Behavior of Turbulence Statistics in the Convective Boundary Layer[1]

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ABSTRACT

Velocity variances in the convective boundary layer are examined using data derived in PBL experiments over land (Minnesota) and ocean (the Coral Sea) supported by data from the Kansas study of the surface boundary layer. In the freely convective limit, the data clearly support scaling of σ_w by the convective velocity w_* instead of u_*, as is common in modern treatments. The freely convective limit appears to be 0.59 for both σ_w/w_* and σ_v/w_*. The available data are compatible with relationships based on additive contributions to total variance by mechanical and buoyant forces. The resulting relations are

$$\sigma_w = (1.2u_*^2 + 0.35w_*^2)^{1/2},$$

$$\sigma_v = (3.6u_*^2 + 0.35w_*^2)^{1/2}.$$

These relations collapse back to the neutral values derived from the Kansas experiment. Similar analyses for temperature variance are somewhat less revealing; however, the available data indicate that scaling by convective properties alone is quite adequate. Hence,

$$\sigma_T \approx 1.8\theta_*,$$

where $\theta_* = H/\rho c_p w_*$. These relations appear to apply over the bulk of the convective PBL, from $z/z_i \approx 0.1$ to at least $z/z_i = 0.6$, but extend downwards to very near the surface in the case of σ_v.

1. Introduction

Knowledge of the turbulence statistics σ_w and σ_v is basic to discussion of dispersion in the atmosphere. The case of plume dispersion is of great practical importance, yet is an example in which the urgency of needs has caused models to be constructed around formulations that are extensions of knowledge rather than expressions of it.

Some especially intriguing questions arise. The relationship between σ_w and external properties is an obvious example. Until recently, the result of research on the behavior of σ_w near the surface was usually summarized by an expression of the kind

$$\sigma_w/u_* = F_w(z/L) \qquad (1)$$

where the form of F_w was determined empirically (see the Appendix for definition of symbols). On the basis of dimensional analysis, the strongly unstable (i.e. freely convective) limit of (1) should be such that

$$F_w \to c(-z/L)^{1/3} \quad \text{as} \quad -z/L \to \infty. \qquad (2)$$

Experimental data obtained in near-neutral conditions indicate that the neutral intercept of (1) is about 1.3. Thus, several relations have been suggested, of the general form

$$F_w(z/L) = a[1 - b(z/L)]^{1/3} \qquad (3)$$

(e.g., Panofsky *et al.*, 1977). Usually, such relations have been suggested as an empirical fit to experimental data, although sometimes they have been developed from theoretical arguments regarding the production rates of turbulent energy by mechanical and buoyant forces. All such relationships have the common feature that they call for σ_w to increase monotonically with increasing instability ($-z/L$), thus implying a monotonic increase with altitude.

Recently, the validity of these expressions has been questioned on the grounds that u_* is a poorly determined, shared variable affecting both axes of plots of σ_w/u_* versus $|-z/L|$ (see Hicks, 1978a, 1978b, 1981). The problems that arise are especially important for the stable case. In stable stratification the *appearance* of a monotonically increasing relationship between σ_w/u_* and z/L can result solely as a consequence of statistical interactions between poorly determined variables, without any physical basis. For the unstable case, however, the analysis of Kansas turbulence data by Hicks (1981) supports the usually accepted monotonic increase of σ_w/u_* with instability, even when the problems of variable interaction are avoided. However, the horizontal-component analogs, σ_u/u_*

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and σ_v/u_*, do not continue to increase in the same way, but appear to maximize at a value of about 3.0.

Scaling in the bulk of the planetary boundary layer (PBL) is now accepted to be such that the convective velocity $w_* \equiv (gHz_i/\rho c_p\theta)^{1/3}$ takes the place of u_* in most convective situations (see Deardorff, 1970). There have been several recent suggestions (e.g., Wyngaard *et al.*, 1971) regarding the form of the relationships that permit σ_w, σ_v and σ_u, to scale as u_* near the surface and as w_* well above it. Many of these formulae address the derivative plume dispersion quantities σ_z, σ_y and σ_x rather than the velocity statistic itself (e.g. Lamb, 1979; Nieuwstadt, 1980; Venkatram, 1980; Deardorff and Willis, 1975), however it is common to assume that velocity variance (or consequent plume dimension) is a result of independent contributions associated with the surface momentum flux (scaled according to u_*^2) and the surface vertical heat flux (scaled according to the convective velocity, and hence involving w_*^2). These variances are assumed to be additive, and hence

