NOTES

Effect of a Change of Atmospheric Stability on the Growth Rate of Puffs Used in Plume Simulation Models

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ABSTRACT

The dependence of plume growth rate on plume size is discussed. It is shown that the growth rates estimated by techniques used in some widely distributed puff models can be in error by as much as two orders of magnitude, although the errors are more commonly on the order of 10%. The concept of virtual travel distance is shown to provide a more realistic representation of the physical processes involved in plume growth after a change in atmospheric stability.

1. Introduction

The behavior of plumes under space- and time-varying meteorological conditions is often approximated by a series of "puffs" or plume segments. At least two different approaches have been suggested for treating the changes in puff dimensions during periods of changing atmospheric stability. Most frequently, the "dimensions" are represented by the standard deviations of a Gaussian distribution, i.e., σ_y and σ_z. Frequently (e.g., Hales *et al.*, 1977), these are approximated by functions of the form:

$$\sigma = a_j x^{\gamma_j}, \tag{1}$$

where a_j and γ_j are constants depending on atmospheric stability class j, and x is downwind travel distance.

The question addressed by this note is: How does one use a function of this form when stability class j (and the corresponding constants a_j and γ_j) changes? Obviously, one cannot simply change the values of the constants while retaining the current travel distance x—this would cause a discontinuous change in σ. I will return to the answers that have been suggested for the above question after I have briefly discussed the nature of puff (plume) growth.

2. The qualitative nature of puff growth

The growth of puffs and plumes depends on the relationship between the size of the plume element and the turbulent spectrum. The expression given in (1) is simply a convenience; the growth rate does not truly depend on the travel distance (or travel time, for that matter). Although the puff size is an intrinsic property of that puff, travel distance (or time) is not. If one observes a puff, there is no way to know how far or how long that puff has traveled. Any number of stability sequences operating over a wide range of travel distances might have produced a puff of the observed size, especially if the vertical dimension (σ_z) and horizontal dimension (σ_y) are considered separately.

In summary, the growth rate depends on the puff's current size; the distance that the puff has traveled is not a unique measure of those properties that influence growth rate. One must be very careful if one defines puff size or growth rate in terms of travel distance, especially during periods of changing stability.

3. Representation of puff dimensions and growth rates

It was noted earlier that (1) could not be used directly to calculate σ after there had been a change in atmospheric stability. This has been recognized by several investigators (e.g., Start and Wendell, 1974; Ludwig *et al.*, 1977), who have arrived at different solutions. Start and Wendell (1974) and several others following in their footsteps (e.g., Hales *et al.*, 1977 Benkley and Bass, 1979) have simply integrated the derivative of (1):

$$\frac{d\sigma}{dx} = \gamma_j a_j x^{\gamma_j - 1}, \tag{2}$$

$$\sigma = \int_0^x \gamma_j(s) a_j(s) s^{\gamma_j(s)-1} ds. \tag{3}$$

Because the integration uses different values of a_j and γ_j whenever the stability class changes, a_j and γ_j are shown as functions of the intermediate travel s. However, as indicated earlier, the growth rate should be represented as a function of size, rather than travel

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TABLE 1. Constants for calculating σ_z (after Benkley and Bass, 1979).

Stability class	Proportionality a factor (m)	Exponent γ
A	2.3×10^{-4}	2.1
D	0.57	0.58

distance. Making the appropriate substitutions from (1), (2) becomes

$$\frac{d\sigma}{dx} = \gamma_j a_j^{1/\gamma_j} \sigma^{(1-1/\gamma_j)}. \quad (4)$$

Eq. (4) presents a growth rate expression that depends only on stability and σ. Later, two examples are presented to show that there can be considerable difference between the growth rates obtained from (2) and (4).

Before proceeding to the examples, the method used by Ludwig *et al.* (1977) will be discussed because it is essentially equivalent to the integration of (4). They use the "virtual travel distance" concept. Virtual travel x_v is defined as the distance that a puff would have traveled to reach its present size, had the stability remained constant at its present category throughout the travel, i.e.,

TABLE 2. Effects of a change in stability at 3 km from the source on growth rates according to two different estimation methods.

