Determination of a Z–R Relationship for Snowfall Using a Radar and High Sensitivity Snow Gauges

YASUSHI FUJIYOSHI, TATSUO ENDOH, TOMOMI YAMADA, KAZUHISA TSUBOKI,
YOSHIHIRO TACHIBANA AND GOROW WAKAHAMA

Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan

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ABSTRACT

A best-fit power-law relationship ($Z = 427R^{1.09}$) between 1-minute integrated averages of snowfall rate (R) and radar reflectivity factor (Z) was determined on the basis of observations made by using high sensitivity snow gauges (accuracy 0.03 mm h^{-1}) and a radar (wavelength 3.2 cm, beamwidth 1.1°) of three 1987 Sapporo snowstorms. The relationship $Z = 554R^{0.88}$, using 30-minute integrated averages of Z and R, produced the best radar estimate of total snowfall. The ratio of the estimated to the observed amount of snowfall decreased with increasing density of new fallen snow ρ, the ratio roughly equaling 1, when $\rho \approx 0.05$ g cm^{-3}.

1. Introduction

Many attempts have been made to obtain Z–R relationships for snowfall since Langille and Thain (1951) first published their results. Two principal methods have been proposed to elucidate the relationships:

1) The Z–R relationships are calculated only from surface observations of snow size distribution (for example, Imai et al. 1955; Gunn and Marshall 1958; Sekhon and Srivastava 1970; Ohtake and Henmi 1970; Yoshida 1975; Harimaya 1978; Yagi et al. 1979). Radar data are not used in this method.

2) The Z–R relationships are obtained from radar data and data from either measurements of snowfall intensity on the ground or the amount of snowfall estimated from snow depths (for example, Langille and Thain 1951; Marshall and Gunn 1952; Kodaira and Inaba 1955; Carlson and Marshall 1972; Jatila 1973; Wilson 1975; Boucher 1978; Collier and Larke 1978; Boucher 1981; Boucher and Wieler 1985). As the radar-determined quantity is the effective reflectivity factor Z_e [see Eq. (1)], this method leads to Z_e–R relationships.

Calculation of Z and R using method 1 requires accurate measurements of the density and falling velocity of each snow particle. In the works cited above, however, these values were derived from an empirical function of particle size. Furthermore, it is not clear to what degree the variability of the Z–R relationship found in each observation is due to the real variation in the relationship within each snowstorm observed, because of the relatively small sampling area. Accuracy of measurements of both Z and R by method 1 is therefore questionable. Another defect of method 1 is that the value of Z is calculated on assumption of the Rayleigh scattering theory, but, it cannot be concluded that Z is always equal to Z_e for every snowfall event. Therefore, some problems still remain in applying the Z–R relationship obtained by method 1 to actual radar observations.

Only Langille and Thain (1951) measured Z and R independently by method 2 to obtain the Z–R relationship. Their result is unsatisfactory, however, because of poor accuracy and an insufficient number of measurements; that is, the time interval of their measurement was 5 minutes, and the output of radar signals was not digitized. In many other studies R was calculated using Z–R relationships obtained already by other investigators. The calculated amount of snowfall was then compared with the actual amount of snowfall and the reasons for the discrepancies between them were discussed.

Measurements of snowfall intensity, R, on the ground and effective reflectivity factor, Z_e, of the falling snow should be made as simultaneously as possible to obtain the highest correlation between these two quantities. In all of the earlier work mentioned above, measurements of Z_e and R have not been made with the high degree of simultaneity and accuracy reported in this work.

Corresponding author address: Dr. Yasushi Fujiyoshi, Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan 060.

2. Data

a. Observation site

Observations were made during three periods of snowstorms in Sapporo (43°03′N, 141°20′E), Japan: Case I (1725 JST 5 February–0300 JST 6 February 1987), Case II (0930 JST 14 February–0200 JST 15 February 1987) and Case III (2109 JST 16 February–0231 JST 17 February 1987). Table 1 summarizes mean values of air temperature, direction and velocity of the wind, the total amount of precipitation, the maximum precipitation intensity and the number of data in each case. The wind direction data indicates that the snowfall observed was derived from band clouds forming in the typical winter monsoon period in Japan. Aggregates composed of rimed dendritic types of snow crystals predominated in the snowfall.

