NOTES

Calibration of a Pressure Sensor and a Radar Receiver Using Behavioral Modeling

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ABSTRACT

An equation used to describe the calibration of an instrument is either of an arbitrary nature or based upon physics. The latter, which we call behavioral modeling, often provides a much better fit to the calibration than traditional, statistical, curve-fitting techniques. In applying behavioral modeling to a pressure transducer and to a radar receiver, we found that the behavioral models gave smaller residuals than even fairly high-order polynomials. Using fewer terms, they provided insight and information about the physical behavior of the devices in question, which otherwise could not have been obtained from arbitrary models. While behavioral modeling cannot be used for all instruments, the advantages in using them may be beneficial.

1. Introduction

In any measuring system, the relationship between input and output quantities, as well as the effects of disturbing inputs, must be precisely known if accurate measurements are to be made. In the past, instruments were usually calibrated through mechanical and/or electrical adjustments; often the adjustable components were a source of instability. Current techniques favor calibrating instruments mathematically with the input/output relation expressed in a model whose coefficients contain the necessary calibration information.

A mathematical model may be arbitrary, e.g., a common polynomial having sufficient terms to express the input/output relationship. In contrast, a behavioral model is derived from the physics that governs the behavior of the instrument. In this paper we will describe the procedures in the development of behavioral models for a pressure transducer and a radar receiver. However, while these applications are specific to devices we are currently using, the concept of behavioral modeling has wider application and should be considered for the calibration of other instruments as well.

2. Pressure transducer calibration

Consider an aneroid pressure transducer whose output is an electrical capacitance. Such a device, often termed a cell or capsule, might consist of a flexible metal diaphragm covering an evacuated chamber, the opposite side being an electrically conducting rigid plate, (see Fig. 1). The diaphragm and plate, which are electrically insulated from each other, form two plates of a variable capacitor with the capacitance being some function of the pressure to which the transducer is subjected.

Because practical aneroid capsule materials expand and contract with changes in temperature, the spacing and the area of the plates change, thereby giving rise to capacitance changes of thermal origin. The pressure measurement made from the electrical capacitance must be corrected for these thermal effects. Our goal is to develop a model relating the capacitance to the applied pressure and temperature of the transducer.

In developing a behavioral model for this capacitance–pressure transducer, we looked at both basic physics and the relationships determined by the design of the device. Capacitance C is proportional to the area A of the plates and inversely proportional to the spacing S between them: $C = KA/S$. For this model, and a relatively small range of pressure measurements, we assume that the diaphragm deflection is proportional to the applied pressure. From the physics of thermal expansion, we find that the diaphragm supports expand linearly with temperature. To a first approximation, the area of the plates also expands linearly with temperature. Using these relationships as a function of the temperature and the applied pressure, we derive an expression for the capacitance of the cell.

It is desirable to solve this expression for the pressure, since it is the variable to be measured. Often such a solution is an ungainly quotient of polynomials that may be simplified by expanding into a series using the division algorithm (Ledermann, 1980). A simple model may be obtained by deleting those terms containing higher order variables. Similar

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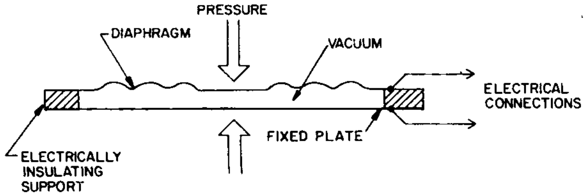

FIG. 1. Conceptual diagram of a capacitance-pressure transducer.

terms are collected and constants are combined into a set of coefficients to be evaluated through calibration and statistical regression techniques.

Using the assumptions stated and following this procedure, we obtained the following behavioral model for a capacitance-pressure cell:

$$P = A_0 + \frac{A_1}{C} + A_2T + \frac{A_3T}{C}, \qquad (1)$$

where C is the capacitance of the transducer, T its temperature, and P is the applied absolute pressure. For (1) A_0, A_1, A_2, and A_3 are determined through least-squares techniques in fitting the model to a set of calibration data. Note that the reciprocal of the capacitance relationship comes directly from the physics of capacitors. The third term expresses a linear temperature relationship, and the fourth term indicates an interdependence among the input variables.

Using a commercially available transducer, we obtained a calibration data set consisting of nine capacitances, measured at three pressures, and repeated at three temperatures. These data are plotted in Fig. 2. This is a brief set of measurements and probably represents a minimum size for the four coefficient model. As shown below, the fit using the behavioral model derived as outlined above is excellent over the limited range of pressure (10 kPa) and temperature (40°C) used for the calibration; it would be quite sufficient for a barometer designed to measure station pressure in a laboratory environment.

