rejecting a valid null hypothesis is ~0.1 after two years, and decreases only slowly to ~0.007 after 10 years. (All proportions are based on Monte Carlo experiments with $n = 1000$. There is considerable sampling error inherent in the tails of the distributions. However, the consistency of values from one run to another can be viewed as confirming the lack of gross sampling errors.)

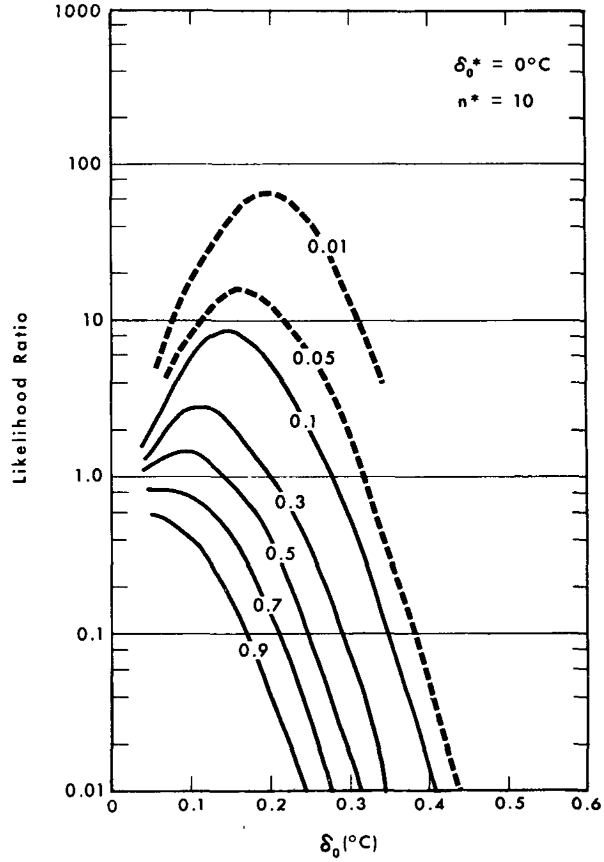

FIG. 8. As in Fig. 6, except after including ten years of additional data.

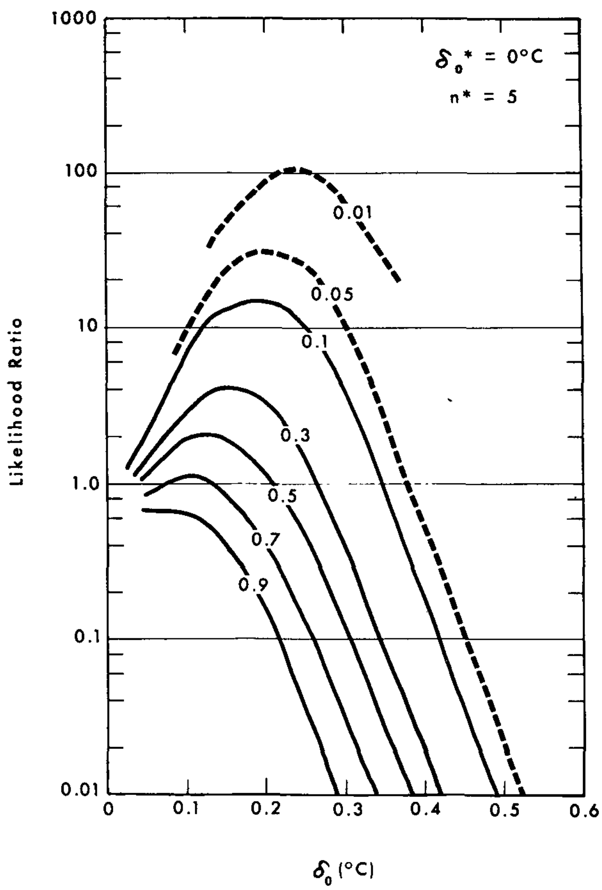

FIG. 7. As in Fig. 6, except after including five years of additional data.