Three sites were selected for snowfall observations. They were spaced at intervals of 100 m in the east–west direction from a center point located at azimuth 358.2° and range 8.7 km from the Institute of Low Temperature Science, Hokkaido University, where the radar was located. Although the three points were set in an area where the wind blew rather weaker than in the surroundings, the amount of snowfall was up to 50 percent lower at the western site than that at the other two sites. This was because that the wind blew harder at the western site than at the other two sites. On the other hand, snowfall data from the other two sites were consistent to within 5%, showing that snowfall occurred in a similar manner at least within a range of 100 m. We used the snowfall data obtained at the central site in the following analysis.

Location of each observation site was identified by the radar, which tracked a radar reflector hanging from a balloon floating 300 m above the ground on a windless day. At the same time the balloon was moved up and down to check whether the radar could exactly follow the radar reflector all the way from the ground to the sky. No barriers, such as tall buildings, existed along the line of sight between the radar site and the observation sites.

TABLE 1. Mean values of surface air temperature (T; °C), wind velocity (WV; m s^{-1}), wind direction (WD; °), maximum precipitation intensity (R_{max}; mm h^{-1}), the water equivalent total amount of precipitation (R_T; mm) and number of data (N) in the cases of each of three snowstorms.

	T (°C)	WV (m s^{-1})	WD (deg)	R_{max} (mm h^{-1})	R_T (mm)	N
Case I	−2	5	300	15.6	11.05	353
Case II	−7	6	320	3.9	6.85	699
Case III	−7	5	310	4.9	3.73	221
				Total	21.63	1273

TABLE 2. Principal characteristics of the radar used in this study.

Transmitting frequency	9,445 MHz
Peak transmitting power	40 kw
Pulse width	0.5 μs
Pulse repetition frequency	2000 pps
Beamwidth	1.1°
Antenna	parabolic-type (2 m in diameter)
Minimum detectable power	−110 dB*m*
Antenna gain	43 dB

b. Measurement of snowfall intensity

A windbreak was formed from frames (3 m × 3 m × 2.5 m) to which wire netting was attached. An electrobalance was set in a wooden box (1 m × 1 m × 0.8 m) put in the windbreak. A light cylindrical container 48 cm in diameter and 15 cm in height was put on the electrobalance. The wall of the container was so thin that falling snow particles did not pile on its edge. The weight of snow particles deposited in the container was measured at one-minute intervals. Water equivalent snowfall intensity (mm h^{-1}) was derived from the change in weight during one minute, and recorded locally on a portable computer (Konishi et al. 1988). The resolution of the electrobalance was 0.1 g, which results in the measurement accuracy of snowfall intensity being 0.03 mm h^{-1}.

c. Data on snow depth

Changes in snow depth were observed with an 8-mm motion picture camera, which took pictures of a pole standing in the snow field every 90 seconds. The camera was positioned at the central observation site. Time data (hour, minute and second) were simultaneously recorded on the film. The density of the newly fallen snow was obtained from the change in snow depth and the amount of snowfall measured by the weighing snow gauge. Temperature, relative humidity, atmospheric pressure, wind direction and velocity, visibility, and solar radiation were automatically monitored at 1 minute intervals at the center observation site.

d. Radar data

Table 2 shows the principal characteristics of the radar used in this study. A more detailed description of the radar may be found in a paper by Fujiyoshi et al. (1986). Even if Rayleigh scattering is not applicable, the received power P_r is practically expressed by

$$P_r = C\frac{1}{r^2}\left|\frac{m^2-1}{m^2+2}\right|^2 Z_e, \tag{1}$$

where C is the radar constant calculated from the data in Table 2; r is the distance of scatterers from the radar and m is the complex refractive index.

