Mapping Frost-Sensitive Areas with a Three-Dimensional Local-Scale Numerical Model.  
Part I: Physical and Numerical Aspects

R. Avissar  
Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado

Y. Mahrer  
Seagram Centre for Soil and Water Sciences, Faculty of Agriculture, Hebrew University of Jerusalem, Rehovot 76100, Israel

(Manuscript received 28 March 1987, in final form 23 October 1987)

ABSTRACT

Radiative frost is one of the most severe weather conditions that affects agricultural activities in many parts of the world. Since various protective methods to reduce frost impact are available, refinements of frost forecasting methodologies should provide economical benefits.

In the present study, a three-dimensional numerical local-scale model for the simulation of the microclimate near the ground surface of nonhomogeneous regions during radiative frost events was developed. The model is based on the equations of motion, heat, humidity and continuity in the atmosphere, and the equations of heat and moisture diffusion in the soil. Emphasis was given in establishing a refined formulation of energy budget equations for soil surface and plant canopy. Additionally, an improved finite difference scheme procedure for approximating horizontal derivatives in a terrain-following coordinate system was introduced.

The sensitivity of the model to various parameters that may affect the nocturnal minimum temperature near ground surface during radiative frost events was tested by using one- and two-dimensional versions of the model. This temperature was found to be sensitive to topography, plant cover, soil moisture content, air specific humidity and wind velocity.

1. Introduction

Frost (especially radiative frost) is one of the most severe weather conditions that affects agricultural activities in many parts of the world. Despite its generally restricted spatial and temporal occurrence and impact on agriculture, frost may cause worldwide economic repercussions as was the case, for example, on the international coffee market after the devastating frosts on the Parana Plateau in Brazil in July 1975.

Various protective methods are available to reduce frost impact (Blanc et al., 1963; Turrell, 1973; Bagdounass et al., 1978). They fall into two main groups: direct or active methods and indirect or passive methods.

The direct methods are mainly aimed at improving the thermal regime of the surface layer of air at the ground and at decreasing the thermal radiation emitted by the soil and the vegetation. They can be divided into four subgroups:

(i) Methods by which radiation (cooling) from the surface of the soil or from plants can be decreased, including use of different types of cover (soil mulches, plastic tunnels), smoke, and the creation of artificial fog clouds.

Corresponding author address: Dr. Roni Avissar, Dept. of Atmospheric Science, Colorado State University, Fort Collins, CO 80523.

© 1988 American Meteorological Society
extended network of observations, especially in complex terrain, over a period of several years and, consequently, are very tedious and expensive. Additionally, the mapping of frost-sensitive areas between the observational sites is generally acquired by using subjective horizontal extrapolations, while neglecting the significant influence of the vegetation on the spatial temperatures (Avissar and Mahrer, 1988; hereafter referred to as Part II). For these reasons this method is not widespread even in frost-vulnerable regions. Since the late 1960s, airborne infrared thermography has been used in topoclimatological studies. Nixon and Hales (1975), Sutherland and Bartholic (1974) and Sutherland et al. (1981) have discussed its use in studies of nocturnal temperatures in orchards. Thermal imagery from satellites has also become available over the last decade and has been used to evaluate nocturnal surface temperatures (Chen et al., 1979; Wiegand et al., 1981; Chen et al., 1982; Kalma et al., 1983; Heinemann and Russo, 1987). However, its use in regional studies is still limited.

In the present study an alternative methodology to map frost-sensitive areas by using a three-dimensional numerical local (meso-γ)-scale model is suggested. This model simulates the microclimate near the ground surface of nonhomogeneous regions during radiative frost events.

The present paper is the first of two parts which describe the study. The model parameterization and sensitivity tests are presented here while the potential use of the model, emphasized by comparing three-dimensional simulations with an observational survey carried out in Hefer Valley (Israel), is provided in Part II.

2. The numerical model

a. Physical aspects

The formulation of the atmospheric module in the present local-scale model is similar to that of the numerical meso-scale model of Mahrer and Pielke (1975, 1977). However, it includes several refinements designed to improve resolution of aspects involved with the specific purposes of the present study.

1) BASIC EQUATIONS

\[
\frac{dq}{dt} = \frac{\partial}{\partial z} \left( K_{z} \frac{dq}{dz} \right) + \text{Fil}(q)
\]

(4)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(5)

\[
\frac{\partial \pi}{\partial z} = -\frac{g}{\theta}
\]

(6)

and

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}
\]

(7)

where \( u, v, \) and \( w \) are the east–west (\( x \)), north–south (\( y \)), and vertical (\( z \)) components of velocity, respectively; \( \Omega \) is the angular velocity of the earth; \( \psi \) the latitude; \( \theta \) the potential temperature; \( q \) the specific humidity; \( g \) the acceleration of gravity; \( \text{[\( \theta / \partial t \)\text{Rad]}} \) the radiative cooling/heating of the atmosphere; and \( \pi \), the Exner function, which is defined as follows:

\[
\pi = c_p (P/P_{\infty})^{\gamma / \gamma},
\]

(8)

where \( c_p \) is the air specific heat at constant pressure; \( R \) the gas constant for dry air; and \( P \) and \( P_{\infty} \) are pressure and the reference pressure, respectively.

2) DIFFUSION

(i) Horizontal diffusion. \( \text{Fil}() \) is a horizontal filter operating on the variable in parentheses which is applied in lieu of explicit horizontal diffusion. The filter of Pepper et al. (1979) is used in the present model:

\[
(1 - \delta) \phi_{i+1}^n + 2(1 + \delta) \phi_i^n + (1 - \delta) \phi_{i-1}^n = \phi_{i+1} + 2 \phi_i + \phi_{i-1},
\]

(9)

where \( \phi \) is the field to be smoothed and \( \phi^* \) the smoothed field. This filter, applied to control the spurious buildup of high-wavenumber energy due to aliasing, eliminates all \( 2 \Delta x \) waves at each application and waves \( 4 \Delta x \) and greater are smoothed according to the value of \( \delta \) (\( 0 < \delta < 1 \)).