Stability change	$d\sigma_z/dx$ Eq. (2)	$d\sigma_z/dx$ Eq. (4)	σ_z (m)	Virtual travel (m)
Class A to D				
Before	3.2	3.2	4610	3000
After	1.1×10^{-2}	4.9×10^{-4}	4610	5.5×10^{6}
Class D to A				
Before	1.1×10^{-2}	1.1×10^{-2}	59	3000
After	3.2	0.33	59	377

$$x_v = (\sigma/a_j)^{1/\gamma_j}, \quad (5)$$

where a_j, γ_j and σ have their current values.

If we substitute x_v for x in (2), we get an expression identical to (4); thus, this approach is equivalent to using the growth rate described in terms of current

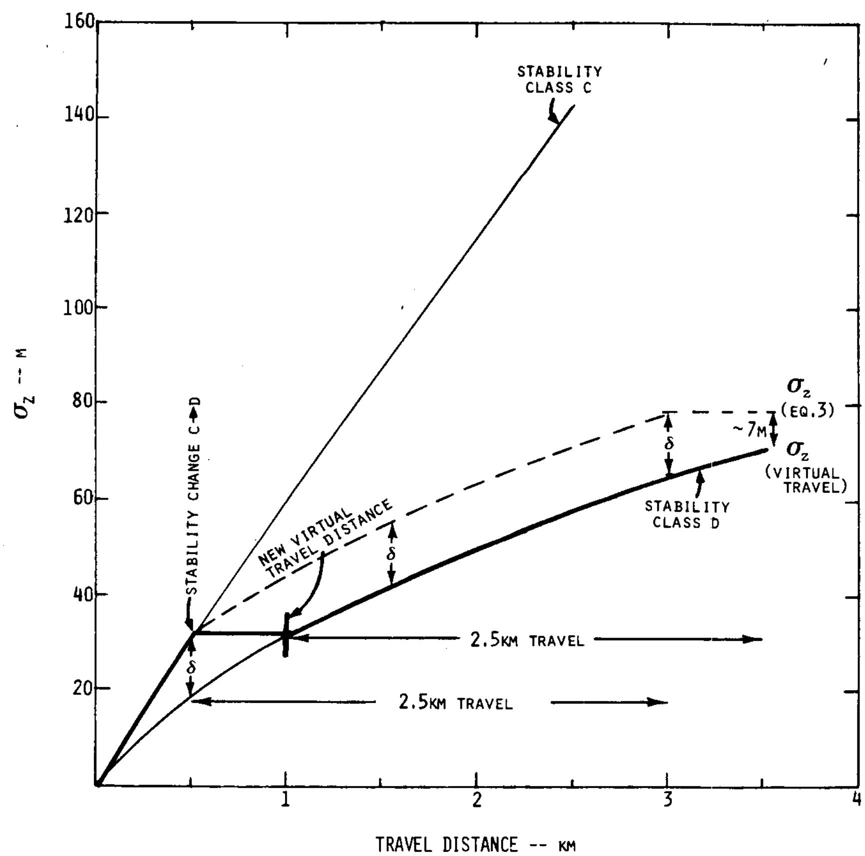

FIG. 1. Schematic diagram showing changes in σ_z, as determined from Eq. (3) and the virtual travel concept.

puff size. The original descriptions of how the virtual travel distance concept is applied (Ruff *et al.*, 1976; Ludwig *et al.*, 1977) should be consulted; it is sufficient to note here that changes in x_v will equal changes in total distance traveled whenever there is no change in stability category. Whenever there is a stability change, the value of x_v must be recalculated before applying the growth rate expression. This is equivalent to a stepwise evaluation of the integral in (3).

