The Surface Sea Breeze: Applicability of Haurwitz-Type Theory

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ABSTRACT

The Haurwitz sea-breeze theory, and modifications by Kusuda and Alpert, are not generally applicable to observed winds in coastal regions, in part because they make no allowance for spatial evolution of wind hodographs. This is demonstrated by integrating the Haurwitz equations of motion to find trajectories of air reaching the shoreline at various times of day through the action of a large-scale pressure gradient force. Hodographs and momentum advection are computed from the trajectories. Hodograph size, shape, and orientation depend on distance from the shoreline, friction, spatial distribution of the time-varying part of the geostrophic wind (that parallel to the shore), and other factors. Differences between the hodographs found here and those found by Haurwitz, or by Kusuda and Alpert, are largely related to nonlinearity.

1. Introduction

Haurwitz’s (1947) treatment of the sea breeze is generally recognized as an important milestone in sea-breeze theory. The mathematical results are expressions for the horizontal wind components at the surface as functions of time, with Coriolis parameter, amplitude of the diurnally oscillating pressure-gradient force, and coefficient of linear friction as parameters. The results are presented as wind hodographs depicting the variation of direction and speed over the diurnal period. Positions and eccentricities of the elliptical hodographs can be adjusted by varying the large-scale geostrophic wind and the friction coefficient.

Since wind hodographs over the diurnal period are readily computed from routinely observed winds, it is not surprising that a number of comparisons have been made with the Haurwitz hodographs. Haurwitz himself compared his theory with mean winds over 40 days at Boston, Massachusetts. Staley (1957) made comparisons with hodographs for many stations in western Washington State. As recently as 1984, Alpert, Kusuda and Abe (hereafter referred to as AKA) analyzed some 26 hodographs from Washington State and 33 stations in California for variations of eccentricity and tilt angle and interpreted the results in terms of variation of friction, or a shifting thermal forcing (or varying amplitudes of forcing in horizontal directions A and B), treated by Kusuda and Alpert (1983) hereafter referred to as KA) in an extension of the Haurwitz theory.

Although AKA recognize the limitations of the Rayleigh friction parameterization, and supplement their theory with a K-theory friction parameterization that predicts change from clockwise to counterclockwise rotation of hodographs with height, many of the remaining features of the Haurwitz theory are retained. The pressure-gradient force is specified as a function of time, and transient features in the solution are allowed to disappear. Thus the extensions and modifications of the Haurwitz theory presented by KA and by AKA preserve the linear treatment and the particle dynamics inherent in the Haurwitz treatment.

The circumstances under which the Haurwitz theory, or its more recent modifications, can be applied to observed diurnal wind variations have never been critically examined. Obviously, particle dynamics involves neglect of the continuity equation, and this was, of course, recognized by Haurwitz. The consequences are difficult to evaluate. It is necessary to solve the problem taking into account the complete dynamical equations (the modern approach), and then comparing with a solution which neglects the continuity equation, but is otherwise dynamically comparable to the Haurwitz dynamics.

Unfortunately, the Haurwitz particle dynamics is of such a nature that it probably cannot reasonably be modeled as a special case of a modern numerical investigation wherein continuity is neglected. Moreover, agreement of the Haurwitz theory with observations is likely to be largely fortuitous. The crux of the matter is that an air parcel is assumed to be forever subject to the same diurnally varying pressure-gradient force, despite its progression away from the coastline in the presence of a large-scale pressure-gradient force. This deficiency is continued in the more recent modifications by KA and AKA.

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Although a numerical solution to a more realistic formulation was not available to Haurwitz, the physical difficulty noted above was not discussed by him. Nor has it been explored by KA or AKA. The question is, how do the wind-velocity variations following an air parcel compare with those observed locally at the coast or at some point off- or on-shore? If advection (non-linearity) can be neglected, the two variations should be the same. If not, they are different, and the hodographs computed from a Haurwitz-type theory cannot be compared meaningfully with observations. It is generally agreed that the sea breeze is a mesoscale phenomenon; hence advection and nonlinearity are to be expected.