Now let us examine the situation if a "true" climate change were occurring. Fig. 9 illustrates the results for $\delta_0^* = 0.2$ with data from two additional years. The median value of the likelihood to be calculated for $\delta_0 = 0.2$ (i.e., the hypothesis that $\delta_0 = 0.2$ not knowing that the "true" value is $\delta_0^* = 0.2$) is now $\lambda_{med} = 20$. There is one chance in 15 that values of λ in excess of 100 will be calculated for δ_0 between ~0.25 and 0.32. There is also, it should be recognized, about one chance in seven that the likelihood ratio for $\delta_0 = 0.2$ will be less than five.

After ten additional years with $\delta_0^* = 0.2$ (Fig. 10) there is only about one chance in six that the likelihood ratio for $\delta_0 = 0.2$ will be *less* than 20, and

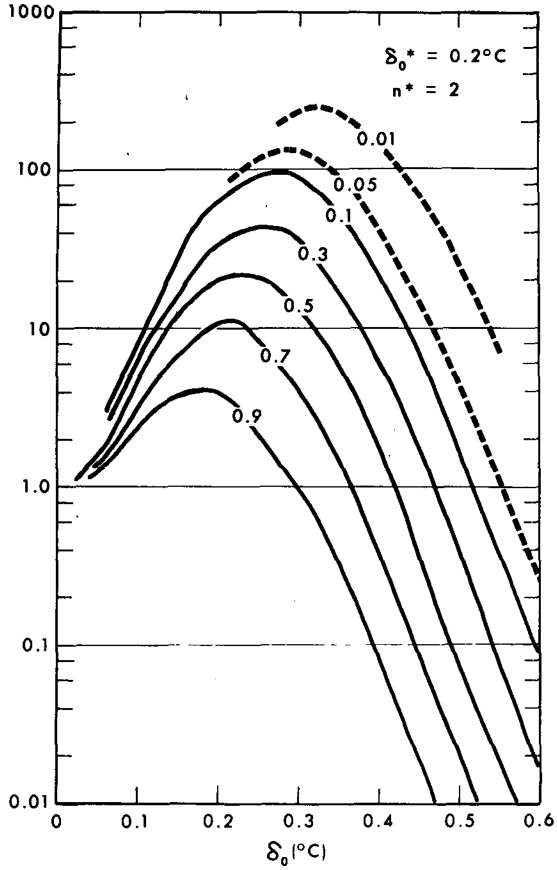

FIG. 9. Probabilities of determining likelihood ratios for Case 1 after two additional years if $\delta_0^* = 0.2°C$.

about a 45% probability that it will be >100. Note also that after 10 years, although the hypothesis that $\delta_0 = 0.2$ will likely be (correctly) accepted, the incorrect hypothesis that $\delta_0 = 0.4$ will probably be rejected.

One of the attributes of Case 1 was that there was some evidence, through 1980, of a real climate change, but the artificiality of the model, and its blatant reflection of what was seen in the data, clearly limit the credibility of the hypothesis. Even after 10 years, it seems, the credibility of these hypotheses may remain in some doubt.

The situation with Case 2 is different in that the model is not so artificial, and there is no positive evidence, according to this model, that climate change is occurring. On the basis of the Monte Carlo experiments, if one assumes $\alpha^* = 0.0$, i.e., no "true" climate change, there is almost no chance of being misled to thinking that change is occurring. No likelihood ratios as large as 10.0 were encountered, for any value of α, at 1, 2, 5 or 10 years, among any of the 1000 Monte Carlo examples. Thus, if a future calculation based on the global mean surface temperatures yields a likelihood ratio as large as 20, it would be very convincing evidence that $\alpha^* = 0$ should be rejected.

On the other hand, even if $\alpha^* \neq 0$, it will require a number of years for sufficient evidence to accumulate to provide the evidence for climate change. If $\alpha^* = 0.2$, then five years from now the maximum probability for $\lambda > 10$ is still only 0.05 (corresponding to the hypothesis that $\alpha = 0.2$). After 10 years, however (Fig. 11), the median likelihood for the hypothesis $\alpha = 0.2$ will exceed 10, and there is more than one chance in 10 that likelihood ratios > 100 will be encountered.