(ii) Vertical diffusion in the surface layer. The vertical exchange coefficients of momentum, heat and moisture are given, in the surface layer, by

\[
K_{z}^m = k_0 u^* z/\phi_m(\xi)
\]

(10)

\[
K_{z}^q = k_0 q^* z/\phi_m(\xi),
\]

(11)

where

\[
\xi = z/L
\]

and \( L \), the length of Monin is

\[
L = \frac{\bar{u}^2}{k_0 \bar{v}^*},
\]

(12)

\( k_0 \) the von Karman constant; \( \bar{v}^* \) the mean vertical temperature profile of the surface layer; and \( u^* \) and \( \bar{v}^* \) the friction velocity and convective temperature, respectively. According to Businger et al. (1971), the expressions for the non-dimensional wind and potential temperature profiles are
\[
\phi_m = \begin{cases} 
(1 - 15\xi)^{-1/4}, & \xi \leq 0 \\
1 + 4.7\xi, & \xi > 0 
\end{cases} 
\]
\[
\phi_H = \begin{cases} 
0.74(1 - 9\xi^{-1/2}), & \xi \leq 0 \\
0.74 + 4.7\xi, & \xi > 0 
\end{cases} 
\]
where
\[
\phi_m = \frac{k_0 z}{u^*} \frac{\partial U}{\partial z}, \quad \phi_H = \frac{k_0 z}{\theta^*} \frac{\partial \theta}{\partial z},
\]  \hspace{1cm} (15)
with \( U \) being the horizontal wind velocity \( (\sqrt{u^2 + v^2}) \).

Integration of Eq. [15] yields the following relations:
\[
u^* = k_0 U / [\ln(z/z_0) - \psi_1],
\]  \hspace{1cm} (16)
\[
\theta^* = k_0 [\theta - \theta(z_0)] / [0.74[\ln(z/z_0) - \psi_2]],
\]  \hspace{1cm} (17)
\[
\psi_1 = \begin{cases} 
2\ln(1 + \phi_m^{-1})/2 + \ln(1 + \phi_m^{-3})/2, & \xi \leq 0 \\
-2 \tan^{-1}(\phi_m^{-1}) + \frac{\pi}{2}, & \xi > 0 
\end{cases}
\]  \hspace{1cm} (19)
and
\[
\psi_2 = \begin{cases} 
2\ln(1 + 0.74\phi_H^{-1})/2, & \xi \leq 0 \\
-6.35\xi, & \xi > 0 
\end{cases}
\]  \hspace{1cm} (20)
are derived from the above equations.

(iii) Vertical diffusion above the surface layer. Above the surface layer, the exchange coefficients were defined according to McNider and Pielke (1981). Following this procedure, the nature of the coefficients depends on the thermodynamical stability of the surface layer. When this layer is stable, local exchange coefficients suggested by Blackadar (1979) are used:
\[
K_z = K_z + [(z_1 - z)^2/(z_1 - z_2)^3] \left[ K_z - K_z^* + (z - z_s) \times \frac{\partial K_z}{\partial z} + 2(K_z - K_z^*)(z_1 - z_s) \right],
\]  \hspace{1cm} (24)
where \( K \) refers to \( K^m \), \( K^c \) or \( K^w \). The depth of the planetary boundary layer (PBL), \( z_s \), is predicted for convective boundary layer conditions (i.e., \( \theta^* < 0 \)) using the prognostic equation of Deardorff (1974). Its form is
\[
\frac{dz_s}{dt} = w_{z_f}^{[\text{calc}]} = \frac{1.8(\omega^2 + 1.1u^2/3.3u^2/2)}{g(z/f)^{1/2} \partial^2 \theta/\partial z^2 + 9.7\omega^2 + 7.2u^2},
\]  \hspace{1cm} (25)
where \( w_{z_f} \) is the vertical velocity at \( z_f \), and the value of \( \omega^* \) is given as
\[
\omega^* = \left[ \frac{-g/\theta \omega^2 \theta^*}{n^2} \right]^{1/3},
\]  \hspace{1cm} (26)
where \( z_i \) \((= 0.04\zeta_0)\) is the depth of the surface layer and \( \partial^2 \theta/\partial z^2 \) the potential temperature gradient immediately above \( z_i \). For the stable surface layer (i.e., \( \theta^* > 0 \)), the PBL is predicted using the formulation suggested by Smeda (1979):
\[
\frac{dz_s}{dt} = 0.06 \frac{\omega^2}{z_i} \left[ 1 - \left( \frac{3.3\zeta_0}{n^2} \right)^3 \right],
\]  \hspace{1cm} (27)
where \( \zeta_0 \) is the model grid spacing in centimeters, and, in the present study, \( A = 0.115 \) and \( B = 0.175 \). The result of this functional relationship is that for the fine vertical grid resolution near the surface, \( R_i \) approaches the theoretical value of 0.25 while away from the surface where the grid resolution is only 500–1000 m, \( R_i \) approaches 1.0. This latter value is consistent with critical Richardson numbers used to predict the onset of clear-air turbulence based on standard rawinsonde observations.

