A New Covariate Ratio Procedure for Estimating Treatment Differences with Applications to Climax I and II Experiments

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ABSTRACT

A new covariate ratio procedure is presented for estimating a treatment-induced effect. The procedure 1) allows for uncontrolled natural variability, 2) adjusts for disproportionate allocation of non-treated and treated experimental units, 3) diminishes the influence in an objective manner of an individual value corresponding to any experimental unit, and 4) accounts for differential treatment effects, i.e., a simple location or scale change is not assumed. This procedure is applied to specific meteorologically defined partitions involving data of the Climax I and II experiments. Results based on the pooled data indicate a 32% precipitation increase for the −20 to −11°C 500 mb temperature partition, a 49% increase for the 190 to 250° 700 mb wind direction partition, and a 13% increase for the total sample. Comparisons based on Monte Carlo simulations (re-randomization) indicate that this procedure yields estimates which are more stable (precise) than corresponding estimates based on the double ratio.

1. Introduction

The purposes of this paper are 1) to describe a ratio procedure for obtaining reasonable estimates of treatment-induced differences and distribution changes which account for control data in a physically more meaningful manner than existing procedures (e.g., double ratios) and 2) to apply this new procedure to data of the Climax I and II experiments (this study was stimulated by the fact that all previous estimates of seeding effects for these experiments seemed inappropriate). A physically meaningful estimate of a treatment-induced effect should 1) allow for the effects of uncontrolled natural variability, 2) compensate for disproportionate non-treatment and treatment allocations to experimental unit classes having similar characteristics, and 3) moderate for the effect of any specific value in an objective manner (i.e., no individual value should have an excessive influence on an estimate). Because a treatment effect on a natural distribution of measurements is often a complex differential effect (e.g., the effect of cloud seeding on precipitation is highly dependent on existing meteorological conditions), the common assumption of representing such an effect by either a simple location or scale change is rarely if ever satisfied. The simple and double ratio estimates of treatment-induced differences used in studies such as the Climax I and II experiments (Mielke *et al.*, 1971, 1981) address at most one of these concerns. The covariate estimation procedure introduced and applied herein addresses each of these concerns. Section 2 contains a technical description of the procedure. The applications of the procedure in Section 3 are confined to the same Climax I and II data partitions considered by Mielke *et al.* (1982).

2. Estimation procedure

A measurement of interest (such as precipitation) from a selected control is employed to assist in accounting for natural variability in some target area measurement of primary interest. If control values are partitioned into a number of disjoint subsets (here intervals based on magnitude), then the corresponding non-treated target values will be distributed according to some natural pattern for each control interval (see Table 1). If a treatment effect occurs, then the distribution of the treated cases should differ from that of the non-treated cases. The distributionally based estimates of this paper are based on partitioned data involving computed proportions of counts within control and target intervals. The proportions are estimated separately from the observed relative frequencies of the non-treated and treated cases within each subset. A treatment effect may then be estimated by using category medians of target measurements where each category is well defined by a specified combination of control and target intervals. Each category's median is based on all target measurements associated with the pooled collection of non-treated

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TABLE 1. Target-control matrix for r target intervals and c control intervals with individual non-treated and treated cell counts and proportions.

	Control				
	1st interval		· · ·	cth interval	
Target	Target non-treated	Target treated	· · ·	Target non-treated	Target treated
1st interval	n_{11} counts $p_{1N1} = n_{11}/n_1$	t_{11} counts $p_{1T1} = t_{11}/t_1$	· · ·	n_{1c} counts $p_{1Nc} = n_{1c}/n_c$	t_{1c} counts $p_{1Tc} = t_{1c}/t_c$
⋮	⋮	⋮		⋮	⋮
rth interval	n_{r1} counts $p_{rN1} = n_{r1}/n_1$	t_{r1} counts $p_{rT1} = t_{r1}/t_1$	· · ·	n_{rc} counts $p_{rNc} = n_{rc}/n_c$	t_{rc} counts $p_{rTc} = t_{rc}/t_c$
	$n_1 = \sum_{i=1}^{r} n_{i1}$	$t_1 = \sum_{i=1}^{r} t_{i1}$	· · ·	$n_c = \sum_{i=1}^{r} n_{ic}$	$t_c = \sum_{i=1}^{r} t_{ic}$

and treated cases in the category. These estimates may then be used to obtain a target treatment effect for each control interval and ultimately an overall treatment effect.

