NOTES

Effect of Deviations from the Marshall–Palmer Drop Size Distribution on the Calculation of Vertical Air Velocity by Rogers's Method

PETER W. BAKER AND M. C. HODSON

National Institute for Telecommunications Research, Johannesburg, South Africa

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ABSTRACT

In 1964, R. R. Rogers proposed a method for estimating vertical air velocity in rainfall which is based on the Marshall–Palmer raindrop size distribution. Many investigators have shown that there are deviations from the Marshall–Palmer distribution such that the intercept N_0 and the shape may vary. Essential to Rogers' method is the relationship between reflectivity Z and Doppler fall velocity $\bar{V}$. A numerical method has been used to obtain $\bar{V}$ versus Z relationships for raindrop size distributions different from those of Marshall and Palmer and to compare these with results obtained using Rogers's method. Thus errors that are likely to occur in estimating vertical air motions have been evaluated and are shown to be a maximum of 1.4 m s^{-1}.

1. Introduction

Of considerable interest to the radio meteorologist is Doppler radar and its application to the study of precipitation environments. Not only may the characteristics of the rainfall be determined but also the air motions within the precipitation. Rogers (1964) determined a simple method for estimating vertical air motions with vertically pointing doppler radar. This method depends on a knowledge of the radar reflectivity Z (mm^6 m^{-3}) and the mean relative Doppler velocity of the rain being observed V_D (m s^{-1}). The theory used by Rogers is based on a standard exponential rain drop size distribution as proposed by Marshall and Palmer (1948). However, many investigators have since observed that deviations from this distribution do occur. The purpose of this paper is to demonstrate the effects of these deviations from the standard form on the determination of vertical motions using Rogers' method, and to show the likely magnitude of errors thereby introduced in typical rainfall situations.

2. Background theory

The exponential distribution for raindrop spectra was first proposed by Marshall and Palmer (1948) in the form

$$N(D) = N_0 \exp(-\Lambda D)$$
$$= 4.1R^{0.21}, \qquad (1)$$

where $N(D)$ represents the number of drops per millimeter diameter interval per cubic meter of air, N is a constant 8000 m^{-3} mm^{-1} and Λ (mm^{-1}) a parameter which depends on the rainfall rate R (mm h^{-1}).

Rogers (1964) derived a simple method in which vertical velocities in rain may be estimated assuming a Marshall–Palmer drop size distribution. This is an expression of the form

$$\bar{V} = 3.8Z^{1/14}, \qquad (2)$$

which relates the reflectivity-weighted mean fall velocity of the rain $\bar{V}$ to the reflectivity factor Z.

When observing precipitation, the mean Doppler velocity V_D is a combination of any vertical air velocity present and the fall velocity of the rain $\bar{V}$ such that

$$V_D = \bar{V} + W, \qquad (3)$$

where velocities are considered positive downwards. Hence it can be shown from (2) and (3) that, knowing Z and V_D from Doppler radar data, an estimate of W will be given by

$$W = V_D - 3.8Z^{1/14}. \qquad (4)$$

There is evidence for different forms of the drop size distribution from that suggested by Marshall and Palmer. There is support for shape effects in the work of Blanchard (1953), Best (1950), Meuller (1965) and Joss and Gori (1978).

Atlas and Ulbrich (1982) suggested fitting a gamma function to the drop size distribution. This is

$$N(D) = N_0 D^m e^{-\Lambda D}, \qquad (5)$$

where the exponent m may be any positive or negative value. Negative values of m correspond to broad concave upward distributions such as those found in orographically induced rainfall in Hawaii, whereas positive m corresponds to a narrow concave downward distribution such as those found by Mueller (1965) for a variety of rainfall types and by Jones (1956) for thunderstorm rain. In general the value of the exponent m will vary between -2 and 6.

Raindrop spectra may well be represented by an exponential distribution in which N_0 is not necessarily a constant. Significant work in terms of showing the variation of N_0 from the value 8000 [$\mathrm{m^{-3}\ mm^{-1}}$] is that by Waldvogel (1974).

