Experiments on the Cloud Droplet Collision Efficiency of Cylinders

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ABSTRACT

The theory of Langmuir and Blodgett for the droplet collision efficiency was verified by growing rime ice accretions on rotating cylinders in a wind tunnel. The results show that the theory is in excellent agreement with the experimental data in the studied range of mean cylinder collision efficiency from 0.07 to 0.63.

1. Introduction

The growth of graupel and hailstones and of ice accretions on structures is largely controlled by the cloud droplet collision efficiency (Macklin and Bailey, 1966, 1968; Beard and Pruppacher, 1971). Albrecht (1931) showed that when the collector is sufficiently large, so that electrical forces and interception are insignificant, the droplet trajectories around the collector are determined by the balance between the droplet inertia and the drag force experienced by the droplets in the air flow. Langmuir and Blodgett (1946) presented a relationship between the droplet drag coefficient and droplet Reynolds number and derived the classical theory of calculating the droplet trajectories and the collision efficiency by solving (with a differential analyzer) the equations of motion in a potential flow.

Since its publication, the theory of Langmuir and Blodgett (1946; hereafter LB) has been the basis of calculations of collision efficiencies, not only in cloud physics, but also in many other applications, such as aerosol studies (Ranz and Wong, 1952) and icing of aircraft and stationary structures (e.g., Lozowski et al., 1983a; Makkonen, 1984). Theoretical calculations of the collision efficiency have also been used as a basis for measuring the liquid water content (Ashworth and Knight, 1978; Rogers et al., 1983) and droplet size (Brun et al., 1955) in clouds. Recently, new numerical schemes have been developed in order to improve the accuracy of calculating the collision efficiency by the theory using modern computers (McComber and Touzot, 1981; Lozowski and Finstad, 1985; Finstad et al., 1987).

Comparisons by Ranz and Wong (1952) between the LB theory and experimental data for small aerosol particles showed reasonable agreement, but considering that the theory of LB is so widely used in cloud physics, it is surprising that it has not been well verified experimentally under conditions of water droplet impingement. In fact, the only experimental study of the validity of the LB theory in water droplet accretion, known to the authors, is that of Macklin and Bailey (1968), in which it was concluded that the theory tends to overestimate the mean collision efficiency of a sphere. This study was hampered, however, by the errors that are involved in measuring the droplet size by the oil slide method (Makkonen and Stallabrass, 1984).

Although the approach by LB is theoretically sound, there are factors that could reduce its practical applicability in calculating the collision efficiency. These include the effects of surface roughness and viscosity on the flow field around the object (List, 1977) and the changes in the droplet drag coefficient due to accelerating flow (Temkin and Mehta, 1982) and turbulence (Torobin and Gauvin, 1960). Therefore, there is clearly a need to experimentally verify the LB theory using well-controlled conditions and the latest measurement techniques. In this study, ice accretions were grown on rotating cylinders in a wind tunnel, and the resulting collision efficiencies are compared with the collision efficiencies modeled numerically, based on the LB theory.

2. Experimental procedures

a. *Icing wind tunnel*

The icing experiments were made in the icing wind tunnel of the Low Temperature Laboratory, National Research Council of Canada. A description of this facility is given in Lozowski et al. (1983b).

A single atomizing spray nozzle placed on the tunnel centerline was employed. The test results were based on average measurements made in the center 10 cm of the 30.5 cm × 30.5 cm test section where the liquid water content varied insignificantly in space.

Since tunnel blockage effects in some of the tests were expected to be present, plenum chambers with perforated walls were installed in place of the test section floor and ceiling, to give a test section porosity of 10%. Two tests with 25% blockage, with and without the porous plenum chambers, but otherwise with identical conditions, showed no significant difference in the icing characteristics on a 7.61 cm diameter cylinder at 36 m s^{-1} airspeed, suggesting that blockage effects do not have a significant effect on droplet trajectories in the conditions of the tests. However, the perforated walls were used for all tests.

b. Determination of the test conditions

A water micromanometer was used to measure the dynamic pressure in the tunnel test section. The error in the wind velocity derived from the dynamic pressure was ±0.15 m s^{-1}. At all test velocities, the turbulence intensity in the tunnel working section was 3.5 ± 0.2%.