The climate change model described by Case 3 implies more rapid future increases for the same small cumulative change to date. Thus the evidence, pro or con, can accumulate more quickly. If there is indeed no change, i.e., if $\beta^* = 0$, then even though the likelihood ratios including the data through 1980 (Fig. 5) are near 10, significantly large likelihood ratios are unlikely to occur. After 1982 (i.e., after two additional years), there is about 0.03 probability of encountering likelihood ratios as large as 20. This decreases to ~0.02 by 1985 and to <0.01 by 1990. The median likelihood ratios are less than 10 for all values of β by 1985, assuming $\beta^* = 0$.

On the other hand, if $\beta^* = 0.6$ (compare Fig. 2) there is more than one chance in 10 that next year—

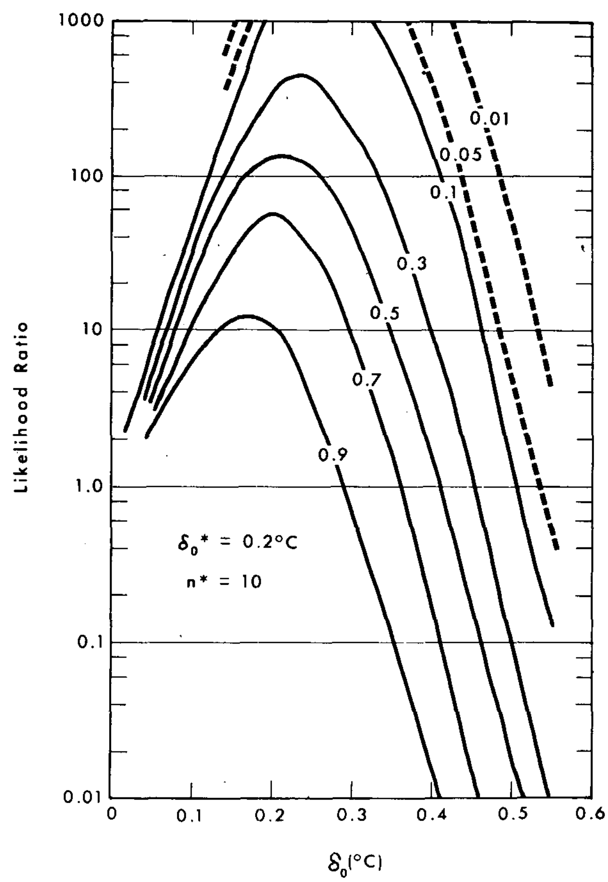

FIG. 10. As in Fig. 9, except after including ten years of additional data.

with only one more year's data—the likelihood ratios corresponding to β's in the interval (0.8–1.1) will exceed 20. After two years, likelihood ratios > 20 will have a 0.3 probability. By the time five years of data accumulate (Fig. 12) the median values of the likelihood ratio will be nearing 100 for β near 0.6 or 0.7. After 10 years, with $\beta^* = 0.6$, it will be unlikely (probability < 0.5) for the likelihood ratios to be <100 for $\beta = 0.6$.

Even for $\beta^* = 0.4$ (half the value shown in Fig. 2) there is a strong possibility that five years' additional data will lead to the correct conclusion that a change is occurring. There is more than one chance in three that $\lambda > 20$ will be calculated (for values of β near 0.5), and the probability that $\lambda > 100$ for β near 0.7 is ~0.14.