Snowfall intensity on the ground corresponds to PPI data (elevation angle 1.5°) obtained every 3 minutes.

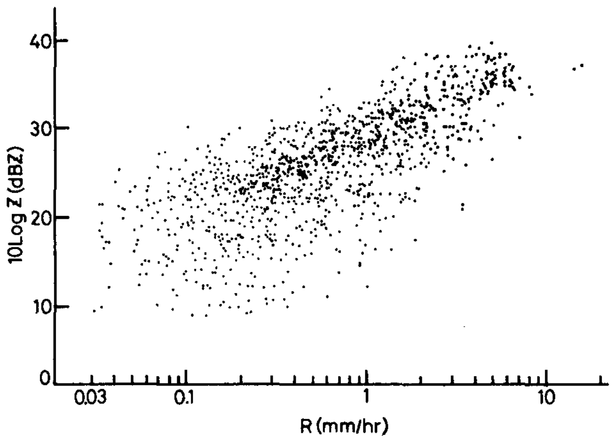

FIG. 1. Comparison of Z obtained by the radar with R found using the high sensitivity snow gauges.

Resolution of received power is better than 1 dB. The elevation and the width of the radar pulse above the snow gauge are 228 and 152 m, respectively. A horizontal distribution of Z_e was printed out every 3 minutes. The one minute interval value of Z_e above the snow gauge was calculated as follows: First, direction and velocity of echo movement were checked every hour and a line was drawn from the position of the observation site. The direction and length of the line correspond to the echo movement and the distance which the echo moved in 3 minutes, respectively. Second, the line was divided into three equal length portions. Last, the values of Z_e were read out and averaged along each segment of the line. This procedure assumes that the Z_e distribution within the snow cloud remained constant for the 3-minute period; our observations suggest that this is a valid assumption.

Series of data on Z_e and R collected every minute were cross-correlated, the time difference giving the best correlation between Z_e and R was less than 3 minutes. Strictly speaking, Z_e is different from Z, but we will write Z_e as Z in the following sections.

3. Results

All 1273 values of Z and R are plotted in Fig. 1. If a regression line is made over the whole range of R, the resultant line will be strongly influenced by the relatively few extreme points existing at both sides of the figure. The primary purpose of this study is to obtain the most reliable Z–R relationship with as small differences as possible between T_R (the total amount of precipitation calculated from Z) and T_B, the total amount of precipitation measured by the snow gauge for the entire observation period. Therefore, we made regression lines over various ranges of R and compared T_R with T_B. Table 3 shows the ratios of T_R with T_B, which were calculated from the Z–R relationships obtained for different ranges of R. As shown in the table, the smallest ratio was obtained when the range of R lies from 0.1 to 3.0 mm h^{-1}. The resultant Z–R relationship obtained by regression on Fig. 1 in this range of R is

$$Z = 427R^{1.09}. \qquad (2)$$

The correlation coefficient of Eq. (2) is 0.64. Table 3 also shows the T_R/T_B calculated from the Z–R relationship given by Gunn and Marshall (1958):

$$Z = 2000R^{2.0}. \qquad (3)$$

Estimated precipitation using Eq. (3) is less than 50% of the measured precipitation.

Figure 2 shows the relationship between the density of new fallen snow ρ (g cm^{-3}) and the ratio T_R (30 min)/T_B (30 min), where T_R (30 min) is the estimated amount of precipitation for each 30-minute period obtained from Z by Eq. (2), and T_B (30 min) is the amount of precipitation measured by the snow gauge. Only cases in which the depth of new fallen snow was larger than 1 cm for 30 minutes are plotted in the figure. Points determined from the Z–R relationship tend to have T_R larger than T_B when ρ is less than 0.05, whereas the reverse tends to be true when ρ is larger than 0.05. This can be explained by snow particles with higher densities falling more rapidly. Although the correlation between the two is weak, the depth of new fallen snow can be roughly estimated by Eq. (2) when 0.05 g cm^{-3} is used as the value of ρ.

