# The Critical Richardson Number and Its Implications for Forecast Problems

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ABSTRACT

It is shown that, in the lowest few hundred meters, nighttime inversions tend to break down due to onset of turbulence when the Richardson number falls between 0.2 and 0.5. Since the Richardson number is statistically related to the wind speed at one to a few hundred meters, it is found that the dew-point depression in the morning, as well as visibility, is related to the wind speed *above* the surface. It follows, that objective forecast techniques for the dew-point depression and visibility can be improved by including the wind speed above the surface as a parameter.

## 1. Introduction

The Richardson number, Ri is defined by:

$$\mathrm{Ri}=\frac{g}{\theta}\frac{\partial\theta}{\partial z}\bigg/\left(\frac{\partial V}{\partial z}\right)^2 \qquad (1)$$

where $g$ is gravity, $\theta$ potential temperature, $z$ height, and $V$ the wind vector.

Except for a factor $K_h/K_m$ (the ratio of the exchange coefficients for heat and momentum), the Richardson number, in stable air, measures the ratio of the rate of destruction of turbulence by buoyancy to the rate of creation of turbulence by the wind shear. Thus, Richardson (1920), assuming $K_h=K_m$, argued that turbulence could not be created when Ri exceeds unity.

Although the detailed characteristics of the ratio of exchange coefficients are still controversial, it seems quite clear that this ratio is less than unity in stable air (see, e.g., Lettau and Davidson, 1957, and Ellison, 1962). Thus the "critical" Richardson number, $\mathrm{Ri}_{crit}$, could exceed one.

On the other hand, even if the production of turbulence exceeds the rate of destruction of turbulence by buoyancy forces, turbulence may still not maintain itself, because of dissipation into heat. This argument leads to small values of $\mathrm{Ri}_{crit}$. Further, the dissipation rate is quite variable and decreases, for example, with increasing height. It is, therefore, quite possible that there is no unique $\mathrm{Ri}_{crit}$, and that $\mathrm{Ri}_{crit}$ will increase with increasing height.

In agreement with this, Taylor (1931) found a "critical" Richardson number of 0.25 on the basis of the perturbation theory; at lower values stratified flow becomes unstable. Taylor assumed an atmosphere bounded below in which the wind shear was constant and the density distribution exponential. On the other hand, layered models did not give definite "critical" Richardson numbers (Goldstein, 1931).

Also, on the observational side, the situation is far from clear. In particular, Wanta (1953) finds that the nocturnal inversion at Brookhaven, N. Y., tends to break down in the morning when the Richardson number drops to between 0.50 and 0.25. McVehil finds from an analysis of winds at low levels (unpublished doctoral dissertation[1] at the Pennsylvania State University, 1962) that for Richardson numbers in excess of 0.14, winds at different levels are essentially decoupled, implying greatly reduced turbulence. Furthermore, in studies of clear-air turbulence (CAT), the Richardson number has frequently been found to discriminate between turbulence and no-turbulence, most recently by Briggs and Roach (1963) and Colson (1963). Zavarina and Yudin (1960) find that a Richardson number of unity discriminates quite well between turbulence and no-turbulence in clear areas, provided that the computations of the Richardson number are based on one-kilometer layers. [*Editor's note:* See also Kronebach, pp. 119–125, this issue.]

Some recent, unpublished work on CAT has shown, however, that a detailed analysis of original radiosonde records leads to critical Richardson numbers lower than unity. The explanation of this apparent discrepancy lies in the fact that, unless temperature and wind distributions are smooth, the numerical value of the Richardson number depends on the resolution of the measurements: the greater the resolution, the larger the magnitudes of wind shear and lapse rate, and the smaller the Richardson number. This is another reason why it seems impossible to settle on a single exact critical value of the Richardson number.

[1] To be published in *Quart. J. R. meteor. Soc.*, 1964.

## 2. Determination of a critical Richardson number at Brookhaven, N. Y.

Durst (1933) drew attention to instances in which the nocturnal inversion is destroyed long before sunrise. Gifford (1952) noted the same phenomenon. The probable mechanism for this breakdown is turbulent mixing induced by vertical wind shear. It should occur in such instances in which the wind above the surface layer gradually increases in speed, so that the wind shear in the lower levels increases. As a result, the Richardson number is lowered. Eventually, a "critical" Richardson number is reached in a layer; mixing sets in; and the inversion is destroyed in that layer. Other layers are affected similarly, and finally no strong inversion remains anywhere.

