NOTES AND CORRESPONDENCE

The Variations of C_n^2 Between 4 and 18 km above Sea Level as Measured over 5 Years

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ABSTRACT

We computed the monthly average backscattered power over a five-year period for the Fleming 50 MHz wind profiler, which is proportional to C_N^2. We found that in addition to seasonal cycle in C_N^2 below the tropopause, there was a year-to-year variation as well. Above the tropopause, the seasonal variations were almost gone; however, there were significant changes with periods longer than one year. We examined a shorter backscattered power record from the Stapleton wind profiler and found similar longer-term trends. These long-term trends will affect the performance of wind profilers.

1. Introduction

During the last several years meteorologists and physicists have studied atmospheric phenomena with radars that scatter signals from refractive index fluctuations. They have found that these fluctuations vary by several orders of magnitude. With the development of clear-air radars for wind profiling, and the installation of a demonstration wind profiler network,[1] understanding the behavior of these variations in backscattered signals is of major importance. In this study, we used five years of data from a 50 MHz profiler situated at Fleming, Colorado, and computed the average backscattered power over 30-day intervals at several heights; we found not only annual variations in the backscattered signals, but longer-term trends as well. We do not attempt to explain the variations we see here; that must be done in a future study. Our objective is to make the community aware of these results to aid in future profiler designs and studies.

2. Background

The radar equation is given by (Probert-Jones 1962)

$$\bar{P}_r = \frac{c}{(1024\pi^2 \ln 2)}(P_t\tau\lambda^2 G_0{}^2\theta\phi)\eta/R^2, \quad (1)$$

where

- $\bar{P}_r$ received power, averaged over many independent realizations of the position of the scatterers (W)
- c velocity of propagation (m s^{-1})
- P_t transmitted power (W)
- τ pulse duration (s)
- λ radar wavelength (m)
- G_0 on-axis gain of the antenna (Gaussian beam pattern assumed)
- θ, ϕ half-power beamwidths of the antenna
- η radar reflectivity (m^{-1}) or radar cross-section per unit volume
- R range to the radar resolution cell; i.e., the volume defined by the angular resolution of the antenna and the radial resolution of the radar pulse (m).

This equation shows that the backscattered power, $\bar{P}_r$, is proportional to the radar reflectivity η. In clear air, the reflectivity can be related to refractive index fluctuations whose spatial components are at a scale of $\lambda/2$ (Tatarskii 1961),

$$D_n(r) = [\overline{n(\mathbf{x}) - n(\mathbf{x}+\mathbf{r})}]^2, \quad (2)$$

where n is the refractive index, $\mathbf{x}$ is the position coordinate, and r is a spatial separation variable. He showed that for homogeneous and isotropic turbulence the structure constant can be expressed as

$$D_n(r) = C_n^2 r^{2/3}, \quad (3)$$

where C_n^2 is the refractive index structure constant.

If the value of $\lambda/2$ lies within the inertial subrange of turbulence, then the radar relflectivity is (Ottersten 1969)

$$\eta = C_n^2\lambda^{-1/3}. \quad (4)$$

Thus, the radar reflectivity is in direct proportion to the intensity of the turbulent fluctuations in the re-

[1] A network of 30 UHF (404.37 MHz) wind profilers is being installed in the central United States. This network should be operational in 1990.

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fractive index if one-half the radar wavelength is within the turbulent inertial subrange. These fluctuations vary by several orders of magnitude, and it is these variations that have been the focus of several investigations. The value of the refractive index structure constant or C_n^2 as a function of height is important in predicting the radar sensitivity needed to detect a signal with a clear-air radar. In addition, since these fluctuations are dependent upon the turbulent intensity and the mean refractive index gradients, knowing the refractive index fluctuations may help in understanding the atmospheric turbulent processes.