$$\sigma^2_{w,v,u} = a^2_{w,v,u}u_*^2 + b^2_{w,v,u}w_*^2. \quad (4)$$

This relation is of a form that suggests a substantially different free convection limit. For the vertical component, for example, it suggests that

$$\sigma_w/u_* \rightarrow A(-z_i/L)^{1/3} \quad (5)$$

as instability increases, rather than (2).

Here, data from three sources will be used to investigate the features of the general relationship. The data sets are selected to provide two greatly dissimilar sets of observations of the PBL, one over land (Izumi and Caughey, 1976) and the other over tropical ocean (Warner, 1972, 1973), so that the applicability of the deduced form over surfaces of different roughness will be assured. The results derived from a comparison of these two data sets will be examined in the light of the Kansas data set (Izumi, 1971) referring to the surface boundary layer. The results of an earlier analysis of Kansas data (Hicks, 1981) will be used as guidance. These results can be summarized as follows:

$$\left.\begin{aligned} \sigma_w/u_* &= 1.1(1-2\,\mathrm{Ri})^{1/3} \\ \sigma_u/u_* &= 2.5 - 1.6\,\mathrm{Ri}; \quad -\mathrm{Ri} < 0.3 \\ &= 3.0; \quad -\mathrm{Ri} > 0.3 \\ \sigma_v/u_* &= 1.9 - 3.5\,\mathrm{Ri}; \quad -\mathrm{Ri} < 0.3 \\ &= 3.0; \quad -\mathrm{Ri} > 0.3 \end{aligned}\right\}. \quad (6)$$

Results obtained using the average data from the two PBL data sets to solve simultaneous equations of the form indicated by (4) will be compared with the results of multiple regression analyses based on the individual observations.

2. The Minnesota data set

Figure 1 shows the variation of velocity turbulence statistics with height in the mixed layer. The data are from the Minnesota turbulence experiment reported by Izumi and Caughey (1976). Following present-day convention, the altitude scale is normalized by the depth of the mixed layer z_i at the time of the experiment. The values plotted are derived from the ratios of velocity standard deviations measured using a tethered sounding system to simultaneous tower-mounted observations of the same quantity at 4 m height. Data are selected to lie in the altitude range between $0.1z_i$ and $0.9z_i$, so as to be above the immediate effects of the surface while also being below the level at which effects of free air entrainment at the top of the mixed layer become obvious (Caughey, 1982, defines the entrainment interfacial layer as $0.8z_i$–$1.2z_i$, for example). Throughout this rather deep layer of the lower atmosphere there is no marked increase in either σ_u or σ_v, both remaining close to the values observed at 4 m height. The vertical velocity statistic seems to be similarly constant with altitude, but it takes this constant value (approximately 2.5 times σ_w at 4 m) at some yet-undetermined level in the lowest 10% of the mixed layer.

It should be noted that the variation with height evident in Fig. 1 is substantially different from that implied by some plots of, for example, σ_w/w_* versus z/z_i. The present analysis is designed specifically to avoid the complication that arises because w_* is a function of z_i (see Hicks, 1978a,b). Nevertheless,

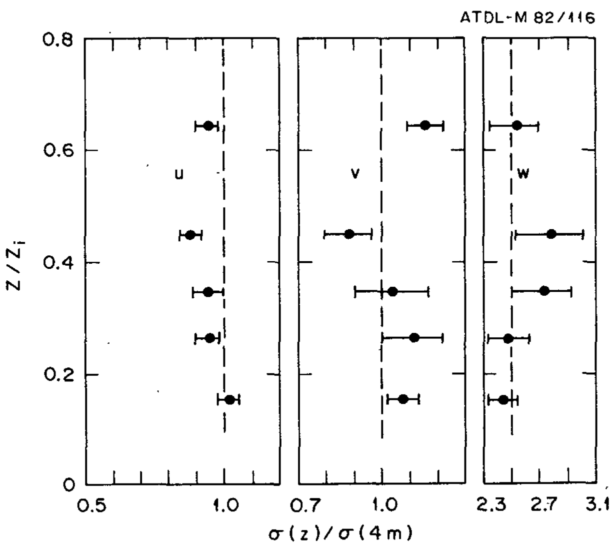

FIG. 1. The variation with normalized height through the daytime mixed layer of the three velocity standard deviations, as derived from the 1973 Minnesota PBL experiment (Izumi and Caughey, 1976). Note that the standard deviations are normalized by simultaneous observations made at a constant 4 m height, and that the figure therefore represents a comparison between SBL and PBL turbulence behavior.