Two methods were used here to judge the onset of turbulence: In eight cases continuous wind speed records were inspected at 5 levels (37, 75, 150, 300, and 355 ft); for 25 additional nights the presence of turbulence was inferred from the absence of strong inversions. Richardson numbers were computed from observations at the same levels; however, in the upper portion of the tower, wind shears are small and lapse rates near adiabatic, so that the Richardson numbers are quite imprecise.

Figs. 1 and 2 give examples for two different types of regime taken from the set of eight clear nights at Brookhaven studied in detail. Fig. 1 shows the vertical distributions and temporal changes of temperature, the wind speed, the Richardson number and turbulence during a night where the winds above the surface were and remained weak until after sunrise. There was no turbulence and the Richardson number generally remained well above 0.25. In contrast, Fig. 2 shows a situation in which the wind at 400 ft speeded up throughout the night. An inversion formed early in the night, but broke about midnight. At nearly the same time, turbulence was noted on all wind speed records, and Richardson numbers dropped below 0.25. Table 1 shows the association between the Richardson number and the turbulence for the same eight clear nights at Brookhaven.

Table 1. Turbulence probability related to the Richardson number at Brookhaven, N. Y.

| Ri | Number of cases | Turbulence, % |
|---|---|---|
| <0.12 | 94 | 91 |
| 0.13–0.24 | 77 | 79 |
| 0.25–0.37 | 49 | 41 |
| 0.38–0.49 | 48 | 31 |
| 0.50–0.99 | 74 | 12 |
| >0.99 | 144 | 10 |

There is no clear "critical" Richardson number defined by the data. However, there is an unmistakable relation between turbulence and the Richardson number. It is perhaps fair to say that if a "critical" Richardson number exists, it must lie between 0.25 and 0.50. For Ri<0.25, turbulence is almost certain; whereas for Ri>0.50, turbulence is quite unlikely. The exception to these rules can be ascribed to difficulties in measuring the Richardson number with sufficient precision.

In most practical situations, winds and temperatures are not available at many levels, so that the details of the distribution of the Richardson number are not known. However, an upper wind may be available from a pilot balloon, or a gradient wind from a synoptic chart. Table 2, therefore, summarizes the relation, for the same 8 nights at Brookhaven, between the wind near the top of the tower and the probability of turbulence. It is clear that, at Brookhaven, turbulence and inversion breakdowns are almost certain when the wind speed at the top of the tower exceeds 8 m sec$^{-1}$.

Table 2. Turbulence probability and the mean wind speed between 300 ft and 355 ft at Brookhaven, N. Y.

| Wind speed, m sec$^{-1}$ | Number of cases | Turbulence, % |
|---|---|---|
| ≤3.9 | 25 | 28 |
| 4–5.9 | 24 | 37 |
| 6–7.9 | 63 | 78 |
| ≥8 | 75 | 98 |

A less precise check on these results was furnished by observations during 25 clear nights at Brookhaven for which hourly mean observations were available on punched cards. In all cases, the cloudiness at 0300 EST was 0.3 or less, and the surface wind speed was 3 m sec$^{-1}$ or less at the same time. Since no continuous records of wind on these 25 clear nights were immediately available, the lapse rate itself served as an indicator of turbulence. Of course, the lapse rate is a factor in the Richardson number. Thus, one would expect an automatic relation between Richardson number and lapse rate. The Richardson number would decrease uniformly with the decreasing absolute value of lapse rate. As will be seen, this is not the case.

Fig. 3. shows average lapse rates in three layers as function of the Richardson number. Reading these graphs from right to left, we begin with strong inversions. As we move further to the left, the lapse rates remain about the same or even slightly increase numerically, but the Richardson numbers decrease due to increased wind shear. Presumably, the lapse rate is dictated by radiative processes. Then, as we continue to increase the wind shear, a "critical" Richardson number is reached; and the lapse rate begins to decrease numerically. The exact numerical value of the "critical" Richardson number appears to show little systematic variation with height. It seems to lie somewhere between 0.3 and 0.5. Actually, individual runs scatter considerably. However, for Richardson numbers below 0.18, no strong inversions exist. In summary, then, it appears that, generally, $Ri_{crit}$ lies between 0.2 and 0.5, in not too bad agreement with previous estimates.