The total temperature of the tunnel air was controlled and measured by a thermostat, which controlled the temperature in the test section to better than ±0.3°C of the set point. The static temperature in the test section (as listed in Table 1) was lower than the total temperature, owing to the adiabatic expansion of the air accelerating within the contraction; the depression was about 0.2°C at 20 m s^{-1} and 0.65°C at 36 m s^{-1}.

Liquid water content was measured using the single rotating cylinder method (Stallabrass, 1978). Tests were run at a series of spray nozzle settings of water flow rate and atomizing air pressure, estimated from prior calibrations to give liquid water contents of about 0.15 and 0.3 g m^{-3} at various droplet sizes. At the chosen spray nozzle settings, confirmatory rotating cylinder measurements were made at tunnel air temperatures lower than −10°C to ensure that the Ludlam limit was not exceeded (Fraser et al., 1953). The liquid water content values employed were the mean of several

TABLE 1. Test conditions and results for the rotating cylinder experiments.

Initial cylinder diameter (cm)	Ultimate cylinder diameter (cm)	Test duration (min)	Wind velocity (m s^{-1})	Air temperature (°C)	Liquid water content (g m^{-3})	Median volume droplet diameter (μm)	Droplet size distribution category (see text)	Ice mass (g/10 cm)	Experimental collision efficiency E_{exp}	Theoretical collision efficiency E_{theory}
1.024	1.55	30	20	−4.5	0.36	17.1	3	9.34	0.56	0.53
1.024	1.48	30	20	−4.5	0.35	14.4	3	7.56	0.48	0.46
1.024	1.42	30	20	−9.5	0.35	14.4	3	6.31	0.41	0.47
1.024	1.60	30	20	−19.3	0.35	14.4	3	7.85	0.46	0.45
1.024	1.43	31	20	−4.5	0.33	13.1	2	6.54	0.46	0.43
3.183	3.65	40	20	−4.5	0.36	17.1	3	18.70	0.32	0.30
3.183	3.54	40	20	−4.5	0.35	14.4	3	14.18	0.25	0.22
3.183	3.50	40	20	−9.5	0.35	14.4	3	12.44	0.23	0.23
3.183	3.70	40	20	−19.3	0.35	14.4	3	13.46	0.24	0.22
3.183	3.47	40	20	−4.5	0.33	13.1	2	10.55	0.20	0.20
4.440	4.85	50	20	−4.5	0.36	17.1	3	23.17	0.23	0.24
4.440	4.81	50	20	−4.5	0.35	14.4	3	17.83	0.19	0.17
4.440	4.82	50	20	−9.5	0.35	14.4	3	16.77	0.18	0.17
4.440	5.05	50	20	−19.5	0.35	14.4	3	19.03	0.20	0.17
4.440	4.71	50	20	−4.5	0.33	13.1	2	11.91	0.14	0.14
7.609	7.95	50	20	−4.5	0.36	17.1	3	29.6	0.18	0.14
7.609	7.98	50	20	−9.5	0.36	17.1	3	31.8	0.19	0.14
7.609	7.81	50	20	−4.5	0.35	14.4	3	15.2	0.10	0.09
7.609	7.89	50	20	−9.5	0.35	14.4	3	17.3	0.11	0.09
7.609	8.13	50	20	−19.3	0.35	14.4	3	19.1	0.12	0.09
7.609	7.80	50	20	−4.5	0.33	13.1	2	10.9	0.07	0.06
1.024	1.48	30	36	−4.9	0.15	15.7	1	7.42	0.63	0.62
1.024	1.40	30	36	−4.9	0.15	13.4	1	6.38	0.56	0.56
1.024	1.35	30	36	−4.9	0.14	12.2	1	5.39	0.50	0.51
3.183	3.60	40	36	−4.9	0.15	15.7	1	19.07	0.45	0.39
3.183	3.50	40	36	−4.9	0.15	13.4	1	13.59	0.33	0.32
3.183	3.45	40	36	−4.9	0.14	12.2	1	11.11	0.28	0.27
4.440	4.86	50	36	−4.9	0.15	15.7	1	26.65	0.35	0.32
4.440	4.75	50	36	−4.9	0.15	13.4	1	17.75	0.25	0.25
4.440	4.66	50	36	−4.9	0.14	12.2	1	12.30	0.18	0.20
7.609	7.91	50	36	−4.9	0.15	15.7	1	27.4	0.22	0.22
7.609	7.78	50	36	−4.3	0.15	13.4	1	16.7	0.14	0.14
7.609	7.77	50	36	−4.9	0.14	12.2	1	12.5	0.11	0.11

measurements (4 to 10) at each spray nozzle setting, and had a standard deviation of ± 0.02 g m^{-3}.