More precisely, let x_k and y_k $(k = 1, \ldots, W)$ respectively denote the observed covariate value (not affected by treatment) and the observed response value (in the target) associated with the kth of W experimental units being investigated. Let n and t respectively designate the number of non-treated and treated experimental units $(n + t = W)$. The W experimental units are exhaustively partitioned into c disjoint subsets by subdividing the x-axis into c finite intervals according to the values of x. The intervals are chosen so that a reasonably large proportion of experimental units is associated with each of the c subsets. Let n_j and t_j respectively designate the number of non-treated and treated experimental units in the jth subset of control values (Table 1). Then $\pi_j = (n_j + t_j)/W$ is the proportion of experimental units in the jth subset and $\sum_{j=1}^{c} \pi_j = 1$. In each of the c subsets for the controls, the non-treated and treated experimental units are placed into r disjoint subsets defined according to values of y (target values). The number of these target subsets, and their respective intervals, will depend on the target (y) data; they may be the same as those for the controls, but need not be. Let p_{iNj} and p_{iTj} respectively denote the proportion of the non-treated and treated experimental units associated with the ith of the r intervals (defined by values of y) for the jth of the c subsets. Thus $\sum_{i=1}^{r} p_{iNj} = \sum_{i=1}^{r} p_{iTj} = 1$. Also let M_{ij} denote the median value of the pooled non-treated and treated values of y in the ith of the r intervals of the jth subset. In particular, M_{ij} is (i) the central value, or (ii) the average of the two central values of y if the number of pooled non-seeded and seeded values of y in the ith interval of the jth subset is even, or (iii) an unknown finite value if $p_{iNj} = p_{iTj} = 0$.

A comparison of the non-treated and treated distributions of y for a given meteorologically defined partition is now described. Let

$$p_{iN} = \sum_{j=1}^{c} \pi_j p_{iNj} \quad \text{and} \quad p_{iT} = \sum_{j=1}^{c} \pi_j p_{iTj}$$

$$(i = 1, \ldots, r).$$

Note that $\sum_{i=1}^{r} p_{iN} = \sum_{i=1}^{r} p_{iT} = 1$. A comparison of $(p_{1N}, \ldots, p_{rN})$ and $(p_{1T}, \ldots, p_{rT})$ provides a description of the differential effects associated with the treatment.

Let

$$U_{jN} = \sum_{i=1}^{r} M_{ij} p_{iNj} \quad \text{and} \quad U_{jT} = \sum_{i=1}^{r} M_{ij} p_{iTj}$$

TABLE 2. Interval associated summary statistics for the −20 to −11°C 500 mb temperature partition based on subset structure A.

Interval	j or i	n_j	t_j	π_j	U_{jN}	U_{jT}	U_{jT}/U_{jN}	P_{iN}	P_{iT}
[0]	1	29	39	0.313	0.000	0.001	∞	0.491	0.435
(0.0, 0.05)	2	42	22	0.295	0.032	0.048	1.492	0.140	0.121
[0.05, 0.1)	3	15	17	0.147	0.092	0.117	1.276	0.081	0.111
[0.1, 0.2)	4	14	15	0.134	0.162	0.241	1.492	0.150	0.105
[0.2, 0.9)	5	9	15	0.111	0.313	0.370	1.182	0.139	0.227

TABLE 3. Interval associated summary statistics for the −39 to −27°C 500 mb temperature partition based on subset structure A.