4. Discussion

Figure 3 summarizes data on the precipitation intensity measured at one minute intervals from 27 January 1985 to 6 May 1987 in Sapporo. Total number of data and total amount of precipitation were 19 202 and 252.6 mm, respectively. The ordinate is the frequency of occurrence of a given precipitation intensity conditional to the occurrence of snow (N_T%) or the

TABLE 3. Ratios of T_R to T_B (T_R/T_B) calculated from Z–R relationships obtained for different ranges of R.

	Range of R (mm h^{-1})							
	0.1–2.0	–2.5	–3.0	–3.5	–4.0	–4.5	All	$2000R^{2.0}$
Case I	1.11	1.11	1.09	1.14	1.14	1.14	1.52	0.30
Case II	1.75.	1.74.	1.73.	1.79.	1.78	1.78	2.15	0.66
Case III	0.83	0.83	0.82	0.85	0.85	0.85	1.01	0.34
I + II + III	1.26	1.26	1.25	1.30	1.29	1.29	1.63	0.42

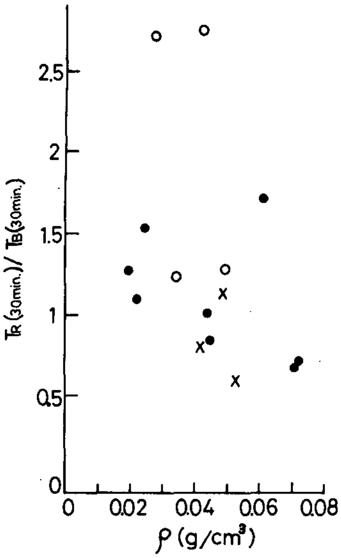

FIG. 2. Variation of ratio T_R (30 min)/T_B (30 min) with density of new fallen snow ρ. ●: Case I; ○: Case II; ×: Case III.

fractional contribution to the total precipitation of intensity in the range $R - 0.1$ to $R + 0.1$ mm h^{-1} (R_T%). When R is lower than 6 mm h^{-1}, then N_T (or R_T) is approximated by the following exponential functions:

$$N_T = \begin{cases} 40.9 \exp(-2.1R), & R \leqslant 1 \\ 12.8 \exp(-0.94R), & R > 1 \end{cases} \quad (4)$$

$$R_T = \begin{cases} 10.3 \exp(-0.49R), & R < 3 \\ 18.7 \exp(-0.69R), & R > 3. \end{cases} \quad (5)$$

It follows from Eqs. (4) and (5) that N_T equals to 0.76% when R is 3.0 mm h^{-1}. And the amount of precipitation in the same range of R contributes to the total amount of precipitation by only 2.4%. The cumulative contribution ratio to the total precipitation attains 80% when $R = 3.0$ mm h^{-1}. This result means that the most of precipitation intensity of snow clouds in Sapporo is less then 3 mm h^{-1} and that the three snowstorms reported here did not show exceptional frequency distributions of R. Naturally Eqs. (4) and (5) are different from place to place. Thus, the range of R should be selected carefully based on ground measurements of precipitation intensity to make an accurate Z–R relationship for any other place.

Figure 4 shows the change in correlation coefficient of the Z–R relationship as a function of period of time over which R and Z are integrated and averaged. In the figure $\bar{Z}$ and $\overline{10 \log Z}$ refer to the averaged values of Z and 10 log$Z$, respectively. That is,

$$10 \log\bar{Z} = 10 \log\{(Z_1 + Z_2 + \cdots + Z_n)/n\} \quad (6)$$

$$\overline{10 \log Z} = 10 \log\{(Z_1 \cdot Z_2 \cdots Z_n)^{1/n}\}. \quad (7)$$