5. Stratospheric and tropospheric temperatures

The extension of the analysis to treat simultaneously more than one climatic variable is reasonably straightforward. Eq. (1) is replaced by an identical equation in which each of the three terms (the "natural" climatic mean, the climate change and the ran-

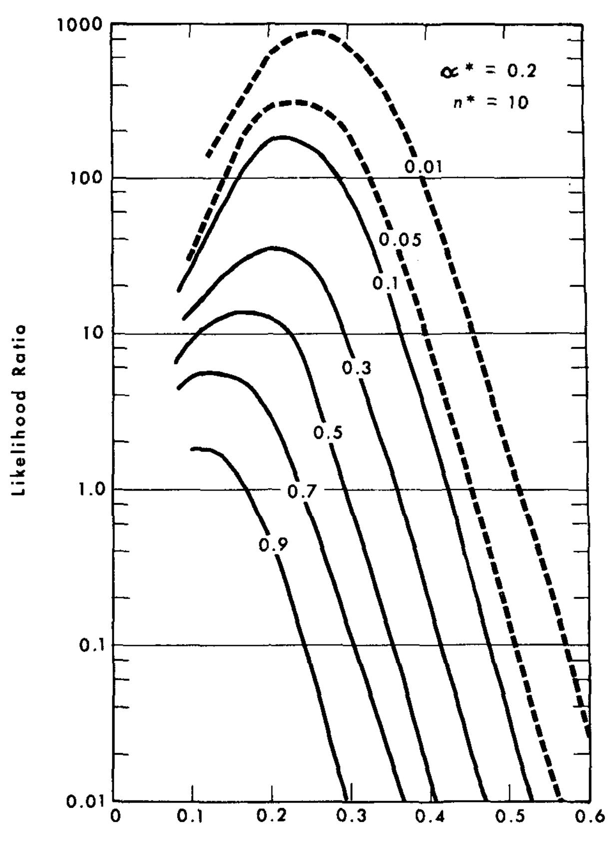

FIG. 11. Probabilities of determining likelihood ratios for Case 2 after ten years of additional data if $\alpha^* = 0.2$ represents the "true" climate change.

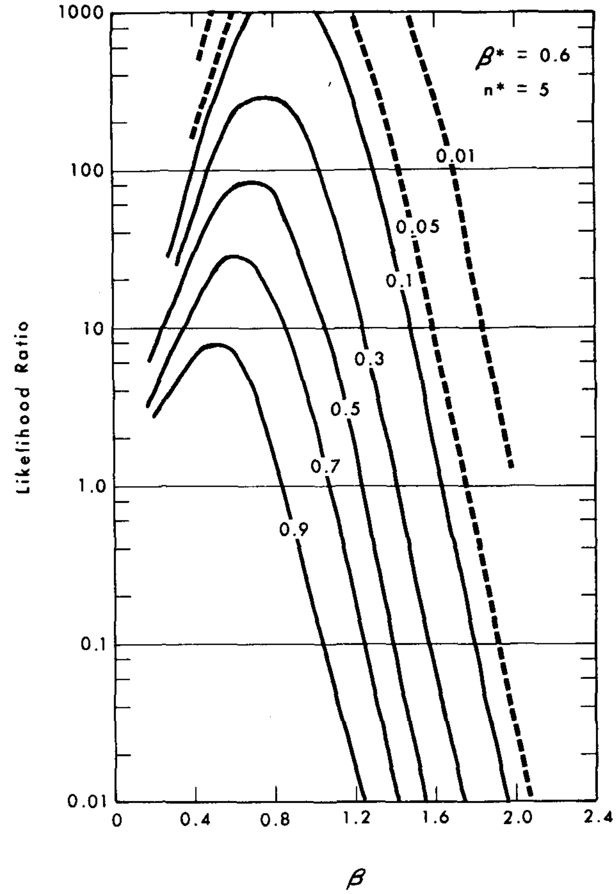

FIG. 12. Probabilities of determining likelihood ratios for Case 3 after five years of additional data if $\beta^* = 0.6$ represents the "true" climate change.

dom element) is replaced by a vector. The random vector ϵ is assumed to be uncorrelated with itself in time, but the components of the random vector may be correlated with one another. In the case we will examine here, the coincident variation of stratospheric and tropospheric global mean temperatures, this means the year-to-year departures may be correlated with one another.