Interval	j or i	n_j	t_j	π_j	U_{jN}	U_{jT}	U_{jT}/U_{jN}	p_{iN}	p_{iT}
[0]	1	2	6	0.062	0.011	0.025	2.241	0.063	0.095
(0.0, 0.05)	2	26	28	0.419	0.064	0.095	1.477	0.187	0.185
[0.05, 0.1)	3	16	14	0.233	0.176	0.122	0.693	0.213	0.152
[0.1, 0.2)	4	19	11	0.233	0.181	0.197	1.091	1.307	0.301
[0.2, 0.9)	5	5	2	0.054	0.199	0.145	0.728	0.230	0.266

respectively denote the non-treated and treated unit estimates for the jth of the c subsets. Then an estimate of proportionate change (e.g., in the unit amount of precipitation) due to the treatment for the jth subset is U_{jT}/U_{jN} ($j = 1, \ldots, c$). If $U_N = \sum_{j=1}^{c} \pi_j U_{jN}$ and $U_T = \sum_{j=1}^{c} \pi_j U_{jT}$, then U_T/U_N is an estimate of proportionate change due to treatment for the W experimental units.

The procedure compensates for differences in the controlled natural variability as well as the disproportionate random allocation of non-treated and treated experimental units. In addition, an excessive influence by any particular value is reduced. Also this ratio procedure accounts for the possibility of complex differential effects, in contrast to the assumption of either a simple location or scale change.

3. Applications

The applications of the ratio estimation procedure are based on the observed covariate (control) and the observed response (target) values of the Climax I and II experiments used by Mielke *et al.* (1982). The treatment in question is the seeding of clouds with silver iodide. A complete description of the data is given by Mielke *et al.* (1981). Unless indicated otherwise, applications are based on the pooled data of the Climax I and II experiments. The item of interest for both the control and target values in the present and previous papers (Mielke *et al.*, 1971, 1981, 1982) is precipitation in inches of water. Most of the illustrations depend on the same five disjoint subsets ($c = 5$) of the observed control values. The five intervals associated with these five subsets are [0], (0.0, 0.05), [0.05, 0.1), [0.1, 0.2) and [0.2, 0.9) where) or] respectively denotes an open or a closed end point. The subset structure of the target data is chosen to correspond with the subset structure of the control data in these applications (thus $r = c$). Tables 2–5 present interval associated summary statistics described in Section 2 for a selection of the meteorological partitions considered by Mielke *et al.* (1982). These include the −20 to −11°C 500 mb temperatures (Table 2), the −39 to −27°C 500 mb temperatures (Table 3), the 190 to 250° 700 mb wind directions (Table 4), and the total sample (Table 5). The summary statistics of Tables 2–5 include the interval identification (j or i), n_j, t_j, π_j, U_{jN}, U_{jT}, U_{jT}/U_{jN}, p_{iN} and p_{iT}.

The overall estimate of proportionate change attributed to a treatment (U_T/U_N) depends on the subset structure, which involves both the number of subsets and the values of x (control) which specify the intervals. To indicate this variability for different subset structures, values of U_T/U_N are presented in Table 6 for the above selected subset structure (A) and six

TABLE 4. Interval associated summary statistics for the 190 to 250 deg 700 mb wind direction partition based on subset structure A.

Interval	j or i	n_j	t_j	π_j	U_{jN}	U_{jT}	U_{jT}/U_{jN}	p_{iN}	p_{iT}
[0]	1	13	13	0.173	0.000	0.008	∞	0.367	0.301
(0.0, 0.05)	2	23	16	0.260	0.031	0.018	0.605	0.219	0.156
[0.05, 0.1)	3	12	15	0.180	0.022	0.113	5.198	0.168	0.119
[0.1, 0.2)	4	15	17	0.213	0.087	0.132	1.522	0.117	0.174
[0.2, 0.9)	5	10	16	0.173	0.248	0.315	1.271	0.130	0.250

TABLE 5. Interval associated summary statistics for the total sample based on subset structure A.