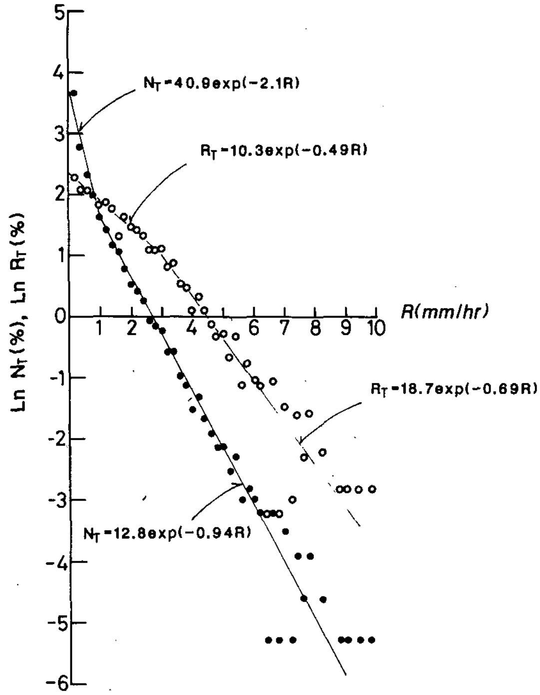

FIG. 3. Frequency of occurrence of a given precipitation intensity conditional to the occurrence of snow (N_T%) and the fractional contribution to the total precipitation of intensity (R_T%) in the range between $R - 0.1$ and $R + 0.1$ mm h^{-1}. The data were obtained from 27 January 1985 to 6 May 1987 in Sapporo.

As expected, correlation coefficients of $\bar{Z}$ are larger than those of $\overline{10 \log Z}$. Correlation coefficients of both $\bar{Z}$ and $\overline{10 \log Z}$ obtained in the whole range of R are higher than those obtained in the range $0.1 \leqslant R \leqslant 3$ mm h^{-1}. The lowest correlation coefficient was obtained in the case where the averaging time was set to be 1 minute. In contrast, the correlation coefficient remained almost constant for averaging times longer than 5 minutes. Short-term variations in vertical air velocity, particle fall velocity, type of snow crystal, particle size distribution, etc., will cause changes in the Z–R relationship. Figure 4 indicates that the effect of variations in these factors can be neglected when the averaging time is longer than 5 minutes.

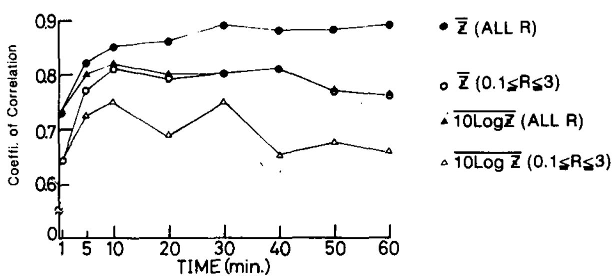

FIG. 4. Changes in correlation coefficient of the Z–R relationship as a function of the averaging time (min) over which both Z and R were integrated and averaged.

TABLE 4. Ratios of T_R to T_B (T_R/T_B) calculated from Z–R relationships made from time-averaged Z and R.

Averaging time (min)	$\overline{10\log Z}$–$\bar{R}$	$\bar{Z}$–$\bar{R}$
1	1.63 (1.25)	1.63 (1.25)
5	1.26	1.20 (1.20)
10	1.17	1.19 (1.22)
20	1.30	1.18 (1.24)
30	1.45	1.13 (1.19)
40	1.26	1.13 (1.19)
50	1.38	1.15 (1.24)
60	1.42	1.11 (1.20)

Different Z–R relationships for the whole range of R were made according to the averaging time. The ratios of T_R to T_B (T_R/T_B) in Table 4 were calculated by converting $\bar{Z}$ to R from these $\bar{Z}$–$\bar{R}$ relationships. Therefore, T_R was calculated from $R(Z)$. Other values of T_R/T_B were calculated by converting Z (1 minute) to R using each Z–R relationship. In this case T_R is calculated from $R[Z(1\ \text{min})]$. The ratios are shown in parentheses in Table 4. As expected from Fig. 4, the ratios obtained from the $\bar{Z}$–$\bar{R}$ relationships are closer to 1 than those obtained from $\overline{10\log Z}$–$\bar{R}$ relationships. The ratios tend to approach 1 with increasing averaging time when we use $\bar{Z}$–$\bar{R}$ relationships; however, the ratio in parentheses is the smallest when the averaging time is 30 minutes.