As suggested by Delage (1974), Brost and Wyngaard (1978) and Blackadar (1979), the mixing length under strong thermal stabilities (except very near the surface) is not dependent on height, and perhaps is dependent on local stability. In the present investigation, the mixing length was calculated following Blackadar (1962):
\[
l = \frac{k_0 z}{1 + (k_0 z)/0.0063u^*},
\]  \hspace{1cm} (23)
When, however, the surface layer is unstable, the profile function of O’Brien (1970) is used above the surface layer:

\[
K_z = K_z + [(z_1 - z)^2/(z_1 - z_2)^3] \left[ K_z - K_z^* + (z - z_s) \times \frac{\partial K_z}{\partial z} + 2(K_z - K_z^*)(z_1 - z_s) \right],
\]  \hspace{1cm} (24)

The $K_{zi}$ and $K_{zi}$ are the exchange coefficients at $z_i$ and $z_s$, respectively.

3) Radiation

The diurnal variation of the solar flux on a horizontal surface at the top of the atmosphere is computed from

$$S = S_0 \cos Z$$

(28)

with

$$\cos Z = \cos \psi \cos \delta \cos H + \sin \psi \sin \delta,$$

(29)

where $S_0$ is the solar constant, $Z$ the zenith, $\psi$ the latitude, $\delta$ the solar declination, and $H$ the solar hour angle. Two empirical functions are used to compute the solar radiation transmission through the atmosphere. One of them accounts for the Rayleigh scattering and the absorption by permanent gases such as oxygen, ozone and carbon dioxide (Atwater and Brown, 1974):

$$G = 0.485 + 0.515 \left[ 1.041 - 0.16 \left( \frac{0.000994P + 0.051}{\cos Z} \right)^{1/2} \right],$$

(30)

where $P$ is the air pressure in millibars. The second empirical function accounts for the absorption of water vapor (McDonald, 1960):

$$a_w = 0.077 \left( \frac{r(z)}{\cos Z} \right)^{0.3},$$

(31)

where $r$ is the optical path length of water vapor above the layer $z$. It is given by

$$r(z) = \int_z^{\text{top}} \rho qdz.$$

(32)

The incident solar radiation on a flat ground surface is given by

$$R_{si} = S(G - a_w).$$

(33)

On inclined surfaces, this radiation is given by

$$R_i = S_0(G - a_w) \cos i,$$

(34)

where $i$ is the angle of incidence of solar rays on the inclined surface and

$$\cos i = \cos \alpha \cos Z + \sin \alpha \sin Z \cos (\beta - \eta).$$

(35)

Here $\alpha$ is the slope angle, and $\beta$ and $\eta$ are solar and slope azimuths, respectively:

$$\alpha = \tan^{-1} \left[ \left( \frac{\partial z_G}{\partial x} \right)^2 + \left( \frac{\partial z_G}{\partial y} \right)^2 \right]^{1/2},$$

(36)

$$\beta = \sin^{-1} \left( \frac{\cos \delta \sin H}{\sin Z} \right),$$

(37)

$$\eta = \tan^{-1} \left( \frac{\partial z_G}{\partial y} / \frac{\partial z_G}{\partial x} \right) - \pi,$$

(38)

with $z_G$ being the topography elevation. Assuming that shortwave absorption in the atmosphere is only due to water vapor, the heating of the atmosphere by this radiation is given by

$$\left( \frac{\partial T}{\partial t} \right)_s = 0.0231 \frac{S_0 \cos Z}{\rho c_p} \left( \frac{r(z)}{\cos Z} \right)^{0.7} \frac{dr}{dz}.$$

(39)

Longwave radiation, and atmospheric heating due to its flux divergence, are calculated according to the carbon dioxide and water vapor concentrations in the atmosphere. The path length for water vapor ($\Delta r_j$) is computed for each layer from the surface to the top of the model by

$$\Delta r_j = - \frac{P_j + 1 - P_j}{g} q_j,$$

(40)

and for the carbon dioxide ($\Delta c_j$):

$$\Delta c_j = -0.4148239(P_j + 1 - P_j).$$

(41)

The total path lengths at a specific level $i$ are obtained by adding up these increments from the first level. The emissivity of water vapor is derived from data of Kuhn (1963) and is given by Jacobs et al. (1974):

$$\begin{align*}
0.11288 \log_{10}(1 + 12.63 \frac{r}{f}) & < -4 \\
10.104 \log_{10} \frac{r}{f} + 0.440 & < \log_{10} \frac{r}{f} < -3 \\
0.121 \log_{10} \frac{r}{f} + 0.491 & < \log_{10} \frac{r}{f} < -1.5 \\
0.146 \log_{10} \frac{r}{f} + 0.527 & < \log_{10} \frac{r}{f} < -1 \\
0.161 \log_{10} \frac{r}{f} + 0.542 & < \log_{10} \frac{r}{f} < 0 \\
0.136 \log_{10} \frac{r}{f} + 0.542 & > \log_{10} \frac{r}{f} > 0
\end{align*}$$

(42)

where $r = |r_i - r_j|$ is the optical path length between the $i$th and $j$th levels. The emissivity of carbon dioxide is given by (Kondratyev, 1969):

$$\epsilon_{CO_2}(i, j) = 0.185[1 - \exp(-0.3919|c_i - c_j|^{0.4})].$$

(43)

The total emissivity at each level is obtained by

$$\epsilon(i, j) = \epsilon_{H_2O}(i, j) + \epsilon_{CO_2}(i, j).$$

(44)

The radiative cooling at a level $N$ is computed from

$$\left( \frac{\partial T}{\partial t} \right)_N = \frac{1}{\rho c_p} \frac{R_u(N + 1) - R_u(N) + R_d(N) - R_d(N + 1)}{z(N + 1) - z(N)},$$

(45)

where the downward and upward fluxes are given by

$$R_d(N) = \sum_{j=N}^{N+1} \sigma (T_{j+1}^4 + T_j^4)[\epsilon(N, j + 1) - \epsilon(N, j)]$$

$$+ \sigma T_{\text{top}}^4[1 - \epsilon(N, \text{top})].$$

(46)
and
\[ R_{n}(N) = \sum_{j=1}^{N-1} \frac{2}{\sigma} (T_{j+1}^4 + T_{j}^4)[\epsilon(N, j) - \epsilon(N, j + 1)] + \sigma T_{G}^4[1 - \epsilon(N, 0)], \tag{47} \]
where \( T_{G} \) and \( T_{top} \) are the temperatures at the ground level and the top of the model, respectively, and \( \sigma \) is the constant of Stefan-Boltzmann. The incident longwave radiation on the ground surface \( (R_{s}) \) is equivalent to the downward flux at the ground level (Eq. 46, \( N = 0 \)).