A primary purpose of this paper is to remove the assumption of the Haurwitz theory that an air parcel is forever subject to the same diurnally varying pressure-gradient force. This will be achieved by computing the trajectories, and winds at points along the trajectories, of air parcels that start at various times from a point well offshore, are carried onshore by the large-scale pressure-gradient force, and are subjected to a diurnally varying mesoscale pressure-gradient force only in the vicinity of the coastline. The Lagrangian derivative, \( dV/\partial t \), is available directly from the trajectory. The Eulerian (local) derivative, \( \partial V/\partial t \), and hodographs, can be determined at the shoreline and various distances inland by recording the times and velocities of air parcels passing by the shore or any other line. The advection, \( -V \cdot \nabla V \), representing the nonlinearity, can be computed from the difference between Eulerian and Lagrangian derivatives.

Hodographs evolve spatially, and the Haurwitz theory appears as a special case. It is hoped that this generalization will clarify the historical position of the Haurwitz theory in the development of sea-breeze theory, and possibly reveal qualitative properties of hodograph spatial evolution that may occur under appropriately simple coastal topography.

A secondary purpose is to demonstrate that, because of spatial evolution of hodographs for a fixed friction coefficient, observed hodographs cannot be used to infer variations of friction or shifting direction of the pressure-gradient force, as attempted by AKA. It must be remarked that observations are most abundant in areas where coastlines are irregular and topography is likely to be a major influence. Clearly, no simple theory can find confirmation in such regions.

The linear (or nonlinear) friction coefficient can be adjusted to yield the appropriate average across-isobaric flow, but does not allow, as does eddy friction, investigation of the height variation of hodographs. The purpose here is not to investigate height variation of hodographs, but to demonstrate horizontal evolution of hodographs and the difficulty of applying to observations a Haurwitz-type theory that does not allow for this evolution. Likewise, a sea-breeze front cannot be simulated, but it does not always occur. In any event, its signature is averaged out when mean hodographs are computed from observations.

A note on units: For the purpose of clarity, metric units appropriate to the scale of the problem, rather than SI units, will be used in what follows.

2. Numerical model

The horizontal equations of motion solved by Haurwitz were:

\[
\frac{du}{dt} = f[v - v_\theta(0)] - ku \quad (1)
\]

\[
\frac{dv}{dt} = f(u_\theta - u) - kv \quad (2)
\]

In these ordinary differential equations, the Coriolis parameter, \( f \), and coefficient of linear friction, \( k \), were treated as constants. The zonal component of the geostrophic wind, \( u_\theta \), was treated as a constant, while the meridional component, \( v_\theta(t) \) (parallel to a north–south coastline), was expressed as a sinusoidal function having a 24-h period and an amplitude based on differential heating between land and sea.

Haurwitz examined the steady-periodic solutions of (1) and (2). The solutions for \( u \) and \( v \) describe elliptical hodographs whose positions, eccentricity and size depend on \( f \), \( k \) and the amplitude of \( v_\theta(t) \).

The present contribution focuses on a problem of physical interpretation of the Haurwitz solution, in the case where \( u_\theta \neq 0 \). In this case, an air parcel eventually moves either far inland or far offshore, as can be confirmed by a time integration of the Haurwitz solutions to get trajectories. The Haurwitz solution implies that the same periodic, \( v_\theta(t) \), indefinitely applies to the air parcel in its travels, regardless of its distance from the shoreline, and despite the formulation of \( v_\theta(t) \) on the basis of differential heating across the shoreline. A possible way out of this difficulty is to neglect horizontal momentum advection and replace \( du/dt \) and \( dv/dt \) by \( \partial u/\partial t \) and \( \partial v/\partial t \), respectively. The steady periodic solution would then apply locally, and everywhere. However, it is not obvious that horizontal velocity advection is negligible, and, indeed, the computations confirm the importance of nonlinearity.