Droplet size measurements were made in the wind tunnel with the Forward Scattering Spectrometer Probe (FSSP), manufactured by Particle Measurement Systems, Boulder, Colorado, and loaned to us by the Atmospheric Environment Service of Canada. This FSSP had undergone considerable calibrations and was one of the subject probes (serial No. 2) in the studies reported by Dye and Baumgardner (1984), thus engendering confidence in the accuracy of its measurements.

The instrument was modified in such a way that the sensor viewed sideways; therefore, when installed, the bulk of the instrument was outside the wind tunnel. Droplet size distribution measurements with the FSSP were made at a temperature just above 0°C to avoid icing of the sensor. Additional measurements using the traditional oil slide method at this temperature and the colder temperatures of the cylinder icing experiments revealed no dependence of the size distribution on temperature. The FSSP measurements were consequently considered to be valid at the temperatures of the icing experiments.

Measurements of the droplet size distributions were made at four wind velocities and nozzle settings. The icing tests on rotating cylinders were made at two wind speeds (20 and 36 m s^{-1}) and three nozzle settings. However, no FSSP measurements were available for the exact nozzle settings used in the icing experiments at 20 m s^{-1}. At this velocity the size distributions were derived from the data at other velocities (15, 30 and 62 m s^{-1}), and also at 20 m s^{-1} but with different nozzle settings. Consequently, the icing test data can be divided into three categories in relation to their accuracy of the droplet size distributions:

1) Data for which the size distribution was measured at the wind velocity and nozzle setting used in the test (12 cases);
2) Data for which the size distribution was interpolated on a channel-by-channel basis from the data at the nozzle setting used but at other wind velocities (four cases);
3) Data for which the size distribution was extrapolated using the relationship between the size distribution and velocity as measured at other nozzle settings (seven cases);

Under the conditions of the tests the effect of the wind velocity on the droplet size distribution was very small. In terms of the median volume diameter (MVD) the difference between 15 and 30 m s^{-1} was around 1 μm. Therefore, it is believed that the interpolation and extrapolation procedures have caused only a very small error in the size distributions. Nevertheless, the three categories are separated in the analysis of the data. Each data point is also identified by its category in Table 1, which shows the test conditions.

c. Rotating cylinders

Icing tests were made on horizontally mounted rotating cylinders of four different diameters, 1.024, 3.183, 4.440 and 7.609 cm. The speed of rotation was 2 rpm. To facilitate measurements—particularly that of weighing a known length of the ice accretion—the cylinders were made in three parts, such that the central part (with 10 cm length) could be separated from the two outer parts.

The duration of the test runs were chosen to give a relatively small, yet measurable ice thickness. Thus, times of 30, 40 and 50 min were chosen for the 1.024, 3.183 and 4.440 cm diameter cylinders, respectively, giving ice thicknesses of between about 1 and 3 mm. A 50-min duration was chosen for the 7.609 cm diameter cylinder also, resulting in slightly smaller ice thicknesses (i.e., between about 0.6 mm and 2.5 mm).

The overall diameter of the ice deposits on the three smaller cylinders was measured with a cooled vernier caliper, while a micrometer was used for the largest cylinder. The ice mass was determined by weighing the central part of the cylinder together with its accumulated ice and subtracting the weight of the cylinder. The three smaller cylinders were measured on a precision balance accurate to 1 mg. The largest cylinder was too heavy for this balance and was weighed on a balance scaled in 1 g increments, but visually interpolated to 0.1 g.

3. Calculation of the collision efficiencies

a. Experimental

The experimental collision efficiency E_{exp} was determined from the equation

$$E_{exp} = \frac{M}{vwDL\tau} \quad (1)$$

where M is the mass of ice on the cylinder, v the wind velocity, w the liquid water content, D the mean cylinder diameter during the experiment, L the length of the ice sample (10 cm in the experiments) and τ the test duration.

b. Theoretical

The theoretical collision efficiencies of LB are based on the solution for the dimensionless equation of motion of a droplet in a potential flow around a cylinder. Langmuir and Blodgett obtained their data using an early type of analog computer. Various functional fits to tabulated LB data are available. Recently, however, Finstad et al. (1987) recalculated the collision efficiency data by integrating the droplet trajectories on a modern computer. They also substituted the droplet drag coefficients by Beard and Pruppacher (1969) for those originally used by LB.