For each calibration point, the difference between the calculated pressure using the model and the actual calibration pressure is termed a residual. The measure of how well the model fits the calibration data is given by the variance of the residuals for all calibration points. Note that this must be calculated with the correct number of degrees of freedom (d.f.), in this case, five, which is the difference between the number of calibration points and the number of model coefficients (Draper and Smith, 1966). For this calibration, the variance of the residuals is 23.2 Pa^2.

Using these same calibration data, we fit a range of applicable first and second order arbitrary models (Draper and Smith, 1966) as shown in Table 1. Several points are quite clear. First, the fit is very poor if temperature is ignored as in Cases 1 and 2. The addition of a linear temperature term, Case 4, makes a large improvement, showing that the device has a significant temperature coefficient. Adding a second order capacitance term, Case 6, again makes a substantial improvement, showing that there is a nonlinear pressure/capacitance relationship. Note that the addition of terms does not always improve the statistics, *viz.*, Cases 1, 3, and 5; this is due to the reduction in the number of degrees of freedom for the limited data set of only nine points. Finally, the addition of a cross-product term, Case 7, results in another substantial improvement. However, even a six-coefficient equation, Case 8, does not quite achieve the low variance of the four-coefficient behavioral model, Case 9. The residuals of the three best fits are plotted in Figs. 3–5.

While the arbitrary modeling exercise revealed facts about the behavior, the behavioral modeling dealt with the fundamental causes of the behavior at the outset and directly achieved a more accurate relationship.

As the range of the input variables is increased, the residuals for any simple model may rise to unacceptable levels. This situation requires an increase in the number of terms, as well as an increase in the number of calibration points. During the derivation of the behavioral model, the higher ordered terms in the series expansion will suggest forms for additional terms. Often it may not be possible to determine from the physics which terms are significant, since the physics of the small effects may not be well known. An empirical determination is then called for, in order to avoid addition of terms that are not good predictors. For any model, the variance of the residuals is a measure of the accuracy of the fit, and the procedure is to determine how much the variance is reduced by the addition of a particular term. The variance with or without use of a term is determined, and an F-test is used to evaluate the coefficient as a predictor, (Draper and Smith, 1966).

3. Radar receiver calibration

The Convective Storms Division (CSD) of the National Center for Atmospheric Research (NCAR) has

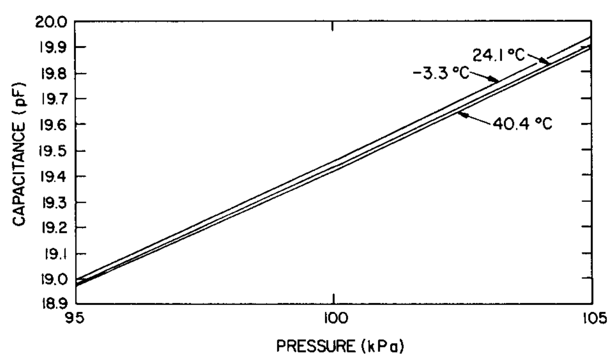

FIG. 2. Capsule transfer function.

TABLE 1. Models tested in order of decreasing variance of the residuals. Cases 4 and 8 are the complete first- and second-order arbitrary models, respectively. Case 9 is the behavioral model. The degrees of freedom for each model are given.

Case	Model	Variance of the residuals (Pa^2)	d.f.
1	$A_0 + A_1C + A_3C^2$	39156	6
2	$A_0 + A_1C$	37870	7
3	$A_0 + A_1C + A_2T + A_4T^2$	5870	5
4	$A_0 + A_1C + A_2T$	4897	6
5	$A_0 + A_1C + A_2T + A_3C^2 + A_4T^2$	153	4
6	$A_0 + A_1C + A_2T + A_3C^2$	129	5
7	$A_0 + A_1C + A_2T + A_3C^2 + A_5CT$	29	4
8	$A_0 + A_1C + A_2T + A_3C^2 + A_4T^2 + A_5CT$	25	3
9	$A_0 + A_1/C + A_2T + A_3T/C$	23	5

used fifth-degree polynomials to fit radar receiver calibration curves for a number of years. This traditional curve-fitting procedure has generally been adequate to describe the shape of calibration curves. Another method used by other groups for radar receiver calibration is a simple piece-wise fit of straight lines between consecutive calibration points. Neither of these methods, however, attempts to describe the actual behavior of the receiver. By considering the basic behavior of a radar receiver in response to an input signal, calibrations might be fit even better than with these other approaches. The application of behavioral modeling to radar receiver calibrations was the motivation that prompted this investigation.