It is of particular interest to examine the coincident behavior of the stratospheric and tropospheric mean temperatures, because the theory of the effect of increasing CO_2 on climate leads one to expect both a cooler stratosphere and a warmer troposphere. The greater information contained in the two time series may make it easier to detect a climate change against the background of climatic noise. Also the attribution of any observed change to CO_2 would be facilitated if the multivariate change were in the direction predicted by theory.

In modeling the bivariate climate change vector δ_i, we have assumed that the same model would apply to both the stratospheric and the tropospheric components (δ_{si} and δ_{ti}). This means that δ_{si} is proportional to δ_{ti}. In fact, physically, this need not be the case. For example, if temperature changes at the surface are being suppressed by the thermal inertia of the oceans, this would tend to retard the warming of the lower atmosphere (Cess and Goldenberg, 1981; Schneider and Thompson, 1981), but the stratospheric cooling, which is primarily a radiative process involving only increased mixing ratio of stratospheric CO_2, would not be retarded. This would suggest using a Case 3 model for the troposphere and a Case 2 model for the stratosphere. There has not been any attempt to pursue such regionally specific models.

If we assume that the two components ϵ_{si} and ϵ_{ti} have a bivariate normal distribution, then the likelihood ratio (relative to the null hypothesis $\delta_{si} = \delta_{ti} = 0$) is given by

$$\lambda = \left[\frac{\mathrm{Var}(T_s - \delta_s)\,\mathrm{Var}(T_t - \delta_t) - \mathrm{Cov}(T_t - \delta_t, T_s - \delta_s)^2}{\mathrm{Var}(T_s)\,\mathrm{Var}(T_t) - \mathrm{Cov}(T_t, T_s)^2}\right]^{-n/2},$$

where $\mathrm{Cov}(A, B) = \overline{AB} - \bar{A}\bar{B}$ and $\mathrm{Var}(A) = \overline{A^2} - \bar{A}^2$. The values of λ are plotted against values of (α_s, α_t) for Case 2 in Fig. 13 and against values of (β_s, β_t) for Case 3 in Fig. 14.

Maximum likelihood ratios are >16 for Case 2 and slightly more than 11 for Case 3. From these graphs one discerns that the data are indicative of recent stratospheric cooling, but less supportive of the suggestion that this is being accompanied by tropospheric warming. In either case, there is not sufficient evidence to support a firm conclusion that climate change has been occurring.

One of the statistics calculated in the course of estimating the likelihood ratios is the coefficient of correlation between the stratospheric and tropospheric temperatures. For the 23 pairs of data a value of 0.22 was found. The sample is small and it is therefore not clear whether a real correlation does or does not exist between ϵ_s and ϵ_t. For the calculations of future likelihood ratio some assumption must be made about the "true" value of this coefficient, ρ. In obtaining the results described below,

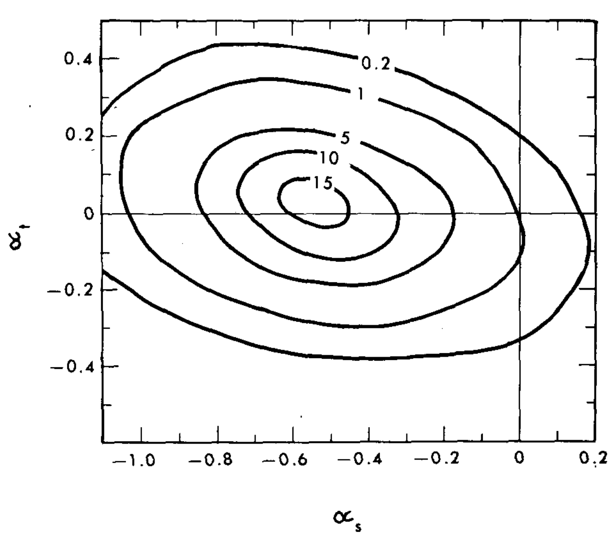

FIG. 13. Likelihood ratios for the data of Figs. 1b and 1c, global mean tropospheric and stratospheric temperatures, as functions of the Case 2 parameters of their joint behavior.

it was assumed that $\rho = 0$. Some supplemental calculations were made using $\rho = 0.3$. The differences were not very noticeable.