In what follows, \( v_\theta(t) \) in (1) is replaced by \( v_\theta(x, t) \), a periodic function having maximum amplitude in the vicinity of the shoreline. In (1) and (2), \( du/dt \) and \( dv/dt \) are replaced by \( d^2u/dx^2 \) and \( d^2v/dy^2 \), respectively.

The origin \((x = 0, y = 0)\) is located far to the west of a coast. At the origin, \( v_\theta(x, t) \) is negligible, and air parcels are started out at various times \( \tau \) \((0 \leq \tau \leq 23 \text{ h})\) with velocity components

\[
u = \frac{f^2 u_\theta}{f^2 + k^2} \quad (3)
\]
\[ v = \frac{k f u_g}{f^2 + k^2} \]  

These solutions follow from (1), (2) for \( v(x, t) = 0 \) and \( du/dt = dv/dt = 0 \), and represent a straightline trajectory making an angle \( \beta = \arctan(k/f) \) with the x axis. Solutions are obtained numerically from (1) and (2) by writing

\[ \frac{dx}{dt} = x, \frac{y(t + \Delta t) - x, y(t - \Delta t)}{2\Delta t} \]  

\[ \frac{d^2(x, y)}{dt^2} = x, \frac{y(t + \Delta t) + x, y(t - \Delta t) - 2x, y(t)}{(\Delta t)^2}. \]  

The numerical solution is started by setting \( x, y(t - \Delta t) = x, y(r) = 0 \) and \( x, y(t) = x, y(r + \Delta t) = (u, v)\Delta t \), where \( u \) and \( v \) are given by (3) and (4). Here \( r \) is arbitrary. The next step is to compute \( x, y(t + \Delta t) \). The difference equations (5) and (6) allow (1) and (2) to be expressed as

\[ x(t + \Delta t) = F_x[x(t - \Delta t), x(t), y(t - \Delta t), y(t + \Delta t)] \]  

and

\[ y(t + \Delta t) = F_y[x(t - \Delta t), x(t + \Delta t), y(t - \Delta t), y(t)] \]  

respectively, where \( F_x \) and \( F_y \) denote functional dependence. Equation (8) can be substituted in (7) to eliminate the unknown \( y(t + \Delta t) \), yielding

\[ x(t + \Delta t) = F_x^*[x(t - \Delta t), x(t), y(t - \Delta t), y(t)] \]  

where \( F_x^* \) is a new function. Now \( x(t + \Delta t) \) can be computed. The result is substituted in Eq. (8), so that \( y(t + \Delta t) \) can be computed. After \( x, y(t + \Delta t) \) have been computed, these and \( x, y(t) \) become the data for computing \( x, y(t + 2\Delta t) \). Library routines, of course, exist for solving the system (1), (2). However, when nonlinear friction is used, as in case d in section 3, the system is no longer linear, and the iterative method used can best be described in terms of modification of the solution procedure outlined above.

The time increment used was \( \Delta t = 15 \text{ min} \). The accuracy of numerical solution was assessed by comparison with analytic solutions of the system (1), (2). The first case considered was one where \( v_g = 0, u_g = \text{constant} = 36 \text{ km h}^{-1} \). Initially \( t = 0 \), the air parcel was assumed to be at \( x, y = 0 \) with (somewhat miraculously) \( u, v = 0 \). The analytic solution is readily obtained and describes a trajectory that starts to the north, is deflected to the right by the Coriolis force, and shows a damped oscillation about an asymptote as the air parcel moves northeastward. To start the numerical solution, the analytic solution was used to determine the position at time \( t = \Delta t \). The numerical and analytic trajectories are thereafter so nearly identical that coordinates of positions after 20 h elapsed time and several hundred kilometers of displacement differ only by the order of 0.1 km. They cannot be distinguished in a plot.

A second test was a comparison of the numerical solution with the Haurwitz analytic solution in a situation where the solutions should be the same. See cases \( b \) and \( g \) in Section 3. For the appropriate conditions, the numerical solution accurately duplicates the analytic solution.