Some of our present understanding of C_n^2 comes from very sensitive clear-air radars. Nastrom et al. (1985) measured C_n^2 with a 50 MHz radar and showed that C_n^2 was approximately lognormally distributed. When they plotted the monthly measurements on a cumulative distribution plot, they found that the distributions had the same slope for each month, but the median values changed by 10 dB over the year. In addition, they found that above 8 km, C_n^2 dropped by about 2 dB per kilometer. There was a strong correlation of C_n^2 with vertical wind and horizontal wind speed, but not much correlation with the vertical shear of the horizontal wind. Warnock et al. (1986) found that during the summer, between 6 and 10 km, C_n^2 may decrease by two orders of magnitude and large convective cells may enhance the reflectivity in the afternoon by two orders of magnitude at some heights. On average, their observed C_n^2 values and variability agree well with C_n^2 model estimates (Warnock et al. 1985); however, during periods of convective activity, the model underestimates the radar reflectivity. Further comparisons are needed in different topographies, climates, and latitudes. In addition, Nastrom et al. (1986) studied the variability of C_n^2 at Poker Flat, Alaska, and Platteville, Colorado, and found that C_n^2 had two maximums during the year. The winter maximum was correlated with increased wind speed and gravity waves. A second maximum occurred in the summer, and they attributed this maximum to increased convection. On daily time scales C_n^2 appeared correlated with wind speed. The monthly means of C_n^2 were correlated with gravity waves, which were parameterized by the standard deviation of vertical velocity.

In this paper we show interannual variation in relative values of C_n^2 from measurements obtained over 5 years with one of the WPL 50 MHz wind profilers. We supported our 50 MHz measurements of C_n^2 with data from the Stapleton 915 MHz wind profiler.

3. Profiler description

The wind profiler technology is similar to that used in conventional Doppler radar systems except that the frequency is generally lower. The 915 MHz profiler uses a solid-state power transmitter and an offset paraboloid atenna; the other profilers use vacuum tube power transmitters and phased array antennas (Tables 1 and 2). We used the 9 μs radar pulse duration in this study. Note that the 50 MHz wind profiler is about 7 dB more sensitive in the 9 μs mode than is the 915 MHz wind profiler.

4. Data processing

The time duration between samples is two-thirds of the pulse duration. This processing includes time domain averaging, power spectral analysis, and averaging of spectra before we compute the spectral power. Each pulse duration mode is used 12 times per hour; the 12 mean Doppler velocity estimates from each pulse and each antenna-pointing direction are averaged by applying a random sample consensus (Fischler and Bolles 1981). At least 4 of the 12 values must form a consensus; i.e., they must lie within a window that is 1/16 of the Nyquist velocity interval (see Tables 1 and 2 for maximum and minimum radial velocities). If fewer than 4 of the 12 velocity measurements pass this consensus test, the data for that height and pulse length are rejected and no data are recorded. In general, if no consensus is reached, there is inadequate signal. If the algorithm finds a velocity consensus, it averages the signal power for that set. If the signal-to-noise ratio is below that needed to compute a reliable estimate of velocity, there is a 10% chance the consensus set will pass a false value for a given beam. These data processing steps are performed at the radar site before the data go to a central computer. We computed the average power from the hourly data supplied by the profilers, without editing the data. In the 50 MHz data, there are few times that there is no signal measurement at 8 km; almost every point within the hour passes the consensus. Above 10–12 km, more points will not pass the consensus. Since the number of points passing the

TABLE 1. Characteristics of the 50 MHz profiler.

Frequency	49.8 MHz
Authorized bandwidth	0.4 MHz
Peak power	30 kW
Average power	400 W
Pulse width	3,9 μs
Pulse repetition period	238.67, 672 μs
Antenna aperture	50 m × 50 m
Antenna pointing	15° off-zenith to north and east (2 antennas)
Antenna type	fixed phase array of colinear-coaxial dipoles
Two-way beamwidth	5°
Data processing (9 μs pulse)	
Time domain averaging	124 pulses
Spectral averages	16
Maximum radial	± 18.06 m s^{-1}
Spectral resolution (64 points)	0.56 m s^{-1}
Height sampling	
First height	2.6 km AGL
Height spacing	870 m
Number of heights	18

TABLE 2. Characteristics of the 915 MHz profiler.