Because of the added dimension, it is difficult to present the results graphically. For each pair of postulated values of the "true" coefficients representing climate change (α_s^*, α_t^*) or (β_s^*, β_t^*), we are estimating probability distributions of λ corresponding to hypothetical coefficient value pairs. Rather than trying to present the full distributions, as could be done in the one-dimensional application, we will instead simply present the probabilities that some critical value of λ would be exceeded, or discuss the results in terms of median values of λ.

First let us consider the results of future calculations of λ if there has been and continues to be no "true" climate change, i.e., if $\alpha_t^* = \alpha_s^* = 0$ or $\beta_t^* = \beta_s^* = 0$. For either case, in only a very few years, it will be unlikely to encounter likelihood ratios large enough to suggest wrongly that a real change is occurring. Statistical anomalies, of course, may occur.

In only two years, basing the analysis on the Case 2 model, there is slightly more than one chance in five that a likelihood ratio will be found >10, and this would correspond to the hypothesis of $\alpha_s \approx -0.5$, $\alpha_t \approx 0$. The probability of a likelihood ratio > 20 after five years with $\alpha_s^* = \alpha_t^* = 0$ is <0.1.

Put in different terms, the maximum median likelihood ratio, after five years with $\alpha_s^* = \alpha_t^* = 0$, is only 2.0 (corresponding to $\alpha_s = -0.2$, $\alpha_t = 0$). For a hypothesis that $\alpha_s = -0.5$ and $\alpha_t = +0.3$, the median likelihood is 0.009 and the probability that $\lambda > 0.1$ is only 15%.

The results for Case 3, with $\beta_s^* = \beta_t^* = 0$, are similar. After two years the probability of encountering $\lambda > 10$ is <0.10, and after five years that probability drops to 0.05.

The results to be expected if there is significant climate change occurring are very different. Figs. 15 and 16 are results for Cases 2 and 3, each after five years of continuing modest stratospheric cooling and tropospheric warming. These graphs contain isolines of probabilities of calculating $\lambda > 100$.

In both cases it becomes more likely than not, after five years, that large likelihood ratios will be encountered. In Case 2 (Fig. 15) the median likelihood ratios exceed 100 for hypotheses in the vicinity of $\alpha_s = -0.5$, $\alpha_t = +0.2$. In Case 3, for the parameters chosen, the probability exceeds 0.8 for a correct hypothesis (i.e., when $\beta_s = \beta_s^* = -1.5$ and $\beta_t = \beta_t^* = 0.5$). Under these conditions, the median likelihood ratio will be ~800; the probability of $\lambda > 20$ is ~0.7.

6. Discussion and conclusions

From a physically naive statistical perspective, appropriate likelihood ratios are reasonable measures for evaluating hypotheses with regard to climate change. There is not yet convincing evidence that climate change has been occurring in recent years, but there is good reason to expect to be able to confirm some climate change hypotheses, in a small number of years, if they are indeed true.

One of the potential pitfalls of statistical approaches is the testing of multiple hypotheses. If enough different hypotheses are examined, then, by chance, it is likely that statistics supporting one of them will be found. We have introduced three models of climate change, and in so doing have increased the chance that we would have found evidence to substantiate one of them. If we had wanted to find one that the data would "confirm," certainly we could have done so.

To avoid this pitfall, hypotheses to be tested in the future should be stated now. We have already stated three hypotheses that may be examined in the future. A fourth, that may be worth pursuing, is to model stratospheric temperatures as Case 2 and tropospheric temperatures as Case 3. But, if investigators want their future claims to be credible, they should state their hypotheses now.