A radar receiver is essentially a device that takes an input power, amplifies it, and sends it on for display, processing, and/or recording. The input signal is composed of both signal and noise from various sources. These noise sources include the sky, the antenna, the detector, the RF amplifier, and any other device preceeding and including the IF amplifier. Other possible sources of noise after the IF amplifier, such as digital processing and averaging, generally add negligible amounts of noise to the resultant signal (Sirmans and Doviak, 1973). The input signal and noise are amplified by the receiver. The receiver itself generates noise, which is added to the amplified signal-plus-noise, and the sum of these is then passed to the next device as the output power.

The weakest signal that a radar can detect is a function of the noise power detected by the radar and the noise generated by the various components of the receiver system. At the other extreme, the strongest signal that a radar receiver can detect is that which just saturates the receiver. Signals stronger than this will result in no increase in output power. Between these limits, receivers are generally well-behaved. The logarithmic receivers used for radar reflectivity measurements by CSD have proven to be quite good at giving an output signal that is linearly proportional to the logarithm of the input power.

Combining the concepts of signal-plus-noise, saturation of strong signals, and the logarithmic output of our receiver system, we arrived at the following equation that models the relationship between the input signal power and the output of our radar signal processor:

$$\log[\underbrace{(G^{(1-P_r/P_s)})}_{A}\underbrace{(P_r + P_n)}_{B}] = \underbrace{A_0 + A_1Q}_{C}, \qquad (2)$$

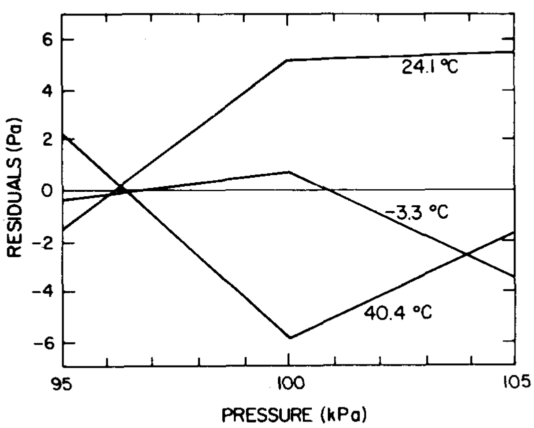

FIG. 3. Plot of residuals from the arbitrary 5-coefficient model.

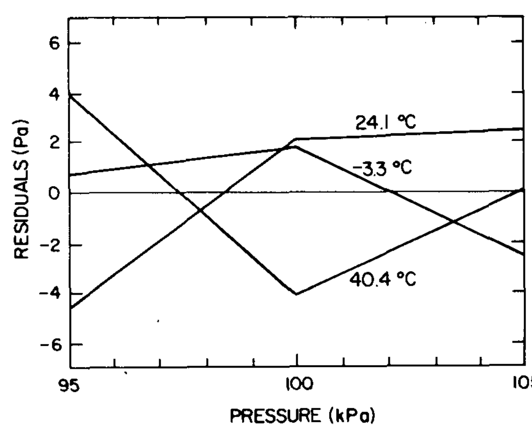

FIG. 4. Plot of residuals from the arbitrary 6-coefficient model.

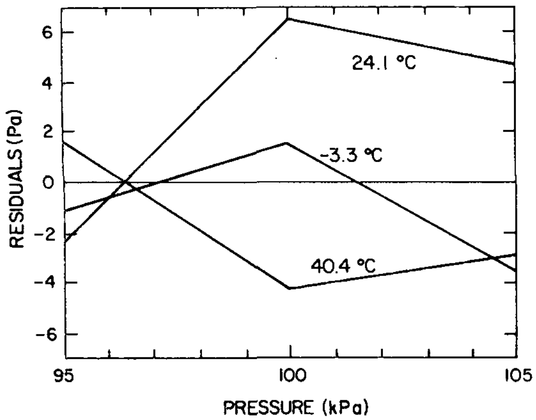

FIG. 5. Plot of residuals from the behavioral 4-coefficient model.

where P_r is the average received power into the receiver, P_n the noise power into the receiver, P_s the power that just saturates the receiver, G is a factor related to the receiver gain, Q the output quanta of the radar digital processor or averager, and A_0 and A_1 are regression coefficients obtained by linearly regressing the output quanta against the expression on the left side of the equal sign using the actual calibration data and standard statistical regression techniques.