Interval	j or i	n_j	t_j	π_j	U_{jN}	U_{jT}	U_{jT}/U_{jN}	p_{iN}	p_{iT}
[0]	1	46	58	0.168	0.003	0.005	2.096	0.275	0.243
(0.0, 0.05)	2	120	96	0.349	0.053	0.066	1.243	0.153	0.175
[0.05, 0.1)	3	58	53	0.179	0.126	0.125	0.996	0.160	0.128
[0.1, 0.2)	4	58	56	0.184	0.185	0.222	1.196	0.216	0.194
[0.2, 0.9)	5	37	37	0.120	0.280	0.301	1.077	0.196	0.260

TABLE 6. Overall estimates of proportionate change in unit amount (U_T/U_N) for each combination of meteorologically defined partition and subset structure (A, B, C, D, E, F, G).

Partition	A	B	C	D	E	F	G
500 mb temperature (−20 to −11°C)	1.325	1.306	1.289	1.304	1.382	1.292	1.370
500 mb temperature (−26 to −21°C)	1.097	1.022	1.010	1.018	1.033	1.104	1.017
500 mb temperature (−39 to −27°C)	1.016	0.946	0.932	0.952	0.932	1.027	0.908
700 mb equivalent potential temperature (308 to 325K)	1.202	1.178	1.256	1.196	1.168	1.197	1.164
700 mb wind direction (190 to 250°)	1.491	1.436	1.386	1.351	1.440	1.518	1.653
700 mb wind velocity (12 to 14 m s^{-1})	1.311	1.297	1.132	1.207	1.254	1.495	1.282
Total sample	1.130	1.100	1.066	1.094	1.134	1.158	1.138

(Columns A–G are under the spanning header "Subset structure".)

others (B, C, D, E, F, G) for each of the seven meteorologically defined partitions considered by Mielke *et al.* (1982). Interval end points for all subset structures are given in Fig. 1. Individual end point inclusion or exclusion in Fig. 1 is defined in a manner completely analogous to subset structure A (i.e., [0] is always closed, the next interval is open on both end points, and all other intervals are closed on the left but open on the right). While the estimated proportionate change does vary for the different subset structures (A, B, C, D, E, F, G), none of the partitions which indicated a treatment effect in the earlier study by Mielke *et al.* (1982) yielded appreciably differing estimates (i.e., estimated proportions for different subset structures which include values above and below unity). However, no difference between nontreated and treated experimental units was indicated by Mielke *et al.* (1982) for either of the two cold 500 mb temperature partitions.

The overall estimates of proportional change (U_T/U_N) based on the selected subset structure (A) along with corresponding 90 and 95% re-randomization interval estimates are presented in Table 7. As previously emphasized in Mielke *et al.* (1981), these re-