### 3. Results

As indicated above, air parcels start out at various times from an origin located far to the west of a west coast. Initially, regardless of the starting time, their motion is northeastward and unaccelerated. As they near the coast, they are deflected northward or southward, depending on the time of day, which determines the sign of \( v_g \). Characteristics of the deflection, and the return to balance of forces after the parcel moves inland, depend on the magnitude, sign, and spatial variation of \( v_g(x, t) \).

In all cases, the geostrophic wind parallel to the coast was assumed to have the following Gaussian and periodic form:

\[ v_g = V_g e^{-[(x-x_c)/L]^2} \cos(2\pi t/24) \]  

Here \( V_g \) is the amplitude of the \( v_g \) fluctuation at \( x = x_c = 150 \text{ km} \), somewhat arbitrarily defined to be the location of the coast. The half-width of the \( v_g \) oscillation is adjusted through the parameter \( X \). Time is expressed in hours.

The formulation (10) makes no allowance for any diurnal displacement of forcing about its mean position, or any change of shape of the forcing. Some displacement must occur, or even a splitting up into a stationary and a moving forcing, especially on occasions when a marked sea breeze front progresses inland. In the absence of observations for guidance, it is too speculative to include such a displacement in any simple Haurwitz-type theory. Obviously it is one more uncertainty contributing to observed hodograph shape, tilt angle, and other properties, and making the averages of hodograph eccentricities and tilt angles from different locations of dubious significance. The results of using (10) are probably most applicable to average hodographs at specific locations, and where sea breeze fronts are exceptional or weakly defined.

Table 1 indicates the parameters and conditions used in the various cases discussed in what follows. In most cases, \( u_g = 9 \text{ km h}^{-1}, f = 0.36 \text{ h}^{-1} \) (at 43° lat.). The values of the parameters are somewhat arbitrary. The friction coefficient was chosen to give a reasonable cross-isobaric flow (26.6° and 14.0° for \( k = f/2 \) and \( f/4 \), respectively). As noted by Haurwitz (1950), linear friction "does not work too badly for surface winds," which are the focus of study here. The parameter \( X \) is more speculative, but chosen to give a finite scale to the differential heating.
### Table 1. Properties and figures relating to cases studied.

<table>
<thead>
<tr>
<th>Case</th>
<th>$u_i$ (km h$^{-1}$)</th>
<th>$V_i$ (km h$^{-1}$)</th>
<th>$X$ (km)</th>
<th>Friction</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>9</td>
<td>9</td>
<td>50</td>
<td>Linear, $k = f/2$</td>
<td>1, 2a-d, 3, 6</td>
</tr>
<tr>
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<tr>
<td>c</td>
<td>9</td>
<td>6</td>
<td>50</td>
<td>Linear, $k = f/4$ ($x &lt; x_c$)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$k = f/2$ ($x &gt; x_c$)</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>9</td>
<td>9</td>
<td>50</td>
<td>Nonlinear, $\gamma = 0.02236$ km$^{-1}$</td>
<td>2m-p, 3</td>
</tr>
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<td>e</td>
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<td>6</td>
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<td>6</td>
<td>100</td>
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<td>4c-d</td>
</tr>
<tr>
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<td>6</td>
<td>500</td>
<td>Linear, $k = f/2$</td>
<td>5a-b</td>
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<tr>
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<td>6</td>
<td>12.5</td>
<td>Linear, $k = f/2$</td>
<td>5c-d</td>
</tr>
</tbody>
</table>

### a. Case a

Figure 1 depicts trajectories of air starting from $x, y = 0$ at times $t = 0, 6, 12, 18$ h. Since the initial velocity is adjusted so that initial forces are in equilibrium, and since $v_i$ is vanishingly small, all initial trajectories are straight and make an angle $[\arctan(k/f)]$ of 26.6$^\circ$ with the $x$-axis. The air parcels arrive at the shoreline at different times of the day and are deflected according to the value of $v_i(x, t)$ in the coupled Eqs. (1) and (2). For example, an air parcel reaching the shore when $v_i$ is reaching its maximum positive value, will experience a decrease of its onshore component, $u$, which, in turn, leads to a northward acceleration. It may be noted that air parcels arriving at various times at some inland point $x, y$ may originate from offshore points separated by as much as 100 km of latitude.

