The Schematics of Balance of Forces in the Planetary Boundary Layer

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ABSTRACT

It is pointed out that in a rotating frame of reference, the friction force on a fluid element may not be parallel and opposite to the velocity vector, as commonly depicted in textbook schematics of the force balance in the friction layer. A better schematic representation of the same is suggested here.

In explaining the effect of friction on velocity distribution in the planetary boundary layer (PBL), authors of many elementary meteorology textbooks, and even some advanced texts on dynamic meteorology, have used schematics of the balance of forces at one or more levels in the PBL. These show a balance between the pressure gradient (**P**), Coriolis (**C**) and friction (**F**) forces on a fluid element in the PBL whereby, intentionally or inadvertently, the friction force is always shown opposite to the direction of the velocity vector (e.g., see Miller and Thompson, 1979, p. 101–102; Ahrens, 1982, p. 277; Neiburger *et al.,* 1982, p. 203; Navarra, 1979, p. 147; Battan, 1979, p. 73; Hess, 1959, p. 179; Atkinson, 1981, p. 19; Holton, 1979, p. 110). This may lead readers to believe that the friction force acts opposite to the direction of motion even in a rotating frame of reference and in the presence of directional wind shear. However, this can be shown to be inconsistent with the equations of horizontal motion given below in the absence of local and advective accelerations,

$$\underset{\textbf{(C)}}{2\rho\boldsymbol{\Omega} \times \mathbf{V}} = \underset{\textbf{(P)}}{-\nabla p} + \underset{\textbf{(F)}}{\partial\boldsymbol{\tau}/\partial z}. \qquad (1)$$

Here $\mathbf{V}$, ∇p and $\boldsymbol{\tau}$ denote the mean velocity, pressure gradient and shear stress vectors, respectively, ρ is the mass density of the fluid, $\boldsymbol{\Omega}$ the earth's rotational velocity, and the forces per unit volume of the fluid element are indicated below the corresponding terms in Eq. (1).

The misleading depiction of friction force in most published schematics of the balance of forces in the PBL arises, perhaps, from a mistaken identification of the net friction force ($\partial\boldsymbol{\tau}/\partial z$) on an element of finite depth Δz with the shearing stress ($\boldsymbol{\tau}$) on a particular horizontal plane (e.g., one face of the element), although even the latter need not be parallel to the velocity vector in the presence of directional shear. Assuming that $\boldsymbol{\tau} = K_m \partial\mathbf{V}/\partial z$, where K_m is eddy viscosity, the friction force is more closely related to $\partial^2\mathbf{V}/\partial z^2$ and cannot, in general, be expected to remain parallel and opposite to $\mathbf{V}$ in the PBL. It can, however, be expected to remain normal to the ageostrophic wind vector ($\mathbf{V} - \mathbf{V}_g$), everywhere in the PBL, which can be seen by rewriting Eq. (1) in the form

$$2\rho\boldsymbol{\Omega} \times (\mathbf{V} - \mathbf{V}_g) = \partial\boldsymbol{\tau}/\partial z. \qquad (2)$$

Only in unidirectional flows can the friction force be expected to remain exactly opposite to the direction of motion.

Figure 1 gives schematics of the balance of forces at different levels in a steady, horizontally-homogeneous, barotropic PBL, which are consistent with the equations of motion. At the surface, where $\mathbf{V} = 0$, the friction force must be equal and opposite to the pressure gradient force. The former makes a large angle with the direction of the surface shear stress, as shown in Fig. 1(a). In the surface layer, as wind speed increases with height without any significant change in wind direction, the magnitude of the Coriolis force increases, but its direction remains unchanged (normal to $\mathbf{V}$). Consequently, as height increases, the end of the friction force vector must move up along the dashed line in Figure 1(b). This, of course, contradicts the concept of the surface layer being a constant stress layer in the strict sense (Tennekes, 1982).

Above the surface layer, as wind veers with height, the Coriolis force vector is expected to rotate anticyclonically, while its magnitude gradually varies in response to changes in the wind speed with height. The force balance then requires the friction force to rotate anticyclonically also and generally decrease in magnitude with increasing height. Figure 1(c) shows the force balance somewhere near the middle of the PBL where winds are often supergeostrophic. Near the top of the PBL, the friction force becomes small in magnitude, but quite variable in direction in response to

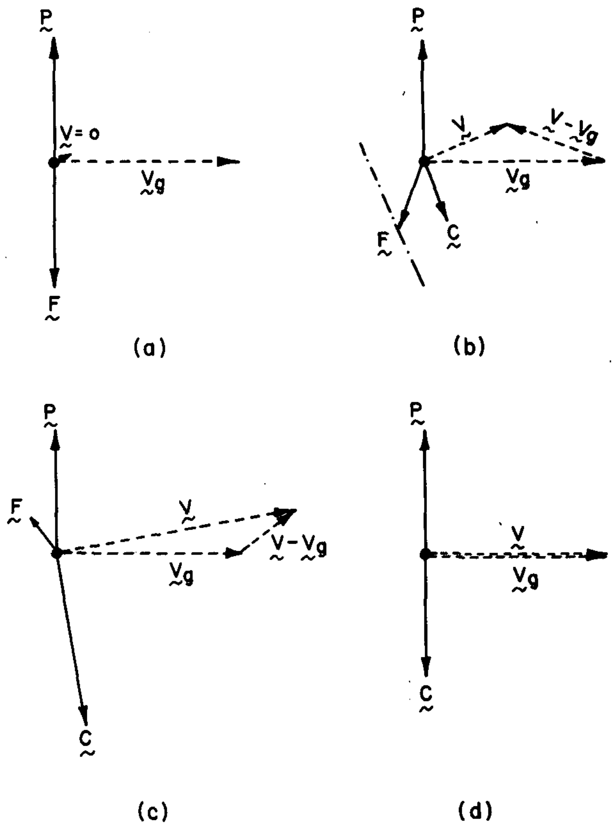

FIG. 1. The schematics of the balance of forces in the PBL: (a) at the surface; (b) in the surface layer; (c) in the middle of the PBL; (d) at the top of the PBL.

the changes in the direction of $\mathbf{V} - \mathbf{V}_g$, as velocity oscillates around the geostrophic equilibrium. Ignoring any such oscillations, the force balance at the top of the PBL is depicted in Fig. 1(d).

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