Figure 5 shows solutions of the equation $Z = BR^{\beta}$ obtained over the whole range of R for different averaging times. As shown in the figure, both B and β tend to decrease with increasing averaging time when $\overline{10\log Z}$ values are used. In contrast, β changes only slightly, while B tends to increase with increasing averaging time when $\bar{Z}$ values are used. Furthermore, both B and β are almost constant and independent of the averaging time for averaging times longer than 30 minutes. Figure 6 shows $\bar{Z}$ (30 minutes) as a function of $\bar{R}$ (30 minutes). In this case, the correlation coefficient is 0.89 and the Z–R relationship is expressed as

$$Z = 554R^{0.88}. \tag{8}$$

Figure 7 shows B and β measured by previous investigators together with the points marked ⊕ and ◎ found from Eqs. (2) and (8). Solid and blank circles

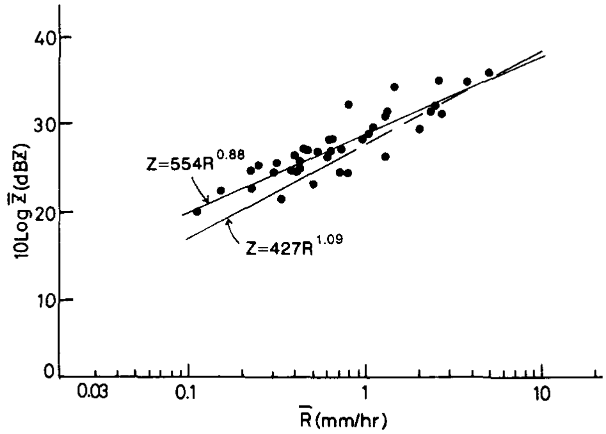

FIG. 6. Same as Fig. 1 except both Z and R are 30-minute integrated averages.

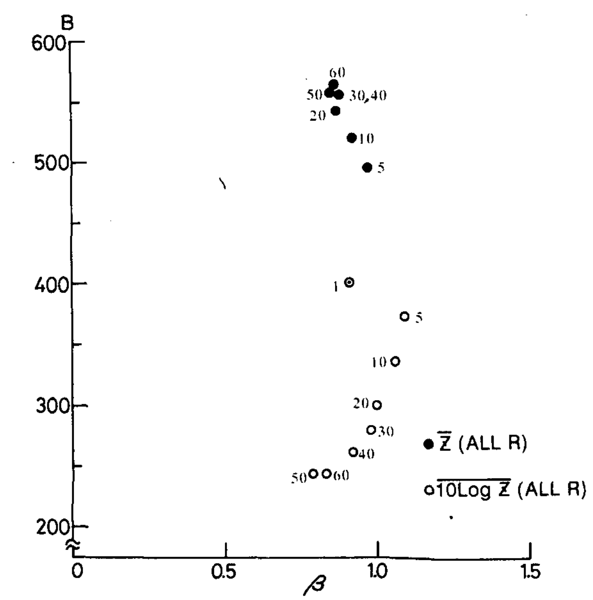

FIG. 5. Changes in B and β as a function of averaging time (min).