The parcel oscillations produced by passage through the field of $v_i(x, t)$ are largely damped out by the time the parcel is 150 km inland ($x = 300$ km).

Figure 2, panels a–d, depicts wind hodographs that would be recorded at various $x$ from 125 km (25 km offshore) to 250 km (100 km inland).

![Figure 1. Four selected trajectories for case a. Parenthetical numbers are local times (in hours) of departure from $x = y = 0$. Numbers along trajectories are local times (in hours) at which those points are reached, with 24 to be subtracted from numbers in excess of 24. On the lower left, all trajectories are the same; only the times are shown for departure at $t = 0$ h.](image)

All hodographs have a roughly elliptical shape with an egglike distortion. The pinching on the lower right-hand side of the hodograph is probably the result of faster passage at these times of predominantly eastward-moving parcels. Eccentricity decreases with increasing $x$. Hodograph size increases with $x$ up to a maximum somewhere between 150 and 175 km, but closer to the latter, which is 25 km beyond the “shore”, where $v_i(x, t)$ reaches a maximum. Beyond 200 km, the amplitude of the diurnal oscillation decreases rapidly, and hodographs become small and more circular.

The end point of the wind vector moves rapidly along the hodograph during times corresponding to when $v_i(t)$ is increasing or decreasing most rapidly (on the sides of the ellipses, corresponding to morning and evening), and most slowly when $v_i(t)$ is varying slowly (at the ends of the ellipses, corresponding to afternoon and late night).

At $x = 125$ and 150 km, the winds deviate more to the left of the undeflected wind $\vec{V}$, which is a vector from the origin to the small triangles.

Figure 2b also depicts the corresponding Haurwitz solution of the linear equations for the same $u_i$ and $v_i = V_x \cos(2\pi t/24)$ with $V_x = 9$ km h$^{-1}$. The elliptical hodograph of the Haurwitz case is substantially larger, as would be expected, since $v_i$ in his case is not spatially concentrated. The hodographs also have somewhat different orientations.

The axes of the hodographs rotate clockwise with increasing $x$. The axial orientation of the Haurwitz ellipse is intermediate.

### b. Case b

The parameters here are the same as in case $a$, except that $v_i$ max = 6 instead of 9 km h$^{-1}$. The results are depicted in Figs. 2e–h.

The hodographs are essentially miniature versions of those in case $a$. The eccentricities differ by no more than 0.02 at each $x$, and are possibly identical if measured more accurately on hodographs computed in more detail.

Figure 2f also depicts the Haurwitz solution for $v_i = V_x \cos(2\pi t/24)$, with $V_x = 6$ km h$^{-1}$. Again, because
$v_r$ in case $b$ is spatially concentrated, the Haurwitz hodograph is much larger.

The Haurwitz solution was also used as a test of the numerical solution. It was reasoned that the numerical solution should be equivalent to the Haurwitz solution if the parcel is subjected to an essentially spatially invariant but sinusoidally varying $v_r$, provided sufficient time has elapsed for effects of initial conditions to disappear. In (10), $X$ was given the value 50 km for $x \leq 150$ km but 5000 km for $x > 150$ km. The hodograph at $x = 250$ km for the numerical solution is indistinguishable from the Haurwitz hodograph.

c. Case $c$

The parameters are the same as in case $b$, except that friction is low over the sea and high over land. The equilibrium angle of cross-isobaric flow is 14.0° over the sea and 26.6° over land.

