NOTES

Comparison of Actual Dividing-Streamline Heights to Height Predictions using the Froude Number

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1. Introduction

A series of papers by Long (1953, 1954, 1955) discussed the relationship between the Froude number and the structure of flow encountering an obstacle. Long predicted that for a Froude number less than 0.32, leewaves, unstable eddies and upwind blocking would occur. Later, Sheppard (1956) started the discussion of how to determine whether flow would go over or around an obstacle. Additional papers such as Drazin (1961), Kao (1965), Hunt et al. (1979) and Snyder (1980) have expanded the discussion concerning use of the Froude number for the prediction of stable flow structure in complex terrain. Ryan and Lamb (1984) present a thorough discussion of Froude number for the prediction of dividing-streamline height with comparison to data obtained at Steptoe Butte. An excellent summary of research on Froude numbers and the concept of the dividing-streamline height (Hd) can be found in Snyder et al. (1985).

The dividing streamline represents a division between two regions of flow that occur in stable stratified flow around a terrain obstacle. Below the dividing streamline, the flow has insufficient energy to surmount the terrain feature and consequently passes around it in a horizontal plane. Above the dividing streamline, the flow passes up and over the terrain feature. A stagnation streamline exists below the dividing streamline but not above it. The concept of the dividing streamline can be illustrated and predicted using the Froude number:

$$F = \left[\frac{eU^2}{-\frac{\partial e}{\partial Z} g(\Delta H)^2} \right]^{1/2} \tag{1}$$

where

e density of the air
g acceleration of gravity
ΔH height to top of terrain
$\frac{\partial e}{\partial Z}$ appropriate density gradient in the vertical
U wind speed

This note examines five ways to utilize the Froude number concept in prediction of dividing-streamline heights and examines the accuracy of the methods by comparison to actual observations.

2. Data discussion

In 1980, the U.S. Environmental Protection Agency (EPA) funded a field program called the Small Hill Impaction Study. Briefly, the program used tracer gases to study the behavior of plumes forced to encounter a small 100 m high hill in southern Idaho called Cinder Cone Butte. The program is described in Holzworth (1980), Spangler and Taylor (1982) and Venkatram et al. (1982), and is depicted in Fig. 1. A number of reports are available on the study, including Lavery et al. (1981) and Strimaitis et al. (1982). In an analysis from Spangler (1986), seven 1-h cases were selected from the database of 100 experimental hours. The cases, all nighttime, represented hours when it was reasonably certain that the tracer gas plume was released below the dividing-streamline height and on the stagnating streamline. These seven 1-h cases can be used to assess a variety of ways to apply the Froude number in the prediction of the dividing-streamline height.

Four Froude numbers were calculated for the seven cases. The four types of Froude numbers are

1) Release-height Froude number (Fr) uses the tracer gas release-height wind, the gradient of density between the release height and the hilltop, and the difference in height between the release height and the hilltop. This Froude number is used directly as an indicator of whether the dividing streamline is above or below the plume height. Release-Height Froude numbers less than 1 indicate the flow at that level has insufficient energy to surmount the terrain obstacle.

2) Bulk Froude number (Fb) uses the wind speed at the 80 m level, the density difference between 10 m and the hilltop, and the total elevation difference between the flat terrain upwind of the terrain and the hilltop. This Froude number must be used in Eq. (2) to actually predict the height of the dividing streamline.

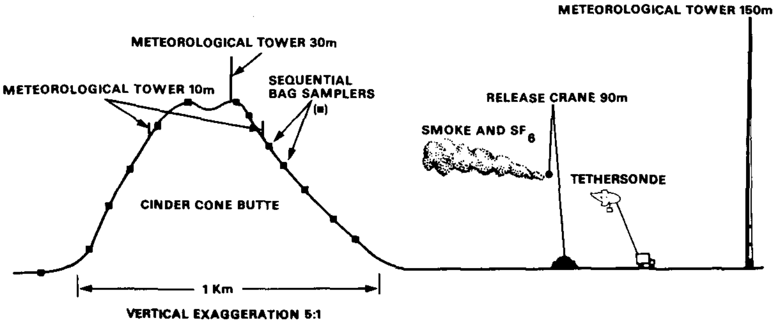

FIG. 1. Depiction of study using tracer gas to study plume behavior.