3. Modeling of V–Z relations

A model is used to calculate by purely numerical means an expression relating $\bar{V}$ to Z. In this a range of drop sizes D between 0.1 and 5.4 mm diameter is assumed. Terminal fall velocities for each drop size $V(D)$ are derived from the work of Wobus *et al.* (1971). An expression in which allowances are made for local factors of atmospheric pressure and temperature is used. Assuming a particular form of the drop size distribution and for a given Λ the number of drops in each size category will be known. Knowing the number of drops in each category $N(D)$, the radar reflectivity is evaluated from $Z(D) + N(D)D^6$ for each interval D to $D + \Delta D$ and hence, by integration over the entire range of drop size, the total reflectivity Z. Similarly, knowing the number of drops $N(D)$ and the fall velocity for each category, the average fall velocity weighted by Z is given by

$$\bar{V} = \frac{\sum_{0.1}^{5.4} [V(D)Z(D)]}{Z}. \qquad (6)$$

Hence for a particular rainfall rate or value of Λ a pair of values $\bar{V}$ and Z are obtained. By considering variations of the value of Λ a complete set of $\bar{V}$ and Z values will be obtained such that a $\bar{V}$ versus Z relation for the particular drop size distribution may be plotted. By assuming variations of the raindrop size distribution (5) where m may vary between typical values and $+2$ and -2 and N_0 is not necessarily a constant, various Z and $\bar{V}$ relations will be obtained, each corresponding to the various drop size distributions.

At Johannesburg, South Africa, a vertically pointing 35 GHz Doppler radar system is used at an altitude of 1800 m above sea level. Working with a similar system at Johannesburg, Pasqualucci (1976) determined empirically from the data of five storms in 1972 and 1973 the expression $\bar{V} = 2.96Z^{0.095}$ to relate the mean fall velocity of rain with radar reflectivity. Pasqualucci regarded the drop size spectra on average to be well represented by the Marshall–Palmer distribution. Assuming this and also raindrop fall velocities appropriate to Johannesburg, a $\bar{V}$–Z relationship is obtained utilizing the numerical model. Therefore, a comparison between empirical and theoretical data, shown in Fig. 1, is obtained.

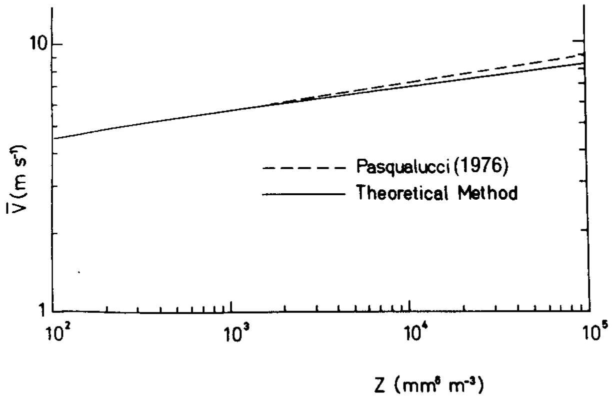

FIG. 1. $\bar{V}$ (m s^{-1}) versus reflectivity Z. Theoretical data for altitude 1800 m compared with that obtained by Pasqualucci (1976), where $\bar{V} = 2.96\ Z^{0.095}$.

It is shown that the results of the numerical model are in good agreement with that established empirically for the same site. Thus the validity of the numerical model is proven. As this institute's radar studies are to be continued at this altitude, all subsequent work in this paper is carried out considering fall velocities at 1800 m.

The aim of this discussion is to show, utilizing this numerical method, the effect upon $\bar{V}$ versus Z relationships of typically occurring non-Marshall–Palmer distributions and thus the effect upon estimating vertical air velocities by Rogers' method.

4. The gamma function

Gamma drop size distributions are shown in Fig. 2 with values of m of $+2$ and -2 and $m = 0$ (standard Marshall–Palmer), where N_0 is 8000 $\mathrm{m^{-3}\ mm^{-1}}$.

The $\bar{V}$–Z relations for these gamma distributions are evaluated and shown in Fig. 3.

The most obvious effect of the gamma distribution is the large deviation of the value of $\bar{V}$ from that obtained by Rogers at the lower values of Z.

For various values of the reflectivity Z, the deviation of the values of $\bar{V}$ for the two gamma distributions from that obtained for the standard Marshall–Palmer distribution is shown in Table 1.

Thus it can be seen that using Rogers' method for determining vertical air motions in precipitation, errors of the order of 1.4 m s^{-1} may arise from a typically occurring gamma distribution. It is observed that the $\bar{V}$–Z relations obtained approach similar values with increased reflectivity Z. At reflectivities greater than 10^4 mm^6 m^{-3} the effect of the gamma function upon $\bar{V}$ values differing from that obtained from a Marshall–Palmer distribution becomes negligible. Above this level the presence of graupel or hail is likely to be a concern.