4) SURFACE PARAMETERIZATION

During radiative frost events, the most pertinent parameter which has to be predicted by the model is the minimum temperature near the ground surface. This is because the agro-topoclimatological mapping of crops and orchards will be executed according to their sensitivity to this parameter. Additionally, the scheduling of frost protective measures application is generally based on this parameter. Therefore, emphasis is given in the present study to the formulation of the lower boundary of the model including the solution of the energy budget equations for the soil surface and the plant canopy.

Deardorff (1978) has suggested a parameterization of the soil–plant–atmosphere system which was later incorporated into a three-dimensional numerical mesoscale model by McCumber and Pielke (1981). This parameterization consists of solving energy balance equations on bare soil surface and a full-covering bulk canopy, and in linearizing the heat fluxes between these two extreme cases for partial vegetated surfaces. McCumber and Pielke (1981) also incorporated in their model prognostic equations for temperature and moisture diffusions in the soil. The hydraulic properties of soil are computed using a procedure suggested by Clapp and Hornberger (1978).

The main difference between Deardorff’s parameterization and the one suggested here consists in the formulation of the sensible and latent heat fluxes from the soil surface and the vegetation to the atmospheric surface layer. The drag coefficients adopted by Deardorff (which are limited to being used only under closely neutral atmospheric conditions) are removed, and the effects of the soil and the vegetation on the heat fluxes are directly incorporated into the equations of Businger et al. (1971) which describe the atmospheric surface layer of the model (Eqs. 13–20).

(i) Vegetation layer. The energy budget of the vegetation is
\[ R_{Nv} + E_v + H_v = 0, \tag{48} \]
where \( R_{Nv} \), the net radiation of the canopy, accounts for the solar radiation \( (R_s) \) and the thermal radiation from the atmosphere \( (R_{a}) \) absorbed directly by the plant and after being reflected by the soil surface. It also accounts for the thermal radiation emitted by the soil surface \( (\epsilon_G \sigma T_G^4) \), the thermal radiation emitted by the vegetation \( (2\epsilon_G \sigma T_v^4) \) and its reflected part by the soil surface. The net radiation is therefore defined as follows:
\[ R_{Nv} = \sigma_f(1 - \alpha_v)[1 + \alpha_G - \sigma_f(1 - \sigma_v)]R_s + \sigma_f\epsilon_v[1 + (1 - \sigma_f)(1 - \epsilon_v)]R_a + \epsilon_G \sigma T_G^4 \]
\[ - \epsilon_G \sigma T_v^4[2 - \epsilon_v \sigma T_v^2(1 - \epsilon_v)]. \tag{49} \]
Here, \( \alpha_v \), \( t_v \), \( \epsilon_v \) and \( T_v \) are the albedo, transmissivity, emissivity and temperature of the vegetation, respectively; \( \alpha_G \) and \( \epsilon_G \) are the albedo and emissivity of the soil surface; and \( \sigma_f \), the shielding factor, represents the fractional coverage of the ground by canopy. This factor is 1 for a complete covered surface and is 0 for a bare soil.

The latent heat flux between the vegetation and the surrounding air, \( E_v \), is given as
\[ E_v = \sigma_f \rho L_a u_a q^*_v, \tag{50} \]
where \( L_a \) is the latent heat of evaporation, and \( \sigma_f \) expresses the relative contribution of the canopy to the total heat fluxes between the surface (composed of soil and vegetation) and the atmosphere:
\[ \sigma_f = \frac{2LAI \sigma_f}{1 + 2LAI \sigma_f}, \tag{51} \]
with LAI being the leaf area index of the vegetation. Using this formulation, the surface specific humidity, \( q(z_0) \), required for the computation of \( q^* \) (Eq. 18) is
\[ q(z_0) = \sigma_f q_{ua} + (1 - \sigma_f)q_G = (2LAI \sigma_f q_{ua} + \sigma_f q_{so})(1 + 2LAI \sigma_f), \tag{52} \]
where \( q_{ua} \), the specific humidity at the leaf–air interface, is computed following a procedure suggested by Avissar et al. (1985):
\[ q_{ua} = d_{sr} q^*_s + (1 - d_{sr})q_{so}, \tag{53} \]
with \( q^*_s \) the saturated specific humidity at the leaf surface; and \( d_{sr} \), the dimensionless relative stomatal conductance defined as follows:
\[ d_{sr} = [d_{sm} + (d_{sm} - d_{sm})f_H f_f f_s f_C f_h]/d_{sm}, \tag{54} \]
where \( d_{sm} \) is the minimal conductance which occurs only through the leaf cuticle when stomata are closed; \( d_{sm} \) is the maximum stomatal conductance obtained when stomata are completely open; and each of the \( f_i \) functions quantifies the influence of a specific environmental factor upon the conductance \( (R \) for solar global radiation; \( T \) for leaf temperature, \( V \) for vapor pressure difference between leaf and ambient air, \( C \) for ambient air carbon dioxide concentration, and \( \psi \) for soil water potential around the roots). The expression used for these functions is
\[ f_i = \frac{1}{1 + \exp[-(\chi_i - b_i)]}, \tag{55} \]
where subscript \( i \) refers to the environmental factor, \( b_i \) is the abscissa at \( f_i = \frac{1}{2} S_i \); the slope of the curve at this point, and \( X_i \) the intensity of the factor \( i \). If one of the environmental factors causes the stomatal closure, then the value of the function referring to this factor is 0 and \( d_{e,i} = d_{e,m}/d_{e,m} \). However, when the given environmental factor does not affect the stomata opening, the value of the related function is 1 and does not influence \( d_{e,i} \). When all the \( f_i \) functions are equal to 1, i.e., there are no environmental effects on the stomatal opening, then \( d_{e,i} = 1 \) and the transpiration reaches its maximum value for the given meteorological conditions. The constants \( d_{e,m}, d_{e,m}, b_i \) and \( S_i \) are empirically determined. For example, Aviszar et al. (1985) gave their value for a tobacco plant. The soil surface specific humidity, \( q_{G} \), is computed following Aviszar and Mahrer (1986):