Let us consider each term in Eq. (2) individually. Term B accounts for the additive nature of signal and noise in radar receivers. This simple rendition essentially lumps all possible sources of noise into a single constant. While it would be possible to consider the contributions from the individual sources already mentioned, the net result of this simple approach, as will be seen shortly, is adequate to give better results than polynomial curve fitting. Term A, derived empirically, accounts for the saturation end of the calibration curve and is constant when the received power is well below the saturation power, but becomes unity at saturation. For received powers greater than that of the saturation power P_s, term A actually becomes less than unity. This is not a serious problem in processing, however, because input powers exceeding P_s give the same output quanta as P_s. When we use the calibration equation to convert quanta into power, the largest power ever produced would be that of P_s. Term C to the right of the equal sign and the entire left side of the equation linearly relates the output quanta to the logarithmic response of the receiver, which in effect has been linearized by the addition of terms A and B.

Equation (2) contains one independent variable (P_r), one dependent variable (Q), and five unknown constants (G, P_s, P_n, A_0, and A_1). The constants P_s and P_n are properties of the receiver being calibrated. While we could use measured values for these, no such measurements were available for our 1981, or earlier, calibrations. Thus, we used iterative techniques to estimate P_s and P_n. In (2) G was arbitrarily set to 1000. Sensitivity tests revealed that changing G by an order of magnitude in either direction resulted in only slight changes in the estimates of P_s (i.e., P_s changed by less than ± 1.5 dB). We then iteratively determined P_s and P_n. For P_s, we used only the top portion of the calibration curve and sequentially tried many values for P_s over a range of reasonable values, choosing as the final value the one which had the best coefficient of determination from the regression analysis. Similarly, using the bottom portion of the calibration curve, we determined P_n. One advantage in using this approach is that we get objectively determined estimates of both the noise power and the saturation power.

Fig. 6 shows a calibration for 12 June 1981 for the CP-2 radar along with the results of the behavioral model and a fifth-degree polynomial fit to the calibration data. Except for very near the top of the curve, the behavioral model curve always lies closer to the calibration data than does the fifth-degree curve. The fifth-degree curve fits the data reasonably well, but it appears to be continually hunting for the correct position, alternately over- and undershooting the true position of the calibration data. Figure 7 shows the residuals between the calibration and the two modeled curves in more detail. Again, in most places the behavioral model results lie much closer to the calibration (i.e., along the zero line) than do the fifth-degree results.

Table 2 shows the results of applying the behavioral model to the same calibration along with the results of all possible polynomials up to the tenth degree. The variances of the residuals shown were calculated

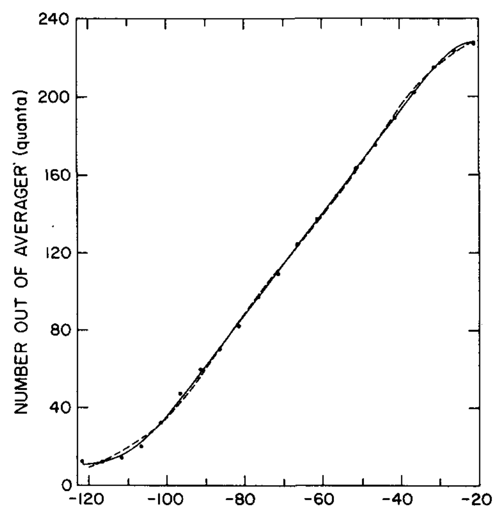

FIG. 6. Radar receiver calibration (dots) for the CP-2 S-band radar from 12 June 1981. P_r is the average received power. The solid curve is the fit from the behavioral model while the dashed curve is the fit from the fifth-degree polynomial.

in identically the same manner as in the last section on the pressure sensor, i.e., they account for the number of degrees of freedom available after determining the regression coefficients. As can be seen, the variance of residuals generally decreases as the degree of the polynomial increases up to the point where the decreasing number of degrees of freedom starts to cause this trend to reverse. The best fit occurred with the seventh-degree polynomial. Even so, the behavioral model gave a better fit to this calibration than any polynomial shown in Table 2.

As a further comparison of the quality of fit between the behavioral model and the traditionally used fifth-degree polynomial regression equation, we fit both models to calibration curves from NCAR's CP-2 and CP-3 radars on 13 separate days. On every day the behavioral model fit the calibration data better than the fifth-degree equation. The average variance of the residuals for the behavioral model was 0.59 dB^2 while that from the fifth-degree equation was 0.98 dB^2.