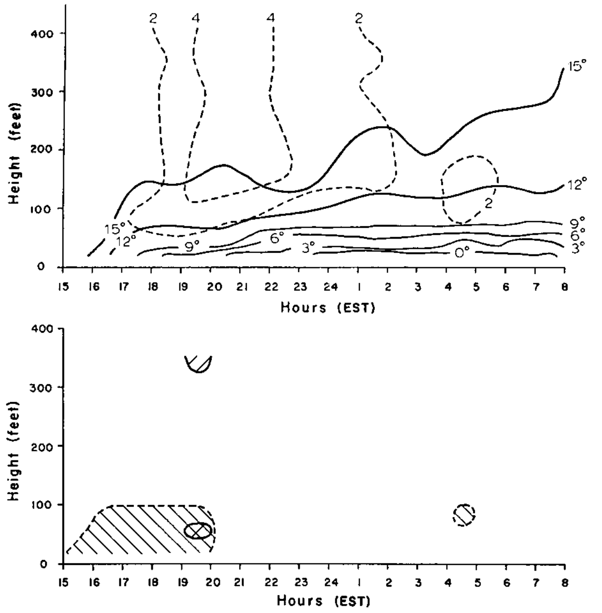


FIG. 1. Upper: Temporal variation of wind speed in m sec$^{-1}$ (dashed) and temperature in C (solid) at Brookhaven, N. Y., November 19–20, 1953. Lower: Turbulence (dashed border) and Richardson number (area with solid border less than 0.25) for the same period.

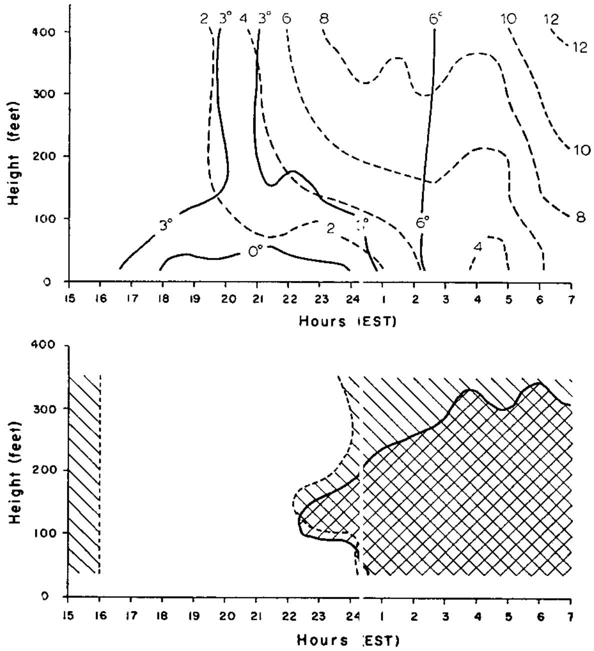


FIG. 2. Upper: Temporal variation of wind speed in m sec$^{-1}$ (dashed) and temperature in C (solid) for Brookhaven, N. Y., December 25–26, 1957. Lower: Turbulence (dashed border) and Richardson number (area with solid border less than 0.25) for same period.