\[
q_{G} = q_{Gw} + (1 - s_w)q_{g0}
\]

(56)

where \( q_{Gw} \) is the upper soil layer specific humidity calculated according to Philip and de Vries (1957):

\[
q_{Gw} = q_{Gw}^{*} \exp(-g |\psi_{Gw}|/R_e T_{Gw}),
\]

(57)

with \( q_{Gw}^{*} \) being the saturation specific humidity and \( \psi_{Gw} \) the matric potential of the upper soil layer. The gas constant of water vapor is \( R_e \).

The surface wetness function, \( s_w \), is defined as follows:

\[
s_w = a + \frac{1 - a}{1 + \exp[b(\theta - \theta_e)]},
\]

(58)

where \( a, b \) and \( \theta_e \) are empirical constants which depend upon soil type. Their values, for Rehovot brown-red sandy soil (Xerorthant), were found to be 0.3, 32.0, and 0.06, respectively (Aviszar and Mahrer, 1986).

The sensible heat flux between the vegetation and the surrounding air, \( H_s \), is

\[
H_s = \sigma_f \rho c_p u_\theta \theta_e
\]

(59)

where \( \theta(z_0) \), needed for the calculation of \( \theta_e \) (Eq. 17), is

\[
\theta(z_0) = \sigma_f \theta_e + (1 - \sigma_f) \theta_G
\]

\[
= (2LAI\sigma_f \theta_v + \theta_o)/(1 + 2LAI\sigma_f),
\]

(60)

with \( \theta_o \) and \( \theta_G \) being the potential temperatures of vegetation and ground, respectively.

(ii) Soil layer. The energy budget of the soil surface is given by

\[
R_{NG} + E_G + H_G + S_G = 0,
\]

(61)

where \( R_{NG} \), the net radiation of the soil surface, accounts for the solar radiation and the thermal radiation from the atmosphere directly absorbed by the soil surface, the solar radiation transmitted by the vegetation, part of the thermal radiation emitted by the vegetation, and the thermal radiation emitted by the soil surface. It also accounts for the reflection, by vegetation, of the thermal radiation emitted by the soil surface. It is therefore defined as follows:

\[
R_{NG} = (1 - \alpha_G)(1 - \sigma_f + \sigma_f t_o)R_s + (1 - \sigma_f)\epsilon_G R_L
\]

\[
+ \sigma_f \epsilon_G \sigma T_e^4 - \sigma_f \epsilon_G \sigma T_e^4(1 - \epsilon_f)(1 - \epsilon_l).
\]

(62)

The latent heat flux between the soil surface and the surrounding air, \( E_G \), is

\[
E_G = (1 - \sigma_f)\rho L u^* \theta^*.
\]

(63)

and \( H_G \), the sensible heat flux between the soil surface and the surrounding air, is

\[
H_G = (1 - \sigma_f)\rho c_p u^* \theta^*.
\]

(64)

The soil heat flux, \( S_G \), is

\[
S_G = -\lambda \frac{\partial T_G}{\partial z} \approx -\lambda \frac{T_G - T_{Gw}}{\Delta z},
\]

(65)

with \( \lambda \) being the soil heat conductivity computed following a procedure suggested by de Vries (1963) which accounts for the texture and the water content of the considered soil layer. The prediction of \( T_{Gw} \) is obtained by solving the soil heat diffusion equation, from the depth at which the temperature is assumed to be constant to the soil surface:

\[
C \frac{\partial T_G}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda \frac{\partial T_G}{\partial z} \right].
\]

(66)

In order to evaluate the soil thermal properties and the wetness of the upper soil layer (which is of primordial importance for a correct evaluation of the latent and sensible heat fluxes at the soil surface) the equation of soil moisture diffusion was also incorporated into the model:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ D_e \frac{\partial \theta}{\partial z} \right] + \frac{\partial}{\partial z} \left[ D_T \frac{\partial T_G}{\partial z} \right] + \frac{\partial K_h}{\partial z} - V_{e},
\]

(67)

where \( D_e \) and \( D_T \) are the isothermal and thermal moisture diffusivities, respectively, (calculated according to Philip and de Vries, 1957). The hydraulic conductivity, \( K_h \), is obtained from an empirical power curve formula based on the generalization of Kozeny and Carman's approach (Wyllie and Gardiner, 1958):

\[
K_h = K_h s_e^n
\]

(68)

where

\[
s_e = \frac{\theta - \theta_r}{\theta_i - \theta_r},
\]

(69)

with \( \psi = \psi_{cr} s_e^{-m} \),

(70)

where \( \psi_{cr} \) is the matrix potential at which the water content begins to be lower than the saturation water content in the \( \psi(\theta) \) curve. The constants \( n \) and \( m \) are specific to each soil and have to be determined empirically.
This system of equations is solved from the soil surface to the soil depth at which the water content is constant (due to the presence of water table) or at which \( \partial \theta / \partial z = 0 \).