A Froude number greater than unity means that no dividing streamline exists.

3) Draw Froude number (Fd) in reference to a saddle or draw at the top of the hill used in the experiment. The draw between two peaks of equal elevation was about 15 m lower than the hill top and could allow air to flow over the hill with less loss of kinetic energy. This number is identical to the release-height Froude number (Fr) except that it uses the height to the draw rather than the very top of the hill. Thus, a Froude number greater than unity implies that the flow will go over the terrain through the draw.

4) Modified release-height Froude number (Fm) is the same as Fr except that the wind at the height of the terrain top is used as a way of incorporating a wind speed closer to the free stream value.

Visual and quantitative tracers suggest that in all seven cases, the plume was actually below the dividing streamline. This fact will allow the comparison of different techniques to predict the dividing-streamline height and the selection of the best approach.

The first technique is to examine whether Fr, Fd and Fm can predict whether the plume was above or below the dividing streamline (see Table 1). Of the three types of Froude numbers, only the release-height Froude number (Fr) correctly predicted that the plume would be below Hd in all seven cases. The draw Froude number (Fd) indicated that for cases 1, 6 and 7, the plume could surmount the terrain through the draw and might not be forced to go around the terrain. Actual observations indicate that some portion of the plumes may have gone through the draw on cases 6 and 7, but the essential features of below dividing-streamline flow were maintained. In every case, the plume was passing around both sides of the hill simultaneously for at least a portion of each hour. The modified release-height Froude number, using winds from near the level of the terrain height, incorrectly predicted the height of the dividing streamline in six of the seven cases.

The Bulk Froude number (Fb) can be used to predict the actual height of the dividing-streamline. The method involves replacing the value of *H* which was the distance from the release height to the top of the hill with the distance from the flat terrain upwind of the hill to the top of the hill. The resulting Froude number is used in an equation first suggested by Hunt et al. (1979):

$$\mathrm{Hd} = H(1\text{-}\mathrm{F}) \qquad (2)$$

where

Hd dividing-streamline height
F Froude number

This relationship is expected to be appropriate when uniform winds and stratification exist in the approach flow. The use of Fb in Eq. (2) correctly predicted that the tracer plume would be below the dividing streamline on only two of the seven cases (see Table 2 also).

TABLE 1. Froude numbers for seven selected cases.

Case	Release height (m)	Fr	Fb	Fd	Fm
1	30	0.76	0.77	1.00	1.25
2	30	0.73	0.59	0.96	1.05
3	30	0.45	0.33	0.58	0.49
4	20	0.54	1.29	0.66	1.74
5	20	0.48	1.12	0.60	1.58
6	24	0.84	1.23	1.07	1.79
7	30	0.90	1.17	1.16	1.85

TABLE 2. Comparison of integrated dividing-streamline estimates relative to heights estimated from isentropic analysis.

Case	Release* height (m)	Isentropic height (m)	Graphical height (m)	Bulk Froude number approach (m)
1	30	40	36	23
2	30	41	38	41
3	30	67	55	67
4	20	54	33	0
5	20	41	33	0
6	24	31	27	0
7	30	32	32	0

* Known to be below Hd.

The principal purpose for calculating Froude numbers in the context of dispersion modeling is to predict the type of flow pattern for stable plumes in complex terrain. Plumes below it fan into a wide wedge and go around both sides of the obstacle simultaneously. The Release-height Froude number performed well in predicting the flow pattern. Predicting the height of Hd however was not performed well using the bulk Froude number (Fb). A better technique was suggested by Snyder (1980) that involves integrating down from the height of the terrain to the level of the dividing streamline. The equation is as follows:

$$\frac{eU^2}{2}=g\int_{\mathrm{Hd}}^{h}(h-Z)\left(-\frac{\partial e}{\partial Z}\right)dZ \qquad (3)$$

where

h height of terrain
e density at height Z
u wind at height Z
Z height above ground
$\frac{\partial e}{\partial Z}$ density gradient from Z to h

To evaluate this technique, an independent estimate of the dividing-streamline height is needed. Two instrumented towers were in place during the Small Hill Impaction Study. The first was 150 m high and located about 1 km north of Cinder Cone Butte. The second tower was 30 m high and located on top of the butte itself. Both towers were instrumented at multiple levels and provided a measure of potential temperature. It was assumed that air passing through the 2 m level of the hill top tower originated at approximately the dividing-streamline height. Thermal effects from the hill are neglected because the air was near the hill surface for only a very brief time. By comparing the potential temperature at the 2 m level to the profile of potential temperature in the upwind flow, the actual height of the dividing streamline was estimated. This approach is illustrated in Fig. 2. The results of the analysis are presented in Table 2.