there is evidence in Fig. 1 of a slight but consistent height variation of all three statistics, with maxima and minima at about $z/z_i = 0.4$. This is similar to the behavior found for σ_w by Warner (1972) for his Coral Sea data (yielding a maximum at $z/z_i = 0.3$), and is much as predicted by laboratory modeling studies (maximum at $z/z_i = 0.35$; Deardorff, 1970). Other workers have concluded that a consistent variation of σ_w with height is typical of the PBL (e.g. see Fig. 9 of Hildebrand and Ackerman, 1984). For the data considered here, the variation is too small and the statistics are too uncertain to divert the present analysis.

The horizontal velocity fluctuations appear to attain fairly constant values at moderate instabilities easily achieved very near the surface [as is seen by inspection of (6)] and then retain these values throughout the mixed layer (see Fig. 1). The vertical component behaves quite differently, as would be expected on physical grounds. The major effect of additional buoyant energy would be expected to appear in the vertical velocity field. Inspection of (6) shows this feature rather nicely; eddies are elongated along the wind near the surface in unstable conditions (i.e. $\sigma_u > \sigma_v$, but quickly become more circular in a horizontal cross section as height increases (i.e. $\sigma_v \rightarrow \sigma_u$ as Ri increases).

Table 1 summarizes the data from the Minnesota experiment used in the following analysis of turbulence in the convective PBL. As in the derivation of Fig. 1, the data are constrained to lie in the range $0.1 < z/z_i < 0.9$.

3. The Coral Sea data set

Conventional relationships based on near-surface observations emphasize the role of u_* and hence of surface roughness. Convective scaling arguments regarding turbulence in the PBL often focus on the role of the sensible heat flux, although the relevant physics demands that the important factor is the virtual heat flux $H_v \approx (H + LE/14)$ because of the buoyancy of water vapor. It is instructive, therefore, to compare the Minnesota data with data derived in an oceanic study, since the oceanic case presents a greatly different circumstance both in surface roughness and in the relative importance of LE. The Coral Sea data set presented by Warner (1972, 1973) provides fairly complete turbulence data, obtained using an instrumented aircraft. In all of the following analyses, the virtual heat flux is used as a direct substitute for the sensible heat flux. Latent heat fluxes exceeded sensible by an order of magnitude throughout the experiment.

Table 1 provides a listing of data obtained in the Coral Sea studies, again constrained to the height interval $0.1 < z/z_i < 0.9$. It should be noted that the horizontal velocity standard deviations reported by Warner are substantially different from the σ_u and σ_v data reported for Minnesota, since the use of an aircraft prohibited accurate decomposing of the observations into precise u and v components. Thus the Coral Sea σ_v data must be expected to be influenced by σ_u. There is no such problem in the case of the vertical component data, which provide the prime target for present attention.

4. Results: Velocity variances

Table 2 summarizes the average conditions of the Minnesota and Coral Sea data sets, and presents the geometric mean values of turbulence statistics (and related quantities) derived from the observations. Even though the surface conditions of the two experiments were markedly different (summer continental grassland versus tropical ocean), there is no strong difference between the mean values of σ_w/w_*, this being the preferred form indicated by the considerations above. It should also be noted that the use of u_* as a normalizing factor for vertical velocity variances in the PBL appears to be far less satisfactory.

For the ratios involving σ_v, the data of Table 2 are less convincing; however, we should note that the ratio observed over the ocean for σ_v/u_* is the same as is indicated by (6), and that the Minnesota data differ by 47% on the average. Scaling by w_* in this instance does not decrease the apparent difference between the two experiments. It should be pointed out that experimental difficulties add to the insecurity associated with these considerations; the Minnesota data were obtained using a tethered balloon system which may not have provided a sufficiently stable platform, and the Coral Sea data were obtained using an aircraft with potentially similar problems.