Except at $x = 250$ km, the hodographs are larger than in case $b$. Because of smaller friction, larger hodographs would be expected offshore and at the coast, but the effect of reduced friction offshore apparently extends for some distance inland. This is perhaps related to an inertial oscillation connected with the abrupt change of friction.

d. Case $d$

A nonlinear friction proportional to the velocity squared is more realistic than linear friction, and nearly as easy to handle in the numerical solution. This was achieved by writing $k$ (heretofore treated as a constant) as
where $\gamma$ is a constant. The initial, unaccelerated wind components, $u, v$ ($t = 0)$ and $k$ (=f/2) for cases a, b were used to calculate a numerical value $2.236 \times 10^{-2}$ km$^{-1}$ for $\gamma$ that produces the same initial straightline trajectory depicted in Fig. 1. When the parcel approaches the shoreline where $v_x \neq 0$, it accelerates, and friction works nonlinearly to produce somewhat different trajectories and hodographs as compared to the linear case.

In the numerical solution, because of the nonlinearity, $x, y(t + \Delta t)$ are not so easily isolated, as indicated by (7), (8), and (9). The relationships for these quantities are now implicit, since they appear in (11) when $u$ and $v$ are expressed by centered differences, and $k$ from (11) is used in (8) and (9). In order to keep the same general procedure, a simple iteration was used. An initial $k$ was computed using $x, y(t - \Delta t)$ and $x, y(t)$ and backward differencing. Then $x, y(t + \Delta t)$ were computed using (8) and (9). The latter values were then used with $x, y(t - \Delta t)$ to recompute $k$ by centered differencing. This was then used in (8) and (9) to compute improved values of $x, y(t + \Delta t)$.

The results are depicted in Figs. 2m–p. They are to be compared with the results of case a (Fig. 2a–d), since, in both cases, $v_x = 9$ km h$^{-1}$. The principal effects of nonlinear friction are a reduction in size of the hodographs and an increase in the eccentricity at larger values of $x$. The nonlinear hodographs are somewhat more elliptical and less egg-shaped.

Again, as in previous cases, the axes of the ellipses show a clockwise rotation moving inland.

Figure 3 summarizes some of the features of case a–d by depicting the variation of eccentricity and major axis length with $x$.

e. Case e

This case is the same as case b, except that $X = 25$ km (instead of 50 km), which concentrates $v_x$ into a narrower range of $x$. Figure 4a, b depicts the hodographs at $x = 150$ and 200 km. At 150 km the hodograph is smaller and very narrow compared to the hodograph in case b. The hodograph axes rotate very rapidly clockwise with increasing $x$; already at 200 km, the hodograph has a very different orientation. The trends noted in the previous cases occur over a much smaller range of $x$.

f. Case f

This case is the same as in cases b and e, except that $X = 100$ km, which spreads $v_x$ over a wide range of $x$. Figure 4c, d depicts the hodographs at $x = 150$ and 200 km. As can be anticipated from the previous cases, the hodographs are larger and the axis of the hodograph at 200 km shows little rotation from that at 150 km.

The hodographs are also more elliptical than in the cases of smaller $X$ (more spatially limited $v_x$ oscillation).

\[ k = \gamma (u^2 + v^2)^{1/2}, \] (11)

\[ \text{Fig. 3. Eccentricity and length of major axis (arbitrary units) as functions of } x \text{ for cases a–d. Points labeled e are major axes for case e at 150 and 200 km.} \]

\[ g. \ Case \ g \]

This case serves as another test of the numerical solution by showing, in the absence of a prevailing wind, the evolution of the Haurwitz solution. The parcel was placed at (150, 0 km) at $t = 0$ and at $t = \Delta t$ later. The quantity $X$ was set equal to 500 km to assure a negligible spatial decrease of $v_x$. Figure 5a depicts the subsequent trajectory. Since $v_x > 0$ at $t = 0$, the parcel at first moves to the west, then is deflected to the right by the Coriolis force. The transient solution quickly vanishes, leaving the steady periodic trajectory and the elliptical Haurwitz hodograph in Fig. 5b. (The elliptical hodograph would be observed both following the parcel and locally.) The rapid vanishing of the transient solution is to be expected, since, where analytic solutions of the linear problem are obtained, the transient terms are proportional to $e^{-\lambda t}$, implying an $e$-folding time $\tau = k^{-1} = 2/f = 0.23 \text{ d} = 5.52 \text{ h}$. 