The function \( V_e \) accounts for soil water extraction by roots due to plant transpiration and is proportional to root distribution in the soil (Avisar and Mahrrer, 1982).

(iii) Water surface. When a water surface (i.e., lake, ponds, sea) is present at the bottom of the model, constant temperature is assumed during the simulation.

5) BOUNDARY CONDITIONS

Zero gradient lateral boundary conditions are specified for the atmospheric variables, i.e.,

\[
\frac{\partial}{\partial x} (u, v, w, \theta, q, \pi) = \frac{\partial}{\partial y} (u, v, w, \theta, q, \pi) = 0. \tag{71}
\]

At the ground surface,

\[
u = v = w = 0. \tag{72}
\]

The conditions imposed at the top of the model are

\[
u = \text{constant}, \quad v = \text{constant}, \quad w = 0
\]

\[
\theta = \text{constant}, \quad q = \text{constant}, \quad \pi = \text{constant}
\]

At the lowest soil level:

\[
\begin{align*}
\theta(1) &= \text{constant at water table or,} \\
\text{if not present}, \quad \partial \theta / \partial z &= 0 \\
T_0(1) &= \text{constant.}
\end{align*} \tag{74}
\]

6) INITIAL CONDITIONS

Within the PBL, the initial velocity profile is determined using an Ekman layer type equation which assumes that shear stress, Coriolis and pressure gradient forces are in balance. Above the PBL, synoptic winds are assumed to be geostrophic. Soil temperature and moisture profiles are assumed to be vertically uniform and equal to their respective lower boundary values. Soil and vegetation types and properties are prescribed.

b. Numerical considerations

1) COORDINATE SYSTEM

The use of terrain-following coordinate systems (TFCS) has been shown to be very effective when topographic features are considered in meteorological numerical models (e.g., Kasahara, 1974; Mahrer and Pielke, 1975; Gal-Chen and Somerville, 1975; Yamada, 1983; among others). Generally, this coordinate system is defined by using the transformation

\[
x^* = x, \quad y^* = y, \quad z^* = \frac{z - z_G}{s - z_G}, \tag{75}
\]

where \( x^* \) and \( y^* \) are the horizontal, and \( z^* \) the vertical coordinates in the TFCS, and \( s \) is a reference height (usually the top of the model). Mahrer (1984), however, has shown that this method introduces inconsistent approximations in the computation of the horizontal gradients of the model variables when the distance between two vertical grid points is smaller than the elevation difference between two horizontal adjacent points (in the TFCS). In order to avoid this problem, an improved finite difference scheme is proposed in which the grid is established using a TFCS and the horizontal gradients of the variables are solved in a local cartesian coordinate system (LCCS) by using a vertical interpolation technique. Mahrer (1984) reduced significantly the error obtained in the horizontal gradients with this vertical interpolation technique. Figure 1 illustrates the combination of TFCS and LCCS. The arrows indicate the LCCS while the dotted lines indicate the TFCS. It must be noticed that, with this method, there is no need to transform the model equations (as requested by using a TFCS) since, actually, the equations are solved in LCCS. Therefore, in addition to simplicity, the computational procedure involved with the description of the equations is somewhat shorter and, consequently, this scheme presents the advantages of both TFCS and LCCS with a better accuracy.

2) NUMERICAL SCHEMES

The advective terms in the model equations are evaluated, optionally, by a simple upstream differencing scheme or by an upstream interpolation with a cubic spline. Although the cubic spline is more accurate than the simple upstream scheme (since it preserves phase and amplitude very well), Mahrer and Pielke (1978), who have compared these two schemes in the numerical simulations of sea breeze, have shown that accurate results may be obtained near the surface also with the simple upstream scheme. Since this method

![Figure 1](https://via.placeholder.com/150)
is much less consumptive in computer resources, it was adopted for most of the numerical experiments. The pressure gradient terms are represented by centered differences.

Vertical diffusions in the atmosphere and the soil are accomplished by using a generalized version of the Crank–Nicholson scheme (Paegle et al., 1976; Avissar and Mahrrer, 1982). No horizontal diffusion is parameterized, although a discriminating low-pass filter is applied, as described to the prognostic atmospheric variables in subsection 2a2. With the value of $\delta$ which was adopted in the present study ($\delta = 0.01$) this filter eliminates all $2\Delta x$ waves but leaves waves $4\Delta x$ and greater essentially unchanged (Pepper et al., 1979; Pielke, 1984).

The Newton–Raphson iterative algorithm is used to solve the energy balance equation for the vegetation layer and the soil surface (Eqs. 48 and 61, respectively).

3. Sensitivity analysis

The model was run in its one- and two-dimensional versions in order to test its sensitivity to different parameters which may influence minimum temperatures near the ground surface during radiative frost events. Initial atmospheric vertical profiles were established using data from 20 radiosondes released at Bet-Dagan (Israel) at 2400 LST during radiative frost events (Fig. 2). These radiosondes were selected because of their similarity and their representativeness of typical radiative frost in Israel. The deviation between each radiosonde and the average value was small and therefore, probably does not affect the microclimate near the ground surface. In order to be consistent with these initial conditions, the model was run starting at 2400 LST 15 February at 32°19'N. In the vertical, 19 levels at 0, 0.5, 2, 5, 10, 25, 50, 100, 200, 400, 600, 800, 1000, 1250, 1500, 1750, 2000, 2500 and 3000 m were considered in the atmosphere and 20 levels of 0.05 m each between soil surface and a depth of 1 m were considered in the soil. For the two-dimensional simulations, a horizontal grid interval of 500 m was adopted. The time step for the integration of the model equations was 300 and 30 s for the one- and the two-dimensional experiments, respectively.