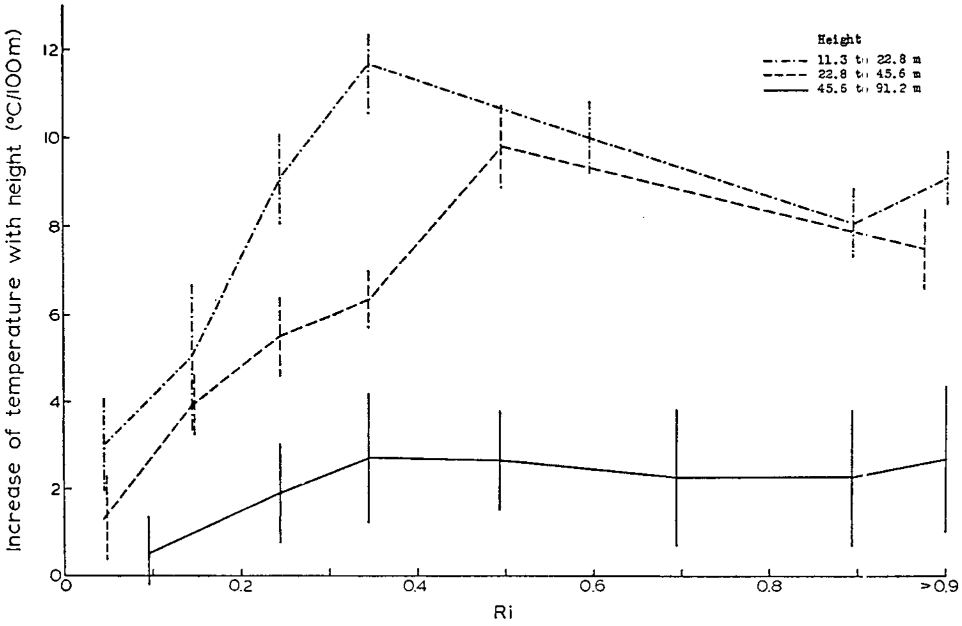


FIG. 3. Variation of lapse rate with Richardson number at Brookhaven, N. Y. averaged over many nights.

## 3. Wind above the surface in relation to surface dew point and temperature

Since the wind some distance above the surface is related to the vertical distribution of the Richardson number and, therefore, to the onset of the turbulence near the ground, it should have a bearing on short range forecasting of temperature, dew point, and visibility at the ground. For example, Carson (1962) developed an objective method for predicting visibility at Miami in which the 1000-ft wind is a predictor.

The relation between wind above the surface and surface conditions was studied at three widely different places: Brookhaven, N. Y., Montgomery, Ala., and Norfolk, Va. At Brookhaven, all available clear nights with light wind were investigated; at the other places, the additional restriction of high initial humidity was imposed. The reason for this additional restriction was the implication the results have for radiation-fog forecasting.

At Brookhaven, 99 situations could be studied with cloudiness less than 0.4 and wind speed 3 m $sec^{-1}$ or less at 0300 EST. Because of the availability of the tower data, the wind representative of the upper level flow was taken as the wind at 300 ft at 0300 EST. Fig. 4 shows the relation between surface dew point and temperature changes with 300-ft wind for light surface wind. It is quite clear that the dew-point depression is much greater, both at 0100 EST and 0400 EST, for strong upper winds than for weak winds. The figure also shows that both dew point and temperature drop rapidly with light upper winds from 0100 EST to 0400 EST, but drop only very little with strong upper winds. This situation is presumably due to the increased mixing with strong 300-ft winds associated with a relatively low Richardson number.

At Montgomery, observations were available for 10 years. Only humid nights were included with dew-point depressions of 3F or less at 0300 CST and with wind speeds less than, or equal to, 5 m $sec^{-1}$. Also, the visibility at that time had to be greater than 4 miles because nights on which fog had already formed were of no interest. Finally, the study was restricted to fog months, October through April. Otherwise, the same conditions prevailed as at Brookhaven. Fig. 5 shows the results. Of course, due to the restriction to moist nights, the dew-point depressions average less than at Brookhaven. But, again, dew-point depressions are somewhat larger on nights with strong winds (this time taken from balloon observations at 500 m) than with weak winds. Here, however, the dew-point depression is greater at 0600 than at 0300 CST. Also, there is a tendency for dew point and temperature to change less with strong winds than with weak winds.

At Norfolk, 59 nights were chosen by the same criteria as at Montgomery, between January 1954 and March 1960. The result is shown in Fig. 6. There seems to be no relation between dew point and temperature structure with wind at 500 m. Nevertheless, as will be seen, fog probability is related to wind. Another peculiar feature at Norfolk is the negligible temperature and dew-point change between 0300 EST and 0600 EST, a fact that must be ascribed to the maritime exposure, particularly during the moist nights selected. A surprising characteristic of the Norfolk data is the relatively large dew-point depression during fogs which is quite inconsistent with the extremely high fog probability to be discussed presently.