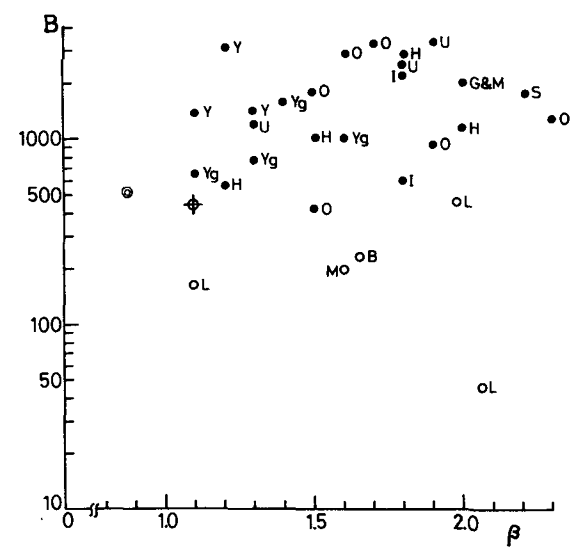

FIG. 7. Values of B and β from the literature cited below. The values obtained by the present study are shown by the symbols ⊕ and ◎. B: Boucher and Wieler 1985; H: Harimaya 1978; L: Langille and Thain 1951; O: Ohtake and Henmi 1979; Y: Yoshida 1975; G&M: Gunn and Marshall 1958; I: Imai et al. 1955; M: Marshall and Gunn 1952; S: Sekhon and Srivastava 1970; Y_g: Yagi et al. 1979.

in the figure were obtained through methods 1 and 2, respectively. Our values for both B and β are rather small when compared with data found by other measurements. The value of β measured by Yoshida (1975) at Sapporo is similar to the values obtained by the present authors. It should be noted again that the Z–R relationship must be constructed considering the precipitation characteristics at any given place of observation.

Fujiyoshi et al. (1983) investigated the relationship between snowfall intensity and visibility (V km) in Sapporo, with the following result:

$$V = 1.0R^{-2/3}. \tag{9}$$

Lowering of the visibility caused by falling snow alone can be estimated remotely using radar by combining Eqs. (2) [or (8)] and (9).

5. Summary

Snowfall intensity was measured every minute by weighing the amount of snowfall with an accuracy of 0.03 mm h^{-1} at a point 8.7 km distant from a radar observation site. Mean surface air temperature ranged between $-2°$ and $-7°$C during the observations (February 1987). Aggregates composed of rimed dendritic snow crystals were predominantly observed. The radar used had the wavelength of 3.2 cm, a pulse width of 0.5 μs, and a beamwidth of 1.1°. The elevation angle of the antenna was 1.5°. Resolution of echo intensity was better than 1 dB.

Regression lines between Z and R were made for different ranges of R. The calculated amount of snowfall, T_R, estimated from the radar data using a Z–R relationship was then compared with the snowfall, T_B, measured on the ground by high sensitivity snow gauges. The ratio between the two values, T_R/T_B, is nearest to 1 when the range of R was from 0.1 to 3.0 mm h^{-1}. The Z–R relationship for this range of R is expressed as the following equation:

$$Z = 427R^{1.09}.$$

The value of T_R/T_B decreased with increase in density of new fallen snow ρ. The value roughly equals 1, when $\rho \approx 0.05$ g cm^{-3}.

Values of both Z (or 10 log$Z$) and R obtained every minute were integrated and averaged for between 5 and 60 minutes to obtain Z–R relationships. The correlation coefficient is the lowest when the averaging time is 1 minute and remains almost constant when the averaging time is longer than 5 minutes. The correlation coefficients between $\bar{Z}$ and $\bar{R}$ ($\bar{Z}$ and $\bar{R}$ are the average values of Z and R, respectively) are always larger than those between $\overline{10 \log Z}$ and $\bar{R}$ ($\overline{10 \log Z}$ is the average value of 10 log$Z$). The closest value of T_R/T_B to 1 was obtained using 30-minute integrated averages of Z and R, when the following equation was found:

$$Z = 554R^{0.88}.$$

From these results we note four important points in determining a Z–R relationship at any given place: 1) the local precipitation characteristics (especially the range and frequency distribution of precipitation intensity) should be studied; 2) Z should be measured at intervals of 1 minute or less (the shorter the better); 3) 30-minute integrated averages of Z and R should be used; and 4) the value of Z (not 10 log$Z$) should be used to calculate the integrated average of Z.

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