When considered in light of the Kansas surface data as summarized by (6), it appears that:

1) The value of σ_v/u_* seems to be about 3.0 at all heights in the mixed layer above the level at which Ri = −0.3, although it should be remembered that the apparent agreement between the Kansas and Coral Sea data sets may be fortuitous.

2) The term σ_w/u_* increases monotonically with increasing instability through the surface boundary layer (SBL) until it reaches a value such that σ_w/w_* attains some constant value, approximately 0.6.

3) Once formulated in terms of u_* and w_*, with appropriate stability dependences in the SBL, there is no evidence of any further influence of surface roughness for the vertical case, and questionable evidence for the lateral case.

5. Results: Temperature variances

Although our purpose is to emphasize the velocity characteristics, the temperature data obtained during these two studies are clearly of high quality and are

TABLE 1. Values of σ_T (°C), σ_w and σ_v (cm s^{-1}) at heights z (m) as reported by Warner (1972, 1973) and Izumi and Caughey (1976). Values of the external properties z_i, H (°C cm s^{-1}) and u_* (cm s^{-1}) are also given. Data are selected so that $0.1 < z/z_i < 0.9$. Note that there are substantial differences between the data sets. For the Coral Sea data, virtual heat fluxes are quoted. No σ_u data are available for the Coral Sea, hence for the present purposes only the transverse velocity statistic σ_v will be considered.

Date and identification	σ_T	σ_w	σ_v	z	z_i	H (surface)	u_* (surface)
				Minnesota			
2A1	0.17	124	139	914	1250	19.6	45
	0.15	106	160	610			
	0.18	93	143	305			
2A2	0.19	110	188	1219	1615	20.9	45
	0.15	129	192	914			
	0.14	125	213	610			
	0.20	103	166	305			
3A1	0.11	140	229	610	2310	18.6	37
	0.12	144	208	457			
	0.13	127	190	305			
3A2	0.10	124	184	610	2300	11.6	32
	0.10	112	165	547			
	0.11	99	148	305			
5A1	0.06	67	108	610	1085	6.9	18
	0.06	67	83	457			
	0.07	71	90	305			
	0.11	73	81	152			
6A1	0.15	157	167	1219	2095	21.0	24
	0.14	163	130	914			
	0.15	145	158	610			
	0.19	123	134	305			
6A2	0.13	151	176	1219	2035	16.2	23
	0.14	156	137	914			
	0.14	125	157	610			
	0.18	116	128	305			
6B1	0.16	112	126	1219	2360	7.2	26
	0.15	114	106	914			
	0.13	103	117	610			
	0.14	101	111	305			
7C1	0.11	134	101	610	1020	22.1	28
	0.12	133	92	457			
	0.15	117	93	305			
	0.26	115	101	152			
7C2	0.10	121	103	610	1140	18.1	30
	0.12	129	94	457			
	0.14	113	83	305			
	0.25	123	102	152			
7D1	0.09	122	59	610	1225	9.9	25
	0.10	131	93	457			
	0.11	110	88	305			
	0.16	125	97	152			
				Coral Sea			
26 June	0.137	71	85	90	600	4.26	36
	0.132	67	96	250			
28 June	0.136	65	92	90	650	3.61	36
	0.127	66	84	250			
29 June	0.149	61	122	90	450	4.34	38
	0.109	63	85	250			
1 July	0.136	51	90	90	570	3.36	32
	0.132	58	81	250			
	0.127	54	75	400			

TABLE 1. (*Continued*)

Date and identification	σ_T	σ_w	σ_v	z	z_i	H (surface)	u_* (surface)
3 July	0.162	47	87	30	210	3.52	28
	0.179	50	44	90			
5 July	0.088	58	64	90	490	2.30	18
	0.099	51	65	250			
6 July	0.109	51	68	90	640	2.21	20
	0.103	56	129	250			
8 July	0.139	70	112	90	830	5.08	32
	0.133	88	132	250			
	0.132	87	112	400			
	0.129	85	159	550			
11 July	0.112	57	78	250	1100	2.87	32
	0.123	55	83	400			
13 July	0.094	45	54	90	670	1.80	24
	0.083	49	55	250			

well worthy of attention. As will be seen, the temperature data serve to reinforce conclusions regarding the overwhelming importance of w_* scaling instead of u_* scaling.