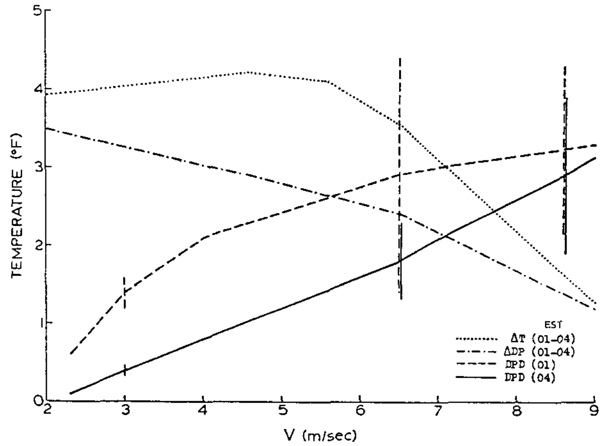


Fig. 4. Wind at 300 ft at Brookhaven, N. Y., related to temperature change from 0100 to 0400 EST, dew-point change from 0100 to 0400 EST, dew-point depression at 0100 EST, and dew-point depression at 0400 EST, averaged over many nights. Vertical lines indicate standard deviations of the means.

## 4. Wind above the surface and fog probability

Since the wind above the surface is related to surface dew-point depression, it should be a useful predictor of fog probability. Table 3 shows the empirical probability of fog or ground fog occurring between 2400 EST and 0600 EST as function of the 300-ft wind at Brookhaven at 0300 EST. The table indicates a clear dependence, in agreement with the previously noted relationship between wind speed and dew-point depression. Both relations are due to the increased turbulence in strong winds (Table 2).

Table 3. Fog probability and the mean wind speed at 300 ft at Brookhaven, N. Y.

| Wind speed, m $sec^{-1}$ | Number of cases | Fog, % |
|---|---|---|
| 0–3.2 | 6 | 67 |
| 3.3–5.2 | 18 | 44 |
| 5.3–6.2 | 14 | 36 |
| 6.3–7.2 | 30 | 33 |
| >7.2 | 30 | 30 |

The sample of observations at Montgomery consisted only of moist nights in which no fog had formed by

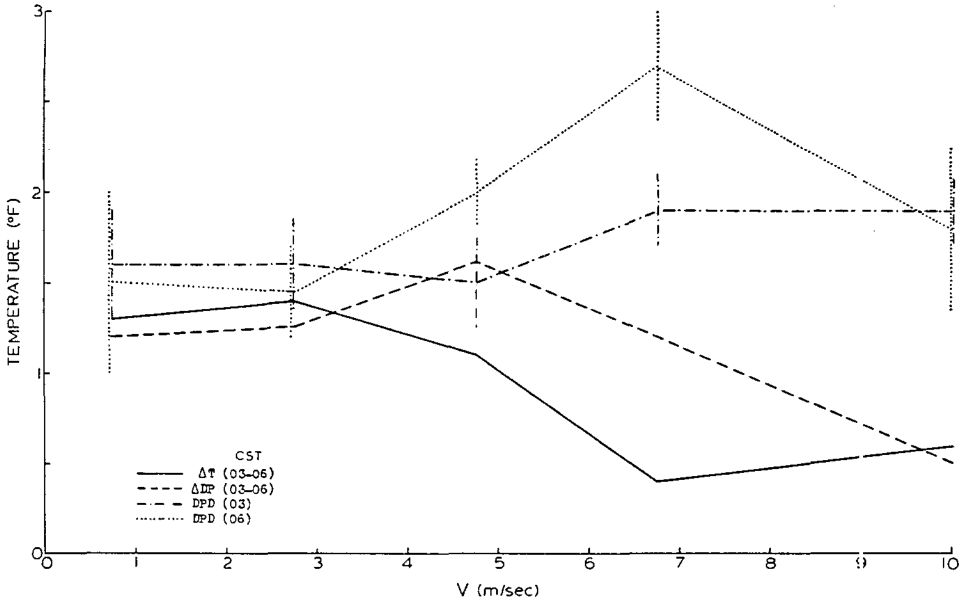


FIG. 5. Wind at 500 m at Montgomery, Ala., related to temperature change from 0300 to 0600 CST, dew-point change from 0300 to 0600 CST, and dew-point depression at 0300 CST and 0600 CST, averaged over many nights.