Frequency	915.0 MHz
Maximum bandwidth	2.0 MHz
Peak power	5.6 kW
Average power	450 W 9 μs mode
Pulse width	1.3,9 μs
Pulse repetition period	110 μs in 99 μs mode
Antenna aperture	10 m × 10 m
Antenna pointing	15° off-zenith to north and east
Antenna type	offset paraboloidal reflector with offset horn feeds
Two-way beamwidth	1.7°
Data processing (9 μs pulse)	
Time domain averaging	46 pulses in 9 μs mode
Spectral averages	32 in 9 μs mode
Maximum radial	±18.06 m s^{-1}
Spectral resolution (64 points)	0.56 m s^{-1}
Height sampling	
First height	2.7 km AGL
Height spacing	870 m
Number of heights	18

consensus will be greater at higher signal-to-noise ratios, the measured average power estimates will be overestimated at low power levels relative to the average power at higher signal-to-noise ratios. This means that the differences in the monthly average power estimates are actually larger than we would estimate with this technique. Because we do not have an absolute calibration of each profiler, and since the backscattered power is proportional to C_n^2, we use our measurements of power in decibels to determine the variability of C_n^2.

5. Results

The archived Fleming data started on 15 April 1984 and ended on 15 January 1989. First, we averaged the power over all range gates in a 2 km interval and then averaged the power over 30 days. This averaged power will be proportional to the average C_n^2 over the same periods. The first data are averaged between 4 and 6 km mean-sea-level (MSL). (The Fleming profiler was at 1337 m above sea level and about 100 km east of the Continental Divide.) The data show (Fig. 1a) that the annual change between maximum and minimum is about 10 dB, and the magnitudes and times of the annual maximums and minimums are similar to the observations of Nastrom et al. (1985). In addition, there is a year-to-year variation in the annual maximums and minimums. For example, the first minimum occurs in October 1984 (month 6) with an average power of 53 dB. The next minimum is 1 year later and is 6 dB higher. The following annual minimums are 59, 60, and 58 dB. The annual maximums are about 65, 67, 68, and 69 dB. Figure 1b shows the backscattered power averaged between 6 and 8 km. The annual minimums show a 5 dB change, and a decrease of 2 dB over the next 3 years. In this height range, the annual amplitude changes are slightly larger, about 12 dB. In addition, the change in the maximum for the first 3 years was larger, about 8 dB compared with the change for the 2 to 4 km interval, which was about 3 dB. The biggest change at both heights occurred between February and April 1985, when the power changed by 13 dB (Fig. 1a) at the 4 to 6 km interval and by 15 dB in the 6 to 8 km interval. In the next height interval (8 to 10 km, Fig. 1c) the character of the backscattered power changes. There is a 3 dB per year increase in the annual minimum over 3 years, and the differences between the annual minimums and maximums are less, about 7 dB. It is in this height region that we found the C_n^2 minimum over most of this five-year period (Frisch et al. 1986). The height interval from 10 to 12 km (Fig. 1d) shows an almost "stair-step" behavior in the backscattered power. There are very large increases in the backscattered power (about 6 dB) in each step. The result is an apparent upward trend, with a 9 dB increase in the minimum over 48 months. In the 12 to 14 km interval (Fig. 1e) the annual variations in backscattered power seem to be gone; a major jump in the backscattered power occurs at about August 1985, where the power increases by 8 dB. Finally, in the 14 to 16 km interval (Fig. 1f) there is no indication of a seasonal variation in the backscattered power. There are large changes of almost 10 dB over a few months but without the seasonal signal as in the lower elevations.

6. Discussion

The long-term variations in backscattered power are significant; however, there are some factors not related to meteorology or climatology that might affect these measurements. First, the input power is not monitored, but there were no hardware or software changes over the five-year period so the sensitivity of the radar should be relatively constant. In addition, there is supporting evidence that the apparent trend is real. The Stapleton 915 MHz profiler data (Fig. 2) show the same type of trend for the first 28 months. In this example, the second annual minimum was 4 dB higher than the first, and the second annual maximum was 3 dB higher than the first. Month 28 shows an even higher value, some 5 dB higher than the first annual maximum. The 915 MHz profiler is of a different design and operated continuously over the period of study; its estimated change in sensitivity is less than 1 dB. Unfortunately, after the first 28 months, the operating mode was changed and the data are no longer comparable. Furthermore, the abrupt 10 dB increase in C_n^2 in the 50 MHz data (Fig. 1c) at 18 months in the 12 to 14 km interval does not occur in the 8 to 10 km interval (Fig. 1b). If there had been a hardware change that affected the sensitivity, this would have been evident at all levels.