a. One-dimensional numerical experiments

Figure 3 illustrates the diurnal variation of predicted soil temperatures (at depths of $-0.1$ and $-0.2$ m), soil surface and air temperatures (at heights of 0.5 and 2 m) for a bare (panel a) and a vegetated (panel b) surface. Since a bulk layer is used to parameterize the vegetation, only one representative temperature of the canopy is computed, independently of the vegetation height. Thus, the temperature at $0.5$ m above the soil surface into the canopy is also equivalent to this representative temperature. Mechanical, physical and photometric

---

**Fig. 2.** Averaged meteorological conditions obtained from 20 radiosondes released at Bet-Dagan, Israel, during radiative frost conditions.
properties of the brown-red sandy soil (Xerorthant) assumed for these simulations are given in Table 1. Dimensions and photometric properties of the vegetation (winter crop) considered here are given in Table 2. From these figures, one can notice that vegetation has attenuated the diurnal soil temperature amplitude while it has increased the diurnal amplitude of the air temperature inside the canopy. This is mainly due to the following:

(i) most of the global radiation is absorbed by the dense vegetation and, consequently, only a small amount of it reaches the soil surface;

(ii) during the night, and especially during radiative frost events, the thermal radiation emitted by soil surface is not compensated by the atmospheric radiation. When, however, vegetation is present, this radiation is absorbed by the leaves and partly emitted back to the soil surface.

The energy transfers occur on the vegetation and soil surface layers according to the shielding factor $\sigma_f$. As the distance from these layers increases the diurnal amplitude of temperature is attenuated and the phase of maximum and minimum comes later. This phenomenon is dependent on the thermal properties of soil and the stability of the atmospheric surface layer.

The sensitivity of the model to different factors which may influence the profile of temperatures near the soil surface is presented in Table 3. Minimum and maximum temperatures of the simulated diurnal cycles are given. Maximum temperatures, although not applicable for frost evaluations, are also presented since they may provide indications relating sensitivity to input parameters. Soil moisture strongly influences both sandy and clay soil temperatures and, the heavier the soil, the more significant the influence. This feature is due to the increase of heat capacity and thermal conductivity of soils with the increase of water content, causing a larger heat conduction to the cooling soil surface during the night. Sandy soils have a relatively higher conduction of moisture to the evaporating surface during the daylight hours. Consequently, larger thermal energy may be converted into latent heat on a relatively wet sandy soil surface than on clay soil surface, at the same water content. As a result the surface temperature of the sandy soil is the lowest.
The maximum soil surface temperature is related to the global radiation absorbed. As the vegetation foliage density increases, a larger part of the radiation is intercepted reducing the soil surface temperature and increasing the temperature of the ambient air in the canopy.

The influence of the specific humidity of the atmosphere on the profile of temperatures near the soil surface was tested by increasing the water content of the atmosphere to saturation. As a result, the longwave radiative flux incident at the ground surface was increased by about 10% compared to the flux obtained with the humidity profile depicted in Fig. 2. As can be seen in Table 3, bare soil surface temperature was consequently increased by 1.6°C. It may be noticed that such radiative flux increase can also be obtained by cirrostratus clouds which fully cover the sky or heavier clouds which partly cover the sky (Oke, 1978). It may therefore be expected that similar influence on the profile of temperatures will be obtained when clouds are present. Reducing the atmosphere relative humidity to 10% at all heights decreased the longwave radiative flux at the surface by about 10%. As a result, the bare soil surface temperature was 4.2°C colder.

The influence of wind speed was tested by running the model with a wind speed ten times stronger than the original one (Fig. 2), i.e., 4 m s⁻¹. Consequently, bare soil surface temperatures increased by 1.5°C and air temperature, at a height of 2 m, decreased by 2.3°C (Table 3). The temperature difference between soil surface and this height was decreased from 5.5°C, for the mean frost conditions, to 1.7°C.

From these one-dimensional numerical experiments, it may be emphasized that vegetation type (characterized by its density, height, roughness and shielding factor), soil moisture content (which depends on soil type), wind speed and specific humidity of the atmosphere have a significant influence on the profile of temperatures near the surface during radiative frost events. These conclusions are in concordance with observational studies (Blanc et al., 1963; Bootsm, 1976; Von Lengerke, 1978; Kalma et al., 1983; Goldsworthy and Shulman, 1984).

b. Two-dimensional numerical experiments

The two-dimensional version of the model was used to test the influence of local topography on the profile of temperatures near the soil surface during the mean radiative frost conditions (Fig. 2). Two topographic features were simulated: a valley 50 m deep, 11 km and 1 km wide at its top and bottom, respectively, and a hill 50 m high, 1 km and 11 km wide at its top and bottom, respectively. Both were east–west oriented. They are schematically represented at the bottom of Figs. 4–7. The small slopes defined in these simulations (1 m/100 m) and the weak winds prevailing during radiative frost enable the use of the hydrostatic assumption (Pielke, 1984). For each case we have checked the influence of vegetation and soil moisture on the predicted temperatures along the topography.

1) VALLEY SIMULATION

Figure 4 illustrates the west–east component of the circulation (u) generated at 0600 LST in the simulated valley. At this hour, the thermally induced flow is at its nocturnal peak, showing a circulation depth of about
150 m above surface with a maximum intensity of 1.1 m s\(^{-1}\) obtained at a height of 10 m above the middle of the west slope. The asymmetry observed between the west and east slopes is due to the interaction between the background wind and the thermally induced circulation. It must be noticed that this simulation was performed using the soil hydraulic and thermal properties of the brown–red sandy soil described in Table 1. The soil was bare and initially wet (water content was prescribed at 0.10 m\(^3\) m\(^{-3}\) at the beginning of the simulation). Additional simulations were performed on the same valley but with a dry soil (0.03 m\(^3\) m\(^{-3}\)), a wet soil covered by small vegetation (\(\sigma_f = 1\), LAI = 3, height = 0.2 m) and wet soil covered by tall vegetation (\(\sigma_f = 1\), LAI = 3, height = 1 m). Similar patterns of circulation were generated for these simulations but the maximum intensity was increased up to 30% for the dry soil case.