The meteorological data collected at Cinder Cone Butte were at discrete levels on a 150 m tower, making a true integration such as Eq. 3 impossible. A graphical technique was employed as an alternative. The technique involved calculating the release-height Froude number at every discrete level where data were available and plotting the points versus height. The level where the Froude number equaled unity was designated the dividing-streamline height. The results of this analysis are also presented in Table 2. The graphical integration technique performed much better than the bulk Froude number approach. The average difference from the isentropic height was 7.6 m.

3. Discussion

In physical modeling and in simple fluid flow problems the dividing-streamline height is often calculated using the velocity in uniform approach flow, and the effective terrain height is calculated from the height of a symmetrical object in the flow. As situations become more complex, the appropriate method for predicting the behavior of flow becomes more difficult. In section 2, five techniques were tested that can be used to predict whether a particular flow level will be above or below the dividing streamline. The results of physical modeling studies presented in Snyder et al. (1985) would suggest that the integration technique [Eq. (3)] is the most appropriate, and that the terrain height should be referenced to the lowest terrain point such as the draw or saddle at Cinder Cone Butte.

The results of the simple analyses presented in this note would suggest a number of conclusions that differ from approaches used in simple fluid flow situations. For predictions of whether the air at a particular level would be prevented from surmounting a terrain obstacle, the release-height Froude number was found to be a correct predictor in all cases. The modified release-height Froude number using a velocity closer to free stream was not a good predictor.

The choice of effective terrain height is also an important component of the Froude number calculation. While some plume material was observed in the Cinder Cone Butte draw during several of the seven cases, the vast majority of the flow was nearly horizontal and

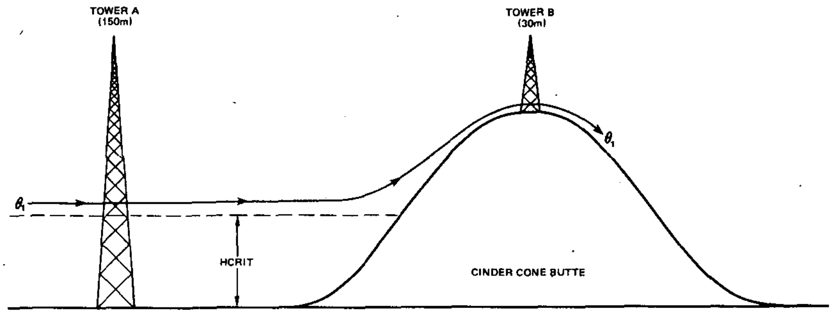

FIG. 2. Depiction of isentropic Hd analysis.

forced to go around both sides of the hill simultaneously. This suggests that the lowest point of the terrain should not be used to predict the general structure of stable flow upwind of a terrain obstacle.

Two methods of predicting the actual dividing-streamline height were tested: a method using the Bulk Froude number and a graphical technique approximating the integration technique [Eq. (3)]. Clearly the integration technique is preferred because it considers only atmospheric structure from the terrain obstacle to the dividing-streamline height, the layer that must be lifted to surmount the obstacle. The comparison between the graphical integration technique and the estimated actual height was quite good.

4. Summary and conclusions

Using seven cases with below dividing-streamline plumes impinging on complex terrain, a number of methods for predicting the dividing-streamline height and the general character of the stable flow were tested. The determination of whether a particular level of the flow would go over or around the terrain was best accomplished using a Froude number, calculated with the wind speed at the plume level and the atmospheric structure from the plume to the terrain height. It was also found that using the lowest terrain height such as a draw or saddle was not a good predictor of upwind flow structure. Actual dividing-streamline heights were best predicted by integrating downward from the terrain height to the level where the energy in the flow equaled the kinetic energy given up in surmounting the terrain.

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