A second potential problem with these measurements is that radar echoes from aircraft could affect

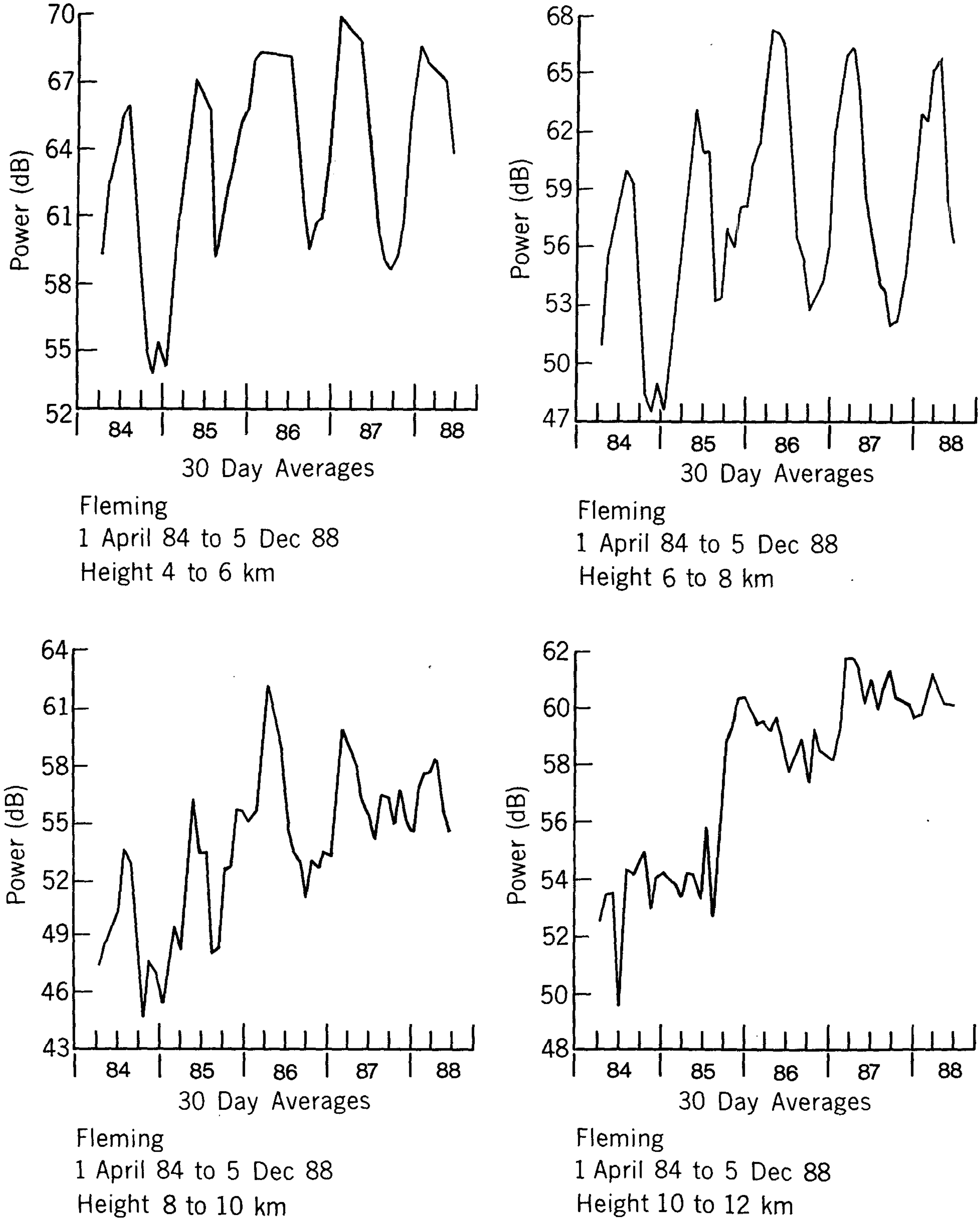

FIG. 1. Averages of backscattered power C_n^2 from data taken at Fleming, Colorado, 1 April 1984–1 March 1988, from a 50 MHz profiler at heights from 4–6 km to 14–16 km.