A qualitative comparison between the behavioral model and the piece-wise calibration technique is somewhat more difficult to make. Since the piece-wise method uses the actual data points as being exactly correct, the "residuals" between the piece-wise fit and the actual calibration data are zero. It is thus impossible to make the same kind of comparison as was made with polynomials of various degrees. We can, however, compare an individual piece-wise calibration for a given radar with the average of all piece-wise calibrations over several days. To do this we fit a straight line (first-degree polynomial) to the middle part of the calibration curves where noise and saturation did not influence the results. We found that the residuals between the average of all curves and the straight line were smaller than the residuals for every individual curve when compared to the straight-line fit. This suggests that individual data points do contain noise that introduce errors into the processed data in the vicinity of such points. Thus, we should expect that individual piece-wise calibration curves would be potentially worse than other techniques that do some averaging of the data along the calibration curve. Because both the fifth-degree polynomial and the behavioral model use curve-fitting procedures, which in effect smooth over a certain amount of noisiness within the data set, both should give a better fit to the true shape of the calibration curve than the piece-wise fit could do.

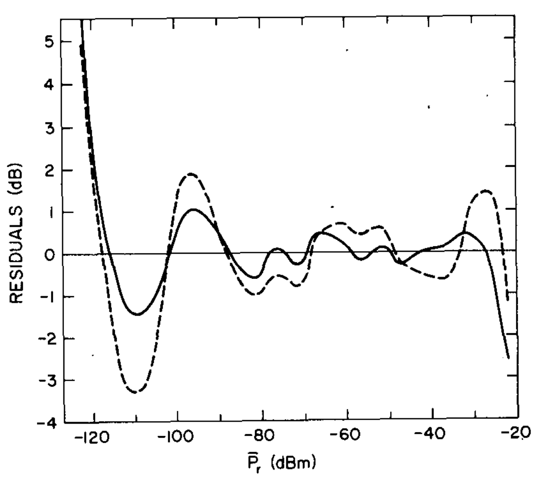

FIG. 7. Differences (residuals) between 12 June 1981 CP-2 calibration data for fifth-degree and behavioral model fits.

TABLE 2. Models tested and their residuals for the 12 June 1981 CP-2 radar receiver calibration shown in Fig. 6. The polynomial models tested are of the form

$$P_r = \sum_{i=0}^{n} (A_i Q^i)$$

where the degree of the polynomial is n, Q is the output quanta from the processor, P_r is the average power injected into the logarithmic receiver and A_i's are the regression coefficients. The variances of residuals were calculated using the number of degrees of freedom (last column) available after determining the regression coefficients.

Model tested	Degree of polynomial	Variance of residuals (dB^2)	d.f.
Polynomial	1	8.00	19
	2	7.51	18
	3	4.75	17
	4	4.84	16
	5	3.69	15
	6	3.76	14
	7	2.46	13
	8	2.47	12
	9	2.72	11
	10	2.62	10
Behavioral	N/A	2.31	19

The behavioral model has provided better fits to CP-2 and CP-3 receiver calibrations than polynomials of even higher order than were used in our previous calibrations. The behavioral model equation is simpler than a correspondingly well-fitting polynomial. Similarly, the behavioral model should give a better fit to radar receiver calibrations than does the piece-wise calibration technique. In spite of the seemingly simple "physics" used to determine this model, the results are sufficiently better than those of polynomials that we are using the behavioral model for our radar receiver calibrations. The behavioral model also provides objective estimates of the saturation power and the noise power, both of which are meaningful parameters of our radar receiver.

4. Summary

Behavioral modeling may range from impossible to pointless in a variety of circumstances. It may fail when the physics is too complex or too poorly understood. On the other hand, the modeling attempt may provide new understanding. In sensors still incorporating devices to modify their behavior, e.g., temperature compensation, behavioral modeling would be more difficult and of little value. Indeed, mathematical formulation of the response of a device allows for the elimination of the need for compensation and its attendant addition of other error sources. Modeling is most successfully applied to an unmodified response. We have found, for instance, that a modeled thermistor response may reduce the errors of a thermistor in a linearizing network by a factor of five or more.

In the examples given, we showed that the results were superior to those obtained with arbitrarily chosen polynomials up through a full second-order model for the pressure sensor and through a tenth-degree polynomial for the radar receiver. The behavioral models not only had lower residuals, but they did so with fewer terms; however, this may not be universally true. For some devices, the behavioral model may have virtually the same form as an arbitrary model, or one may elect not to develop a behavioral model when an arbitrary fit suffices. In some circumstances, however, the gain from behavioral modeling is spectacular. In those cases, the physics gives a highly accurate relationship negating the need for empirical terms.

Behavioral modeling opens insights into the physical processes underlying the function of a device and makes it possible to quantify them. In addition, fundamental sensor research and new system development can be moved further from empiricism into purposeful design.

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