The most important reason for using either Eq. (4) or, equivalently, the virtual travel distance concept, is that the growth rate becomes a function of the current dimensions of the diffusing cloud. This is much more physically realistic than a growth rate that depends on distance or time of travel since the material was emitted. If one were trying to simulate cloud growth in a continuously changing turbulent medium, then Eq. (4), directly applied, with the parameters γ and a being continuous functions of the turbulent state of the atmosphere, would be the method of choice. However, when discrete stability classes are used, the concept of virtual travel distance has substantial computational advantages, because virtual travel distance increases directly with actual travel distance, except when the stability class changes.

4. Examples

Two examples have been chosen to illustrate the magnitude of the difference between the growth rates. Both involve a change of stability category at 3 km from the source. In the first example, the change is from A (extremely unstable) to D (neutral); this change, while extreme, is quite possible: It would be associated with high solar elevations, light winds, and a change from clear skies to overcast. The other example given relates to the reverse case, a change in stability from D to A.

Table 1 shows values of a and γ given for σ_z by Benkley and Bass (1979). Table 2 shows the values of virtual travel distance, and $d\sigma_z/dx$ according to (2) and (4), before and after the stability change. According to Table 2, the differences in growth rate are not trivial; they can amount to more than two orders of magnitude. In both the cases shown, (2) overestimates the growth rate. This apparent paradox can be understood if one uses (2) and invokes the concept of virtual travel distance. According to (2), the *growth rate* increases with downwind distance for values of $\gamma > 1$ and decreases with distance if γ is <1. Table 2 shows that the virtual travel distance increases abruptly when stability changes from A to D. Because γ for D stability < 1, this results in a growth rate that is less than that based on the actual travel distance. For the reverse change (from D to A), the virtual travel distance becomes less than the actual travel and the appropriate value of γ for A stability is >1; again, the growth rate based on actual travel overestimates the correct growth rate.

A diagram like that shown in Fig. 1 should help explain the concepts involved. It will also provide a more realistic example. In the case shown, the stability is assumed to change from Class C to Class D after 500 m of travel and then to continue for an additional 2.5 km. To help understand the process, the σ_z curves have been plotted linearly against distance, rather than the usual log–log plot. The heavy solid line in the figure shows the transition from one curve to another represented by the virtual travel distance approach. The dashed line illustrates the growth dictated by Eq. (3). At the end of 3 km travel, the approach using Eq. (3) would yield a value for σ_z that is $\sim$7 m ($\sim$10%) larger than that obtained using the more realistic virtual travel distance.

For a change from more to less stable conditions, the virtual travel distance for both σ_y and σ_z will always decrease. In the case of σ_y, where γ is 0.9 for all stability classes (see, e.g., Benkley and Bass, 1979), this means that (2) will underestimate growth for changes from more stable to less stable and overestimate for opposite changes (as illustrated in Fig. 1). Of course, the differences are not as pronounced as those given in Table 2. For instance, the corresponding (2) and (4) growth rates for σ_y after transition from A to D stability at 3 km are 5.2×10^{-2} and 4.2×10^{-2}, respectively.

Qualitatively, σ_z estimates obtained from the two equations bear the same relationship as those for σ_y for all transitions involving stability categories from C through F, where values of γ are <1. For transitions involving categories A and B, the relationships for σ_z are qualitatively the reverse of those described above for σ_y. That is, for a transition to less stable conditions (e.g., D to A), virtual travel distance decreases and (2) overestimates growth rate and vice versa.

5. Conclusions

The impetus for writing this note came from the discovery that a puff model (Benkley and Bass, 1979) now being used for regulatory decisions (e.g., Schock, 1981) uses (2) to calculate σ_y and σ_z. It is apparent from the preceding discussion that this formulation can lead to significant errors in calculated concentration. The errors in growth rate can be an order of magnitude. The example given in Table 1 is admittedly chosen to represent an extreme (but possible) case. A value of σ_z of more than 4 km is very unlikely, but the change from Class D to Class A involves a more reasonable σ_z of 59 m and still produces an order of magnitude difference in growth rate between the two methods. In general, the errors will be on the order of 10% or less. The potential for important errors is great enough to warrant correction before the applications of models using (2) become more widespread. The use of virtual travel distance does not require more complicated calculations.

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