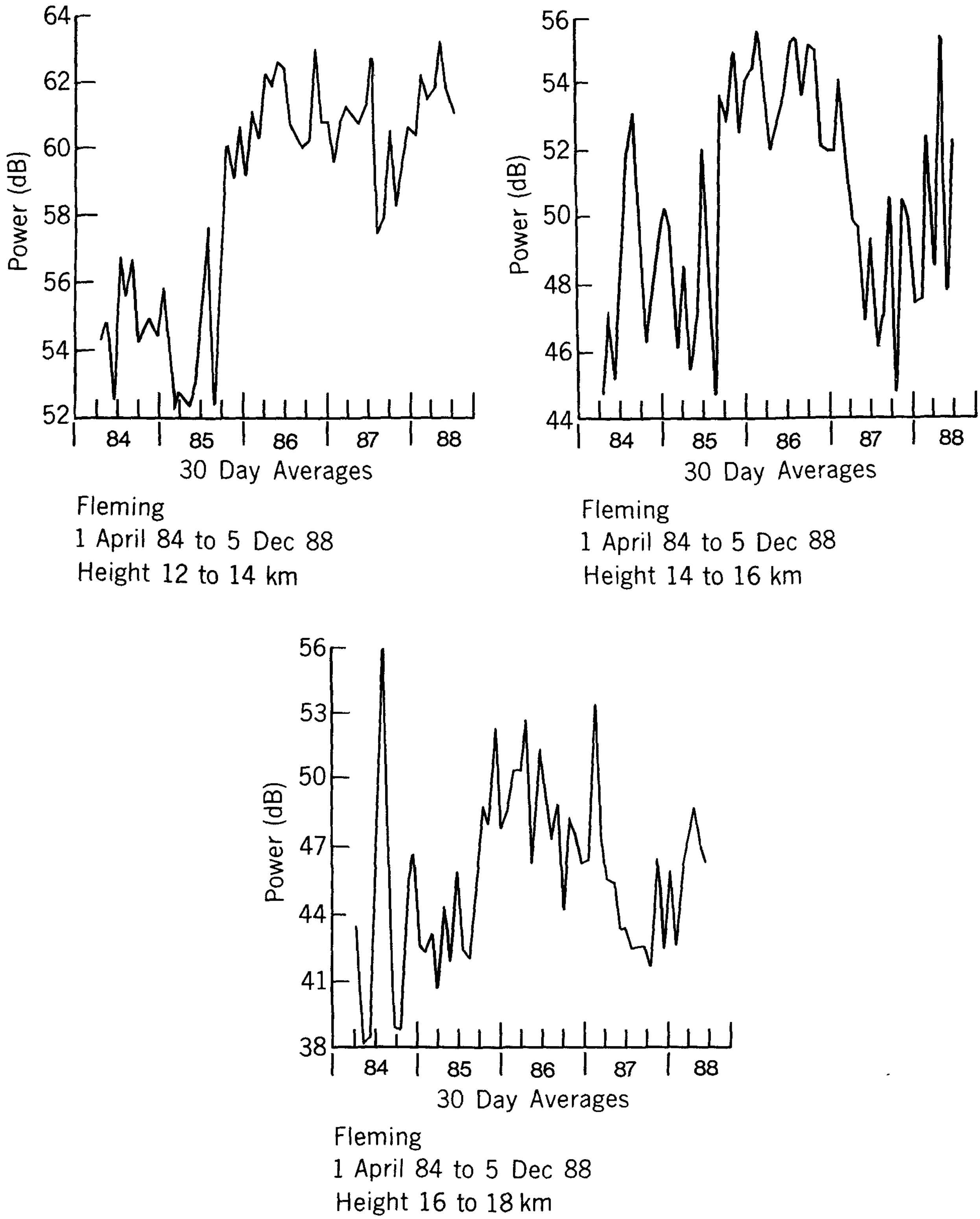

FIG. 1. (*Continued*)