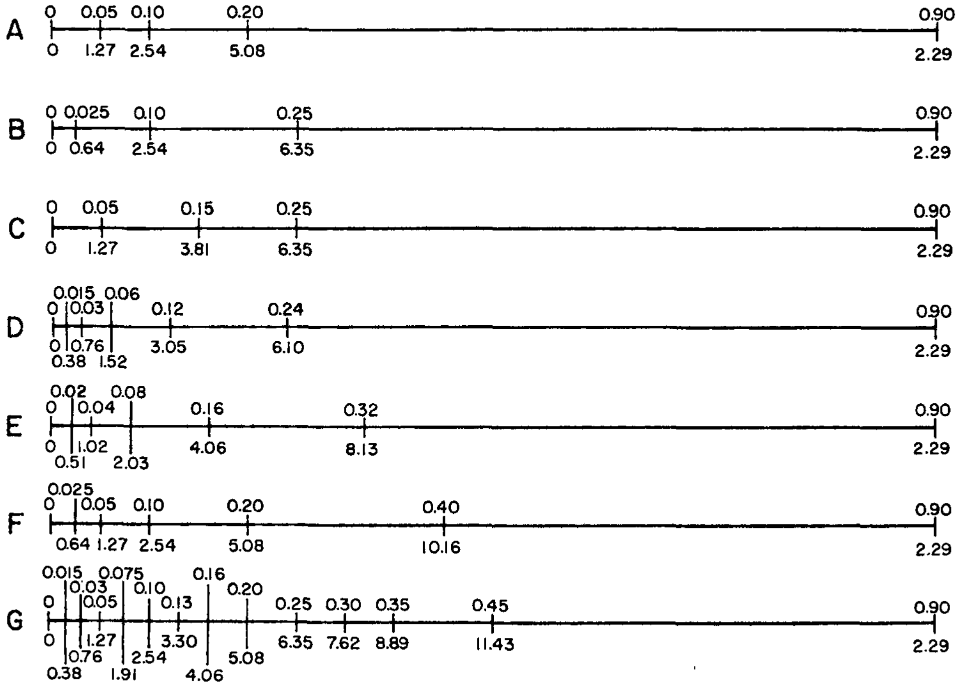

FIG. 1. Interval definitions for subset structures A, B, C, D, E, F and G in inches (above) and millimeters (below).

TABLE 7. Overall estimate of proportionate change in unit amount (U_T/U_N) and corresponding 90% and 95% re-randomization interval estimates (RIE) based on subset structure A for each meteorologically defined partition.

Partition	U_T/U_N	90% RIE	95% RIE
500 mb temperature (−20 to −11°C)	1.325	(0.809, 1.257)	(0.790, 1.310)
500 mb temperature (−26 to −21°C)	1.097	(0.872, 1.139)	(0.855, 1.159)
500 mb temperature (−39 to −27°C)	1.016	(0.826, 1.198)	(0.804, 1.226)
700 mb equivalent potential temperature (308 to 325 K)	1.202	(0.847, 1.198)	(0.812, 1.231)
700 mb wind direction (190 to 250°)	1.491	(0.821, 1.229)	(0.795, 1.259)
700 mb wind velocity (12 to 14 m s^{-1})	1.311	(0.842, 1.196)	(0.809, 1.249)
Total sample	1.130	(0.896, 1.103)	(0.875, 1.123)

randomization interval estimates cannot be objectively reproduced since they are realizations of a random sampling process (i.e., depend on an additional type I statistical error). Other than being based on 500 rather than 1000 random separations of the pooled seeded and non-seeded experimental units of a specified partition, the procedure of this paper parallels the re-randomization interval estimate procedure described by Mielke *et al.* (1981).

Of interest in Table 7 is the fact that the observed estimates of U_T/U_N exceed the 95% re-randomization interval estimates for the −20 to −11°C 500 mb temperatures, the 190 to 250° 700 mb wind directions, the 12 to 14 m s^{-1} 700 mb wind velocities, and the total sample. While the observed estimate of U_T/U_N for the 308 to 325 K 700 mb equivalent potential temperatures exceeds the 90% re-randomization interval estimate, values for the two colder 500 mb temperature partitions are both contained in their respective 90% re-randomization interval estimates.

The double ratio (DR) is the only previous estimate of a seeding induced precipitation difference which allowed for the effects of uncontrolled natural variability (cf. Mielke *et al.,* 1981). However, a double ratio neither (i) compensates for a disproportionate allocation of non-treated and treated experimental units nor (ii) moderates for excessive influence by any specific value in an objective manner. As a consequence of not compensating for (i), a double ratio is not dependent on any subset structure (i.e., the objectivity of a double ratio is also an inherent deficiency in this instance). Furthermore, the double ratio's dependence on arithmetic means permits a small number of values to have a potentially overwhelming effect on a subsequent estimate.