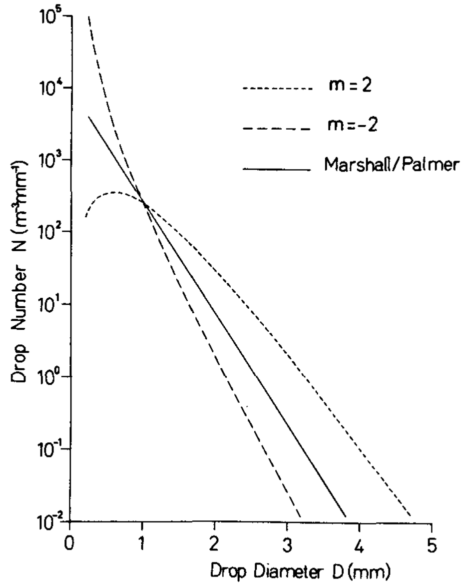

FIG. 2. The form of positive and negative gamma drop size distributions, represented by the equation $N(D) = N_0 D^m e^{-\Lambda D}$ compared with the Marshall–Palmer distribution.

5. Variability of N

Waldvogel (1974) shows how during reasonably steady periods of rainfall N_0 undergoes significant variations by as much as a factor of 10 although the rainfall rate remains nearly constant. One instance described is for 6 June 1968, in which during a period of nearly steady rainfall of 10 mm h^{-1}, N_0 changed from 50 000 to 5000 m^{-3} mm^{-1}. This change was associated with a change from a cold front to more widespread stratiform rain. Using each of the above values as typically occurring examples of N_0 variations whilst keeping the shape factor m as zero, the numerical method is used to derive $\bar{V}$ versus Z relations, which are plotted in Fig. 4.

From the graphs shown it is apparent that there is a significant change in the $\bar{V}$–Z relationship caused by variations in the value of N_0. As would be expected, the greater value of N_0 (50 000 m^{-3} mm^{-1}) corresponds to smaller values of $\bar{V}$, since this distribution is weighted towards smaller drop sizes. Note from the graphs of Fig. 4 that over a large range of reflectivity there is a significant deviation from the curve representing a Marshall–Palmer distribution; for example, a reflectivity of 6300 mm^6 m^{-3} corresponding roughly to a rainfall rate of 10 mm h^{-1} gives errors of 0.4 and −1 m s^{-1} for N_0 values of 5000 and 50 000 respectively.

TABLE 1. Deviations of $\bar{V}$ in m s^{-1}.

	Z (mm^6 m^{-3})				
m	10^2	3×10^2	10^3	3×10^3	10^4
2	0.9	0.8	0.7	0.5	0.5
−2	−1.4	−1.3	−1.1	1	−0.6

6. Summary

The method that Rogers (1964) proposed for estimating vertical air motions in rainfall is valid for the Marshall–Palmer distribution. The work of some investigators, however, shows that drop size spectra deviate from this. Essential to Rogers' method is a knowledge of the relationship between reflectivity Z and mean Doppler velocity $\bar{V}$ (in still air). A numerical method has been used to calculate Z and $\bar{V}$ for different parameters of the drop size distribution

$$N(D) = N_0 D^m \exp(-\Lambda D).$$

Curves for extreme values of N_0 and for three different values of m have been plotted to show that the maximum error in the estimate of the vertical air velocity is 1.4 m s^{-1}.

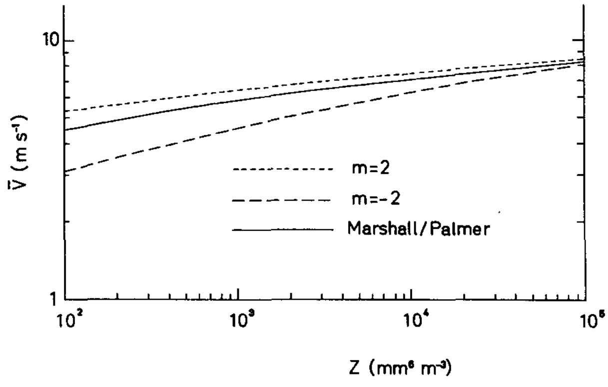

FIG. 3. The effect of positive and negative gamma drop size distributions on the $\bar{V}$ versus Z relationship.

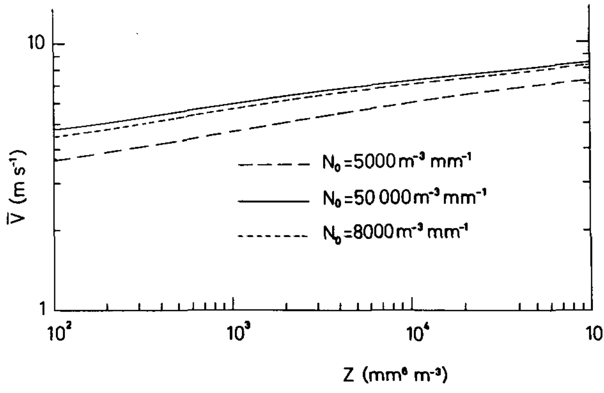

FIG. 4. The effect of variation of N_0 on the $\bar{V}$ versus Z relationship.

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