the power measurements. If the speed of the aircraft were within ±9 m s^{-1} of the measured wind speed, the power measurement from the aircraft would be averaged into the clear-air power measurement. We can check for aircraft scattered power in the C_n^2 distribution function on a lognormal plot to determine if there is anything other than the lognormal distribution. Figure 3a shows the January and July 1985 and 1986 cumulative distribution functions measured at 8.3 km MSL versus power in decibels, uncorrected for range dependence. The distribution functions in all four months are almost linear and have the same slope, which is similar to the slopes reported by Nastrom et al. (1986). The median values of C_n^2 are at the 50 percent point on this plot. This figure shows that the median value of July 1986 was several decibels higher than the median value of July 1985, and the median value of January 1986 was higher than the median value of January 1985. Thus, using different methods to estimate the monthly value of C_n^2, we show an increase of several decibels from 1985 to 1986.

Figure 3b shows the cumulative power distribution at 11.8 km MSL. The distribution is almost linear on this plot until the 90th percentile, where the slope becomes much steeper. One could speculate that this change is due to aircraft; however, the aircraft would have to be in the beam or sidelobe 10% of the time and have just the right velocity to fold to the wind measurement value within 9 m s^{-1}. We believe that this is unlikely. In addition, the backscatter cross-section of an aircraft, even in a sidelobe, would probably not make a smooth transition at the 90% level. Figure 4 shows a sample of what we believe the backscattered power from the atmosphere and aircraft might look like. In this instance the backscattered power distribution function changes slope very rapidly, over a fraction of a percent of the distribution function. Even so, the power at the 50% level still shows a several-decibel annual increase from one January and July to the next.

At this time we do not know the reasons for the long-term trends in C_n^2. There does seem to be decoupling of the height behavior when the frequencies are

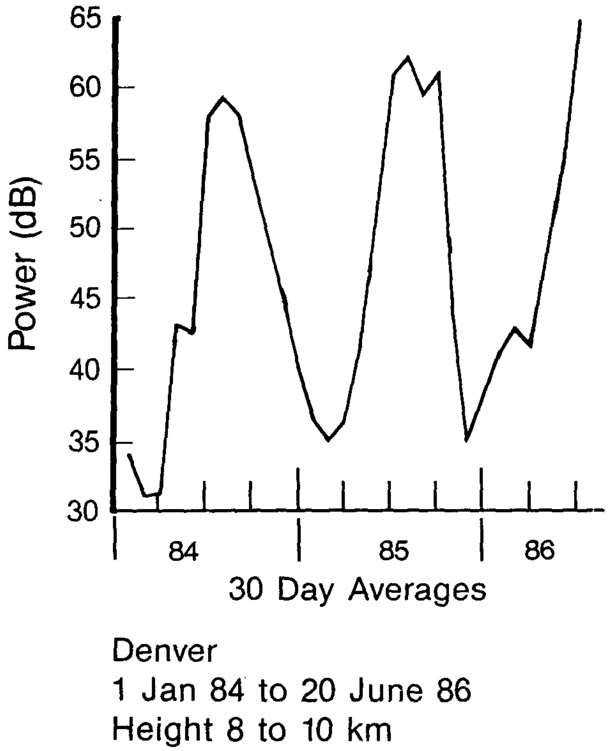

FIG. 2. Average backscattered power computed from data taken at Stapleton Airport, Denver, Colorado, 1 January 1984–20 June 1986, from a 915 MHz profiler at 8 to 10 km.