All of the tables given here include temperature data, analyzed in a manner paralleling the velocity considerations. When these data are plotted as in Fig. 1, little height variability is evident (somewhat in contrast to the implication of Caughey, 1982; see his Fig. 4.19), hence once again z/L stability scaling appears inappropriate in the altitude range $0.1 < z/z_i < 0.9$. Inspection of the mean values listed in Table 2 indicates that there is more merit in scaling σ_T by θ_* ($\equiv H/\rho c_p w_*$) than in scaling by T_* ($\equiv H/\rho c_p u_*$) over the selected height interval. (That is, scaling by T_* fails to bring the two data sets together as much as does scaling by θ_*.) However, we must note the great differences between the two data sets and question whether any such scaling argument should be expected to resolve all the differences.

6. Simultaneous equation analyses

Table 3 summarizes results obtained using the averages of Table 2 to solve pairs of simultaneous

TABLE 2. Comparisons between geometric mean results obtained in the Minnesota (41 values) and Coral Sea (23 values) experiments.

	Minnesota	Coral Sea	Ratio M:C
σ_T (°C)	0.1316	0.1227	1.07
σ_T/T_*	0.2621	1.0789	0.24
σ_T/θ_*	1.7834	3.2124	0.56
σ_w (cm s^{-1})	115.96	60.27	1.92
σ_w/u_*	4.050	2.065	1.96
σ_w/w_*	0.595	0.694	0.86
σ_v (cm s^{-1})	126.21	87.65	1.44
σ_v/u_*	4.408	3.003	1.47
σ_v/w_*	0.648	1.009	0.64
z (m)	476.5	168.2	
z_i (m)	1573.0	594.4	
H (°C cm s^{-1})	14.37	3.32	
u_* (cm s^{-1})	28.63	29.18	
f (s^{-1})	1.09×10^{-4}	4.14×10^{-5}	
w_* (cm s^{-1})	194.81	86.89	
$-z/L$	39.12	3.06	
$-z_i/L$	129.1	10.83	
u_*/fL	215.6	120.5	

TABLE 3. Results of solving for the constants a^2 and b^2 in relations of the general form of (5), using simultaneous equations to satisfy geometric mean data summarized in Table 2 and using multiple regression to examine the individual observations of Table 1, constrained to the height interval $0.1\ z_i < z < 0.9\ z_i$.

	Simultaneous equations	Regression analyses: Minnesota	Coral Sea	Combined
		$\sigma_w^2 = a^2u_*^2 + b^2w_*^2$		
$r_{12.3}$	—	−0.36	−0.08	−0.09
$r_{13.2}$	—	0.92	0.84	0.92
a^2	1.40	−2.76 ± 2.7	0.22 ± 0.10	−0.4 ± 2.7
b^2	0.32	0.41 ± 0.01	0.51 ± 0.03	0.37 ± 0.01
		$\sigma_v^2 = a^2u_*^2 + b^2w_*^2$		
$r_{12.3}$	—	0.42	−0.10	0.50
$r_{13.2}$	—	0.64	0.64	0.72
a^2	6.56	7.1 ± 3.8	−1.3 ± 2.4	6.4 ± 3.1
b^2	0.28	0.34 ± 0.07	1.28 ± 0.16	0.35 ± 0.06
		$\sigma_T^2 = a^2T_*^2 + b^2\theta_*^2$		
$r_{12.3}$	—	0.10	0.15	−0.06
$r_{13.2}$	—	0.56	0.84	0.61
a^2	−0.2	0.01 ± 0.01	0.1 ± 0.2	−0.01 ± 0.02
b^2	12.00	2.5 ± 0.5	8.3 ± 0.6	3.3 ± 0.5

equations of the form indicated by (4). For the vertical component, the results indicate a near-neutral limit corresponding to $\sigma_w/u_* = 1.18$, quite near the Kansas result of about 1.1 as indicated by (6). For the lateral component, the data indicate a neutral limit corresponding to $\sigma_v/u_* = 2.56$. The uncertainties associated with the latter evaluation will become apparent later. For the present, we might note that the Kansas data indicate a considerably lower neutral limit, and a maximum value of about 3.0 for σ_v/u_* when $-$Ri exceeds about 0.3. If we hypothesize, for temperature, an additive-variance relationship paralleling (4),