Under the conditions of the present icing tests, the analytical expression given by Finstad et al. (1987) to approximate the new recalculated collision efficiencies is

$$\left.\begin{aligned} E_{\text{theory}} &= A - 0.028 - C(B - 0.0454) \\ \text{where} & \\ A &= 1.066K^{-0.00616}\exp(-1.103K^{-0.688}) \\ B &= 3.641K^{-0.498}\exp(-1.497K^{-0.694}) \\ C &= 0.00637(\phi - 100)^{0.381} \end{aligned}\right\} \quad (2)$$

In Eq. (2), K is the nondimensional inertia parameter (Stokes number)

$$K = \rho_w v d^2/9\mu D \quad (3)$$

and

$$\phi = \mathrm{Re}^2/K \quad (4)$$

with the droplet Reynolds number based on the free stream velocity v:

$$\mathrm{Re} = \rho_a d v/\mu. \quad (5)$$

Here, d is the droplet diameter, D the cylinder diameter, ρ_w the water density, μ the absolute viscosity of air, and ρ_a the air density.

The theoretical collision efficiency was determined by Eq. (2) separately for each droplet size category. Finally, the theoretical mean collision efficiency E_{theory} was derived as the sum of the collision efficiencies for each size category multiplied by the fraction of the total liquid water content represented by that size.

4. Results and discussion

The test conditions and the results for the rotating cylinder experiments are given in Table 1. The number of the rotating cylinder tests with separate experimental conditions is 33. In a few cases, two or three tests were run to check the repeatability of the experiments. In these cases, the mean values are given in Table 1 and are used in the analysis of the results. The deviation in the resulting ice amount in the tests that were repeated was within 15%. These deviations originate from measurement errors and the fact that, although the test conditions were continuously monitored, there may have been small variations in the spray nozzle air and water pressures.

All the experiments on the rotating cylinders were made in dry growth conditions where the resulting ice was rime and no shedding from the cylinder took place. The relative increase in the cylinder diameter over the course of each test was low so that the time dependence in the experiments was small.

Photographs of the ice accretions on a cylinder at two temperatures are shown in Fig. 1. The ice surface was generally quite smooth. At lower air temperatures, however, clearly visible roughness elements started to form on the surface (Fig. 1, lower panel).

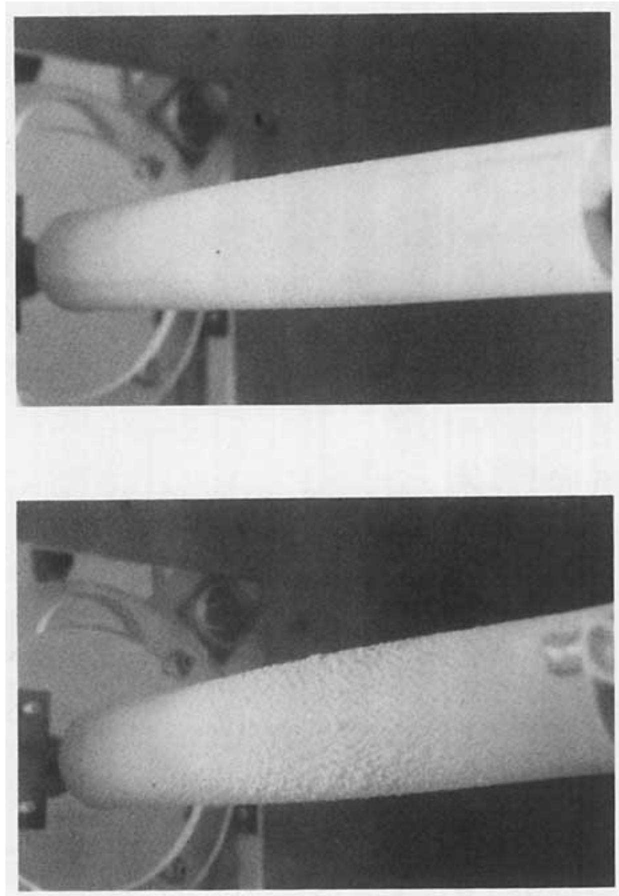

FIG. 1. Icing on a 4.44 cm diameter cylinder at the air temperature of −9.5°C (above) and −19.5°C (below). Wind velocity is 20 m s^{-1}, liquid water content is 0.35 g m^{-3} and median volume diameter of the droplets is 14.4 μm.