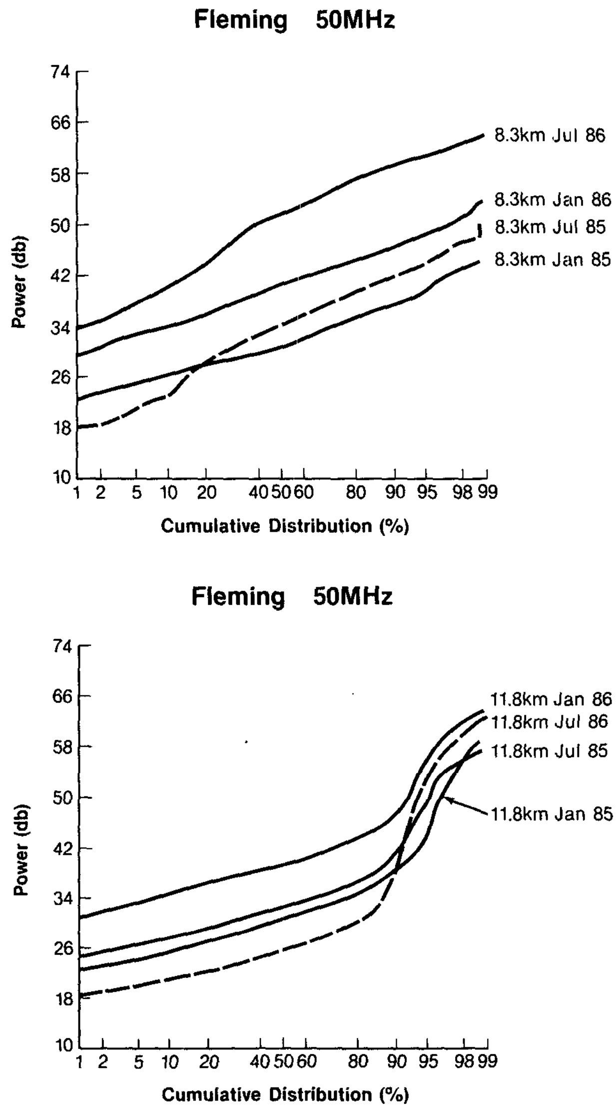

FIG. 3. Cumulative distribution function of backscattered power at 8.3 and 11.8 km at Fleming, Colorado.

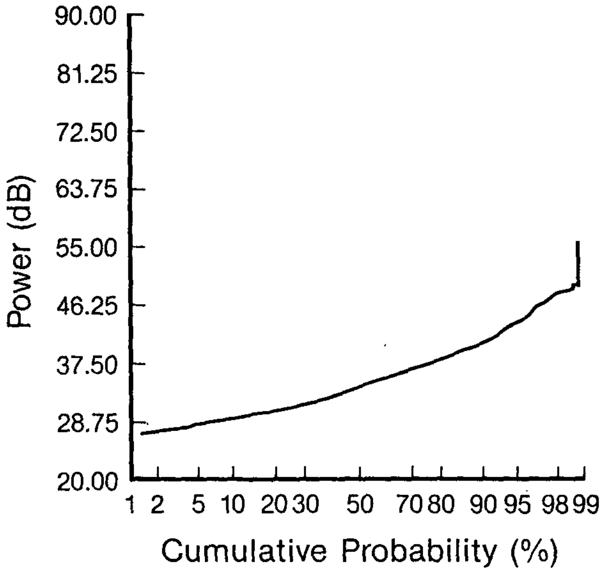

FIG. 4. Cumulative distribution function of backscattered power at Stapleton Airport, Denver, Colorado.

shorter than one year. The tropospheric measurements of C_n^2 show a strong annual signal superimposed on a longer time-scale. Above the tropopause ($\sim$8 to 10 km) the annual variations become small and eventually disappear with height. The annual signal is undoubtedly related to the seasonal variations in the lower atmosphere. We can speculate that the longer trends must be related to other phenomena such as the position of the jet stream and year-to-year variations in tropopause height.

These long-term variations of C_n^2 can be important in designing wind profilers. For example, Frisch et al. (1986) used a measured distribution function for C_n^2 to estimate the change in profiler outages with a change in profiler sensitivity. In their example, an increase of 5 dB in sensitivity corresponded to an outage decrease from 38% to 17%. If instead of a change in sensitivity in the profiler, the monthly average of C_n^2 increased by 5 dB, there would be a corresponding decrease in profiler outages, from 38% to 17% in the example given by Frisch et al. (1986). However, there is no reason to believe that the average of C_n^2 would not drop in the future by at least as much as it went up in the past. This means that for fewer outages in the future, present day profilers will have to be designed with extra sensitivity of at least 5 to 10 dB.

7. Conclusion

We have shown that there can be 2 dB or larger changes in the interannual variation of C_n^2. The C_n^2 trends at lower altitudes (less than 10 km MSL) are different from those at higher altitudes. This long-term variation can have a significant effect on the design of the wind profilers if outages are a problem at certain altitudes.

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