$$\sigma_T^2 = a_T^2 T_*^2 + b_T^2 \theta_*^2, \qquad (7)$$

and then solve for the values of a_T^2 and b_T^2 required to explain the Minnesota and Coral Sea data sets (i.e. solve the simultaneous equations that are defined by the means listed in Table 2), then we find $a_T^2 = -0.19$ and $b_T^2 = 12.00$. Not only does the value of b_T^2 cause the θ_* term to dominate, but the sign of a_T^2 is contrary to expectations based on SBL data alone.

7. Multiple regression analyses

The discussion above centers on the need for any general relationship to satisfy the extremes represented by the Minnesota and Coral Sea data sets. Regression analyses based on the values listed in Table 1 are also informative. Results are tabulated in Table 3 for each experiment independently and for both combined.

Results are given for partial correlation coefficients $r_{12.3}$ and $r_{13.2}$ yielded from consideration of equations of the form of (4), where variable 1 is the turbulence variance, variable 2 is the appropriate SBL scaling quality (u_*^2 or T_*^2), and variable 3 is the corresponding PBL quantity (w_*^2 or θ_*^2). The analysis differs from convention only insofar as the results are constrained to pass through the origin; a conventional statistical analysis would provide results passing through mean values defined by the data sets themselves without recognizing the external physical constraint that in all velocity cases $\sigma \rightarrow 0$ as u_* and $w_* \rightarrow 0$, and $\sigma_T \rightarrow 0$ as $H \rightarrow 0$.

The quantities a^2 and b^2 listed in Table 3 are the multiple regression coefficients yielded by the partial correlation analysis, with standard errors of estimates calculated accordingly. In statistical terms, these values correspond to slopes $b_{12.3}$ and $b_{13.2}$ in the analysis procedure outlined above.

8. Discussion

In every instance, the results of Table 3 point to a dominance of PBL properties and scaling. The most significant correlation coefficients involving SBL quantities are for the cases of velocity variances as reported by the Minnesota study, which are sufficient to impose a statistically significant correlation in the combined data set. The possibility of errors arising from the use of a tethered balloon to collect these data has already been mentioned. In this particular case, it is clear that considerable weight must be given to the results provided by the Kansas experiment, which can be used as a firm foundation for evaluating Table 3. In all cases, it must be remembered that Table 3 is derived from examination of data purposefully selected to be above the SBL, and thus the listed values should not be considered as evaluations of SBL quantities but rather as guidance on how to extend proven SBL relations.

For σ_w, the Kansas data indicate that $\sigma_w \rightarrow 1.10u_*$ at neutral [see (6)]; Table 3 provides no statistically significant evidence that might cause reconsideration of this conclusion. The value 1.40 indicated by consideration of the simultaneous equations based on Minnesota and Coral Sea data is poorly based, as indicated by the statistics of the independent data sets. On the other hand, the conclusion regarding the PBL component is strongly significant. An optimal conclusion appears to be that $b_w^2 \approx 0.35$, as indicated by the overall statistical analysis (0.37 ± 0.01) and by examination of the mean values of Table 2 if $\sigma_w/u_* = 1.1$ is assumed to be imposed as a neutral limit (yielding 0.33 for Minnesota and 0.34 for the Coral Sea).

For σ_v^2, Table 3 again contributes little of statistical significance concerning SBL behavior. Hence, the well-determined Kansas result $\sigma_v = 1.9u_*$ at neutral [see (6)] can be used as a basis. A firm conclusion regarding an optimal value for b_v^2 is somewhat more difficult to derive in this instance, since the Coral Sea data differ greatly from the Minnesota data. However, the overall statistical analysis yields $b_v^2 = 0.35 \pm 0.06$, and the assumption of $\sigma_v/u_* = 1.9$ implies that $b_v^2 = 0.34$ and 0.61 for the Minnesota and Coral Sea data respectively (derived from Table 2). The appearance of numerical equality (0.35) to the case of σ_w is appealing. In this regard, it should be remembered that the Coral Sea data were obtained using an aircraft, imposing a basic difficulty in separating σ_u from σ_v. This matter is emphasized in the original reports (Warner, 1972, 1973), and hence the Coral Sea σ_v data are not weighted highly here.