Separate Climax I and II comparisons between estimates of proportionate change in precipitation (U_T/U_N) based on the selected subset structure (A) and double ratios (DR) along with corresponding 90% re-randomization interval estimates are presented in Table 8 for each meteorologically defined partition. The double ratios and their corresponding 90% re-randomization interval estimates are taken from Mielke *et al.* (1981). An obvious feature of Table 8 is the somewhat better agreement between Climax I and II for the estimates of proportionate change in comparison with the double ratios. The −20 to −11°C 500 mb temperature partition and the 12 to

TABLE 8. Separate Climax I and II comparisons between estimates of proportionate change in unit amount (U_T/U_N) based on subset structure A and double ratios (DR) and their corresponding 90% rerandomization interval estimates (in parentheses below each estimate) for each meteorologically defined partition.

	Climax I		Climax II	
Partition	U_T/U_N	DR	U_T/U_N	DR
500 mb temperature (−20 to −11°C)	1.343 (0.776, 1,342)	1.520 (0.683, 1.475)	1.271 (0.748, 1.342)	1.089 (0.692, 1.398)
500 mb temperature (−26 to −21°C)	1.123 (0.807, 1.204)	0.985 (0.757, 1.302)	1.055 (0.830, 1.236)	0.985 (0.791, 1.261)
500 mb temperature (−39 to −27°C)	0.923 (0.794, 1.237)	1.008 (0.741, 1.379)	1.101 (0.735, 1.308)	1.372 (0.692, 1.433)
700 mb equivalent potential temperature (308 to 325 K)	1.093 (0.781, 1.275)	1.220 (0.721, 1.387)	1.270 (0.780, 1.213)	1.160 (0.786, 1.274)
700 mb wind direction (190 to 250°)	1.352 (0.748, 1.452)	1.552 (0.685, 1.483)	1.521 (0.792, 1.264)	1.504 (0.757, 1.320)
700 mb wind velocity (12 to 14 m s^{-1})	1.624 (0.558, 1.361)	2.266 (0.446, 2.211)	1.233 (0.833, 1,260)	1.022 (0.765, 1.288)
Total sample	1.093 (0.864, 1.153)	1.088 (0.835, 1.215)	1.171 (0.880, 1,147)	1.066 (0.845, 1.173)

14 m s^{-1} 700 mb wind velocity partition emphasize this feature. The Table 8 results cast serious doubt on the use of cloud treatment effect estimates based on double ratios for weather modification experiments. The basis for this doubt is that all but one of the 90% re-randomization interval estimates involving U_T/U_N are totally contained in the corresponding 90% re-randomization interval estimates involving double ratios (the one exception is the 308 to 325 K 700 mb equivalent potential temperature partition for Climax II). Assuming equally likely events, the exact two-sided P-value of observing only one exception among the 28 comparisons of the U_T/U_N and DR re-randomization interval end points (in Table 8) is $58/2^{28} \approx 2.161 \times 10^{-7}$.

The above applications of this new ratio procedure suggest a useful technique when large samples are involved. Its utility with relatively small samples is an open question requiring subsequent attention. The number of cases within each interval needed to attain stable results may vary with the number of intervals, as suggested in Table 6. The use of medians in the described procedure is a simple consequence of their intuitive appeal. Variations of this ratio procedure may also have potential providing they do not lead to undefined or unstable quantities.

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REFERENCES

Mielke, P. W., K. J. Berry and J. G. Medina, 1982: Climax I and II: Distortion resistant residual analyses. *J. Appl. Meteor.*, **21**, 788–792.

——, G. W. Brier, L. O. Grant, G. J. Mulvey and P. N. Rosenzweig, 1981: A statistical reanalysis of the replicated Climax I and II wintertime orographic cloud seeding experiments. *J. Appl. Meteor.*, **20**, 643–659.

——, L. O. Grant and C. F. Chappell, 1971: An independent replication of the Climax wintertime orographic cloud seeding experiment. *J. Appl. Meteor.*, **10**, 1198–1212; Corrigendum: **15**, 801.