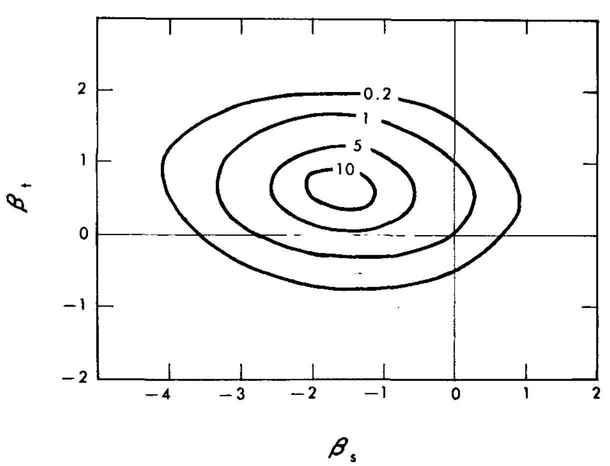

FIG. 14. Likelihood ratios for the tropospheric and stratospheric temperatures as functions of the Case 3 parameters of their joint behavior.

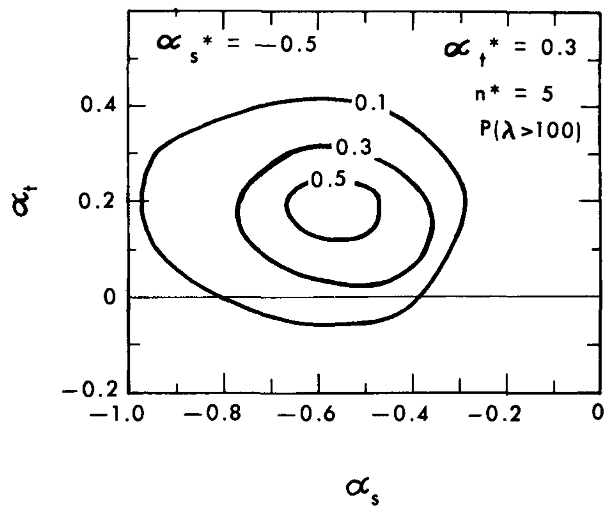

FIG. 15. Probabilities that the likelihood ratios will exceed 100 after five years of additional data if the "true" climate change is represented by Case 2 with $\alpha_t^* = 0.3$ and $\alpha_s^* = -0.5$.

In examining the results of future calculations, we have dealt with cumulative likelihood ratios. This represents a Bayesian viewpoint that all the relevant data, past and present, should be considered. A more classical approach would be to examine only the likelihood ratio of future data. This is equivalent to dividing the future likelihood ratios as calculated above by the values of λ as shown in Figs. 3, 4, 5, 13 and 14. The general nature of the conclusions will not be very different.

If one examines only the global mean surface data of Angell and Korshover (1977), then it will probably require at least ten years before evidence of a continuing modest climate change will be convincing. There is a possibility that five years will be sufficient but that possibility is small, unless the rate of climate change is accelerating quite rapidly.

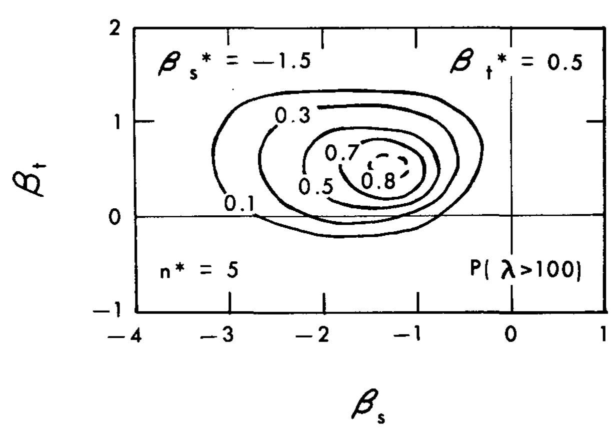

FIG. 16. As in Fig. 15, but for Case 3 with $\beta_t^* = 0.5$ and $\beta_s^* = -1.5$.

The joint behavior of the tropospheric and stratospheric temperatures is more likely to offer evidence that discriminates between change and no change. There is good reason to expect convincing statistical evidence by 1986, assuming that no extraneous events of major climatic impact occur. Furthermore, the evidence provided by the joint behavior of the stratosphere and troposphere will make it easier to argue whether or not any change should be attributed to CO_2.

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