These circulations, as weak as they are, however, have a significant influence on the minimum temperature predicted at a height of 0.5 m above soil surface, along the topography (Fig. 5). As was already noted in the one-dimensional numerical experiments, dry soil and dense vegetation significantly decrease minimum temperatures near the soil surface during frost radiative events. Observing cold-night temperatures of agricultural landscapes with an airplane-mounted radiation thermometer, Nixon and Hales (1975) obtained similar results. The influence of the local circulation generated in the valley is particularly noticeable in the tall vegetation: the stronger the wind velocity (at the lowest atmospheric level of the model), the more the air near the soil and around the canopy is mixed and, consequently, the higher the temperature around the vegetation. From these results, it seems that the higher the vegetation, the lower its minimal temperature is during

![Fig. 4. West–east component of the circulation generated at 0600 LST during radiative frost events by a valley 50 m deep.](image)

![Fig. 5. Predicted minimum air temperatures during radiative frost events at a height of 0.5 m above soil surface in a valley 50 m deep.](image)
radiative frost events. However, it should be kept in mind that the represented temperatures of Fig. 5 are obtained at a height of 0.5 m above soil surface. While, for the 1 m height vegetation, the air temperature at this level is essentially influenced by the leaf temperature, it is also influenced by the temperature of the above air layer which is warmer than the canopy for the 0.2 m height vegetation. Actually, the canopy temperature, itself, was almost identical in the two considered cases.

2) HILL SIMULATION

In this series of numerical experiments, four simulations analogous to the valley simulations were carried out. Figures 6 and 7 are the corresponding results of the west–east component of the wind generated by the hill 50 m high (at 0600 LST) and the minimum temperatures at a height of 0.5 m above the soil surface. Although the slopes of the hill and the valley are identical, the horizontal component of the wind was about 50% greater in the present simulations. This is attributed to the fact that, in the valley, convergence of the downslope-induced flows reduces the flow intensity. This process is not involved with the hill simulation.

It may be again noted that higher temperatures are predicted, near the ground, in domains with stronger wind velocity. Consequently, the temperature of the vegetation canopy is higher on the slopes than on the top and the bottom of the hill.

3) TWO-DIMENSIONAL INHOMOGENEITY

The two-dimensional version of the model was also utilized to test if any interaction exists between two adjacent flat areas differing by their vegetation cover, soil type or soil moisture content. However, due to strong atmospheric stability and weak winds prevailing during radiative frost events, the temperature distribution near the ground was mainly influenced by the surface properties of the specific area, and no significant interaction was obtained in the simulated domain.

From these two-dimensional numerical experiments, it may be emphasized that, during radiative frost events, the temperatures near the surface are influenced by topography. This is mainly due to the variation of the wind flow according to topography. Higher temperatures are generally obtained on the slopes and the higher parts of the domain, while lower temperatures are found in the valleys. This conclusion, also in agreement with observational studies (Blanc et al., 1963; Lomas and Gat, 1971; Bagdonass et al., 1978; Suzuki et al., 1982), has led to the suggestion of growing the most sensitive plants on the slopes of hilly and mountainous terrains (i.e., Blanc et al., 1963).

4. Summary and conclusions

The numerical model presented in this study was developed in order to simulate the microclimate of local-scale regions during radiative frosts. The use of such a model, for agricultural planning and for application of frost protection methods in frost sensitive regions, seems to be a good alternative to previous methodologies like the expensive and tedious topoclimatological surveys of minimum temperature at a height of 0.5 m above soil surface. This model was parameterized using the state of the art of mesoscale numerical modeling where a refined version of the soil–plant–lower atmosphere system and an improved finite difference scheme for solving horizontal gradients in a terrain following coordinate system were incorporated.

Using one- and two-dimensional versions of this model, it was shown that the temperature distribution near the ground is sensitive to plant cover, soil type and moisture content, air specific humidity, wind speed and topography. Similar conclusions, based on observations, were already drawn and therefore emphasize

![Graph](image-url)
the model's ability to correctly predict the general trends of the microclimate near the soil of local regions under radiative frost conditions. It is important to note that, in addition to minimum air temperature generally provided with other methods, the model gives a detailed description of the microclimate near the soil surface. From a practical point of view, the model computes the leaf temperature which is the parameter to be considered for mapping frost-sensitive areas, instead of air temperature which may be very different from it, as shown in the different numerical experiments presented here.

However, in order to be applicable with a good degree of reliability, this model should be verified and compared with field experimental data. In Part II, the model is run in its three-dimensional version and predicted minimum temperatures are compared with an observational survey, emphasizing its potential use.

**Acknowledgments.** This research was supported by the Israel Academy of Sciences and Humanities (Basic Research Foundation) and by the National Science Foundation under Grants ATM-8414181 and ATM-8616662. Computations were partly performed using the National Center for Atmospheric Research CRAY Computer (NCAR is supported by the NSF). The authors wish to thank Roger A. Pielke, Moti Segal, George Kallos and Ofir Naot for their review of the manuscript and for making valuable suggestions. Linda Jensen and Susan Einarsen are thanked for typing the manuscript and Judy Sorbie for preparing the graphs.

**REFERENCES**


de Vries, D. A., 1963: Thermal properties of soils, in *Physics of Plant
Schnelle, F., 1950: Local climatic surveys according to frost damage to fruit culture. Deutscher Wetterdienst in der U.S. zone, Berichte No. 12, 99–104.