For σ_T, Table 3 indicates a similar dominance of the PBL terms. As stability tends to neutral, it is obvious that σ_T tends to zero (assuming that humidity enters as required to relate virtual temperatures to virtual heat fluxes). All aspects of the present analyses indicate that the complexity indicated by (7) is not warranted, and that a_T^2 can be taken to be zero. In this case, a "best value" for b_T^2 remains difficult to determine. In the lack of other guidance, the result of the overall statistical analysis appears to be the best-determined statistic: $b_T^2 = 3.3 \pm 0.5$.

Table 4 compares the present results with evaluations presented elsewhere. It should be noted that many of the values listed are derived from relationships involving vertical and lateral plume dispersion, and direct comparison with values based on turbulence data is therefore difficult.

9. Conclusions

The analysis of PBL turbulence data obtained over land (Minnesota) and over ocean (Coral Sea) suggests the following forms for σ_w and σ_v in the convective mixed layer:

$$\sigma_w = (1.2u_*^2 + 0.35w_*^2)^{1/2},$$

$$\sigma_v = (3.6u_*^2 + 0.35w_*^2)^{1/2}. \quad (8)$$

These relations are constrained to conform with the requirements for neutral stability deduced independently from the Kansas experiment.

For σ_T, scaling by θ_* appears to be far superior to scaling by T_*. The value of σ_T/θ_* is not well determined, but appears to be about 1.8. The data do not appear to warrant a complexity paralleling (8) in this case.

The statistical analyses carried out here are confined to the interval $0.1 < z/z_i < 0.9$, yet Fig. 1 indicates that for σ_u and σ_v the results obtained should extend to within 4 m of the surface. Clearly the present results should not be expected to extend into the near-surface region where effects of individual surface roughness elements or surface discontinuities can be detected, however it is equally obvious that the PBL w_*-scaling provides an excellent description of horizontal-component turbulence characteristics well below levels normally considered to be in the "surface boundary layer."

Surface boundary layer relations are frequently assumed to extend upwards from the surface to a height such that $-z/L \approx 1$. However, the results summarized in Table 3 indicate that w_* contributions to σ_w outweigh those of u_* throughout the unstable PBL, provided $-z_i/L > 4.0$ approximately.

From the individual data listing of Table 1, the averages of Table 2 and statistical analyses like those leading to Table 3, no major role for the Coriolis parameter f in explaining turbulence statistics for either velocity or temperature is indicated.

TABLE 4. Results of analyses yielding free-convection limits for σ_v and σ_w.

	Neutral		Convective	
Analysis	σ_v/u_*	σ_w/u_*	σ_v/w_*	σ_w/w_*
Present analysis	1.9	1.1	0.59	0.59
Deardorff and Willis (1975)	—	—	0.51	1.34
Misra (1982) (attributed to D. H. Lenschow)	—	—	0.4†	0.37
Nieuwstadt and van Duren (1979)	1.31	—	0.72	—
Venkatram (1980)	—	—	0.45†	~1.0
Lamb (1979)	—	—	0.33†	0.3†
Nieuwstadt (1980)	—	—	—	0.9†

† Inferred from σ_y and σ_z relationships.

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APPENDIX

Nomenclature

a, b, c	constants
A	constant
$b_{12.3}$	multiple regression coefficient
c_p	specific heat of air at constant pressure
f	Coriolis parameter
g	acceleration due to gravity
H	sensible heat flux
H_v	virtual heat flux
L	Monin–Obukhov length scale
LE	latent heat of evaporation
$r_{12.3}$	partial correlation coefficient
Ri	gradient Richardson number
T_*	temperature scaling parameter, surface
u, v, w	velocity components
u_*	friction velocity
w_*	convective velocity scale
x, y, z	Cartesian coordinates
z_i	mixed-layer depth
θ	potential temperature
θ_*	temperature scaling parameter, PBL
σ_T	temperature standard deviation
$\sigma_{u,v,w}$	velocity standard deviations
$\sigma_{x,y,z}$	plume dimension standard deviations
PBL	planetary boundary layer
SBL	surface boundary layer

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