The mean collision efficiency based on the measured ice amounts is plotted against the collision efficiency predicted by the theory [Eq. (2)] in Fig. 2. Figure 2 shows excellent agreement between the experimental results and the theory. The standard error of the theoretical E computed with respect to experimental data is 0.02, and the linear correlation coefficient between the two collision efficiencies is 0.99. As a conclusion, the basic theory of the collision efficiency of cloud droplets by Langmuir and Blodgett (1946) is quantitatively verified in the studied range of E from 0.07 to 0.63. Considering this result and the assumptions made in the LB theory, it is likely that the theory is valid at values of E higher than 0.63, as well. At lower values of E, however, air turbulence, surface roughness, electrical forces and other factors not included in the theory, may become more significant (Ranz and Wong, 1952; Finstad et al., 1987).

The theoretical collision efficiencies for these data were also calculated using the original data by LB. The resulting agreement with the experimental values was

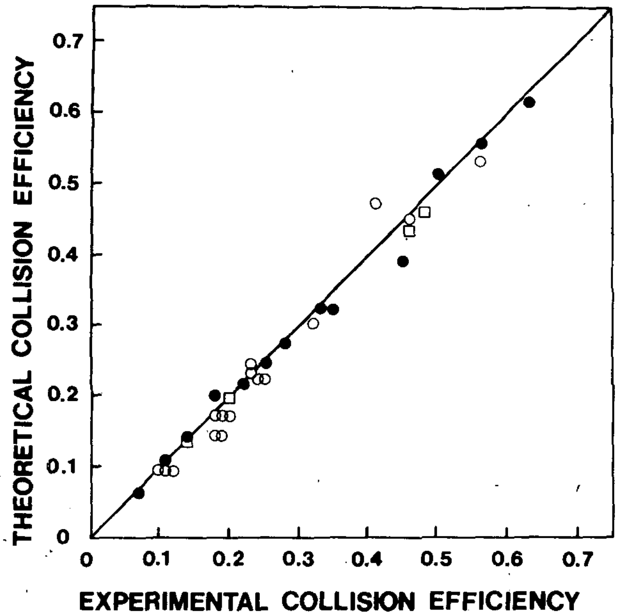

FIG. 2. Mean collision efficiency on rotating cylinders as predicted by the theory [Eq. (2)] versus the experimental mean collision efficiency. Test conditions are given in Table 1. Droplet size distributions measured: solid circle, interpolated; square, extrapolated; open circle, see text.

poorer than that in Fig. 2. This suggests that the recalculated data by Finstad et al. (1987) should be used for all future cylinder impingement studies.

From a practical point of view, it is interesting to note that air temperature does not seem to significantly affect the collision efficiency, although the surface of the icing cylinder became clearly rougher at low temperatures, as shown by Fig. 1. This indicates that the mean collision efficiency by the LB theory is a useful tool in studying icing in natural conditions, in which resulting ice accretion surfaces are often rough. The good agreement between the theoretical and experimental ice accretion weights on simulated transmission line cables, found by Makkonen and Stallabrass (1984), also supports this conclusion.

The good agreement between the theoretical and measured collision efficiencies is an indirect verification of the droplet trajectory calculations. Therefore, the results of this study also suggest that the droplet impact speeds calculated by the LB theory (e.g., Rasmussen and Heymsfield, 1985) are reliable. This is significant, because the density of rime depends on the droplet impact speed (Macklin, 1968). Rime density, on the other hand, affects the amount of ice accreted within a given time interval (Makkonen, 1984).

The results of this study show that the various attempts that have been made to model the ice accretion process using the LB theory are well justified. Strictly speaking, the results are, however, limited to the mean collision efficiency of the cylinder, and more experimental studies should be made on objects of other shapes, as well as on local variations of the collision efficiency on the object surface. The validity of the LB theory at values of E lower than those included in this study should also be examined, although it should be noted that the use of LB theory in practice is limited by the high sensitivity of E to the errors in the input parameters at very low collision efficiencies.

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