h. Case h

This case differs from g by severely limiting the spatial distribution of $v_x$ by taking $X = 12.5$ km. The results are depicted in Fig. 5c, d. The trajectory is now smaller, reflecting the smaller $v_x$ everywhere except at $x = 150$ km. The hodograph in this case is again derived following the trajectory, but cannot be interpreted as that to be observed locally at $x = 150$ km, as in case g, because of the strong spatial dependence of $v_x$. There
is no obvious way to get a coastline hodograph by particle dynamics when \( u_x = 0 \) and \( v_x \) is strongly dependent on \( x \).

For different starting times, the transient solution varies, but the final, steady-periodic solution remains the same, with two crossings of the coastline over the diurnal period. What local winds at the coastline might be at other times cannot be determined.

4. Momentum advection

Departures of the hodographs from the Haurwitz solutions are related, in part, to momentum advection. The \( x \) and \( y \) components are

\[
\begin{align*}
-u_x &= \frac{\partial u}{\partial t} - \frac{du}{dt} \\
-u_x &= \frac{\partial v}{\partial t} - \frac{dv}{dt}.
\end{align*}
\]  

The local derivatives can be obtained from the winds recorded at different times at a specified \( x \), in effect from the data used to construct the hodographs. The individual derivatives are computed from the changes following the trajectories. The components of momentum advection are then obtained from the differences on the right-hand side of (12) and (13).

The variations over 24 h at \( x = 150 \) km ("shoreline") and 200 km of the quantities in (12) and (13) for case \( u \) are depicted in Fig. 6. Panels a and b show that, at 150 km, momentum advection is, on average, smaller than the local and individual changes, but contributes substantially and importantly to their difference. For example, negative advection of \( u \) contributes substantially to making \( \partial u/\partial t \) less than \( du/dt \) from \( t = 5 \) to 17 h.

At 200 km (panels c, d), the advection of \( u \) is comparable in magnitude to \( \partial u/\partial t \) but a little less than \( du/dt \). On the other hand, the advection of \( v \) is only a small difference between \( \partial v/\partial t \) and \( dv/dt \).
Comparison of 150 and 200 km curves indicates a much greater inland variation of advection and individual change than local wind change. The latter, of course, is related to the hodographs, and, although they evolve inland, the general features are not drastically changed between 150 and 200 km.

5. Concluding remarks

The character of observed wind hodographs may reflect numerous factors not envisioned in the theory of Haurwitz and the extended theory of KA. Apart from the difficulty of assessing the consequences of complex terrain and particle dynamics on the predicted hodographs, that theory does not take into account the localized nature of the sea breeze and the spatial evolution of wind hodographs. This evolution includes variation in size, shape, eccentricity, and tilt angle. The statistics of AKA are developed partly from data from eastern Washington State and central California. Although diurnal wind variations are pronounced in these topographically complex regions, they reflect diverse mesoscale forcings largely unrelated to each other or those in the distant coastal regions. Specifically, the directions and phases of the forcings are different, and the position of stations relative to directions of maximum forcing are even more questionable than for stations near a coast. Thus the averaging of data from these diverse regions to obtain hodograph eccentricities and tilt angles leads to results of uncertain significance.
The emphasis here has been on clarification of the dynamical nature of Haurwitz-type sea-breeze theory. It has been shown that it has deficiencies limiting its application that go beyond the long-appreciated deficiencies of particle dynamics, friction, and one-dimensionality. On the other hand, few, if any, modern sea-breeze theories attempt to account for the ubiquitous mean diurnal wind variations in complicated terrain. Most modern studies, like that of Haurwitz, are episodic, looking at evolution throughout one diurnal period, or assuming an unchanging large-scale pressure field such that one day’s wind variations are like any other. For complicated terrain and a variable large-scale pressure gradient force, it is doubtful if present models account for mean (over one month) hodographs any better than do simpler particle-dynamics models.

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