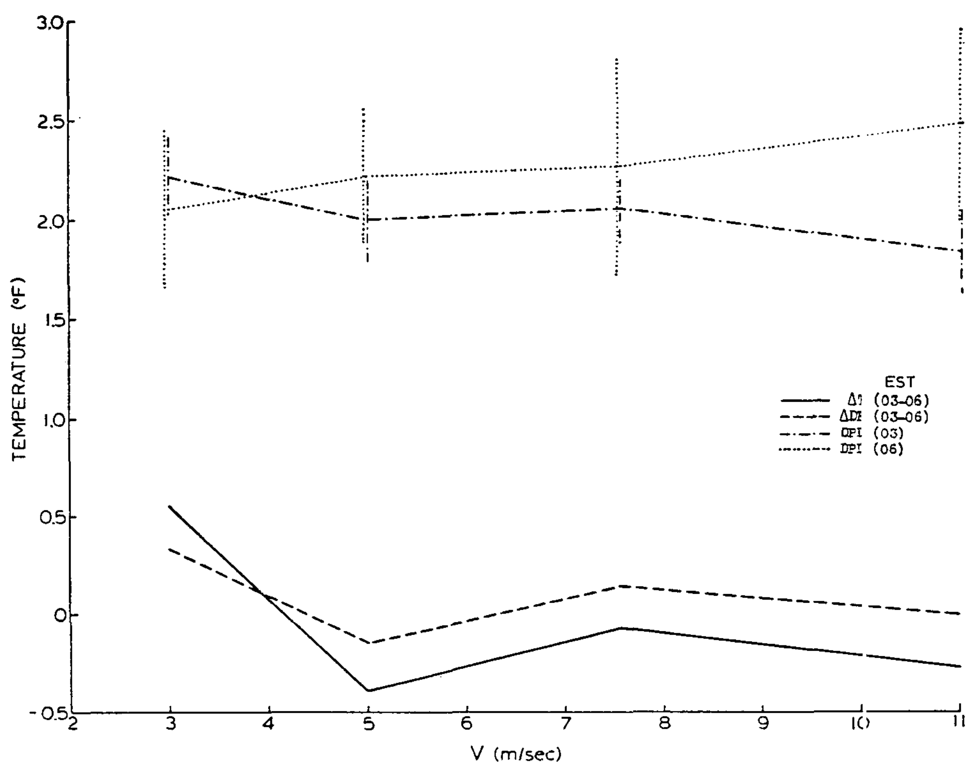


FIG. 6. Same as Fig. 5, but for Norfolk, Va. Time in EST.

0300 CST. Out of a total of 151 cases, 16 developed visibilities below 2 miles by 0600 CST; 23 developed visibilities between 2 and 4 miles, and good visibilities remained in the other 112 situations. Fig. 7 contrasts the progress of wind speed at 500 m for these three groups. Certainly, for the cases of good visibility, the average wind speed was and remained higher than for the other two categories. Further, the cases with the worst visibility had the slowest wind at 0300 CST, although the difference between the two low-visibility groups is not statistically significant. The relation between the wind speed at 0300 CST and the probability of three categories of visibility is contained in Table 4.

Here, quite a striking relationship between wind speed and the probability of fog appears. For example, other things being equal, the probability of fog with winds at 500 m of 8 m $sec^{-1}$ or more is quite negligible. Further, for the lightest winds, substantial fog is more common than light fog, whereas the reverse is true for all other wind speeds.

For Norfolk, Va., 111 cases were selected with surface winds of less than 3 m $sec^{-1}$, dew-point depressions at or below 4F, cloudiness of less than 0.4, and visibility greater than 4 miles, all at 0300 EST. Table 5 shows the association of fog between 0400 and 0900 EST with the wind speed at 0300 EST.

It may be noted that, under the conditions specified, the probability of fog is 70 per cent, although most of these cases are described as ground fog. The difference between the mean wind speeds for two categories in Table 5, although hardly striking, is statistically significant at the 5 per cent level. This is interesting, because no relation between dew-point depression and wind at 500 m had been noted. The same data are shown in a different form in Table 6, which relates fog probability to wind speed.

TABLE 4. Visibility probability and the mean wind speed at 500 m at Montgomery, Ala.

| Wind speed, m $sec^{-1}$ | Number of cases | Probability of visibility, % $\leq$2 mi | 2–4 mi | $\geq$4 mi |
|---|---|---|---|---|
| 0–1.5 | 12 | 42 | 16 | 42 |
| 2–3.5 | 32 | 13 | 31 | 56 |
| 4–5.5 | 44 | 9 | 13 | 78 |
| 6–7.5 | 33 | 6 | 12 | 82 |
| >7.5 | 32 | 3 | 3 | 94 |

TABLE 5. Fog probability and the mean wind speed at 500 m at Norfolk, Va.

| Weather | Number of cases | Wind speed, m $sec^{-1}$ |
|---|---|---|
| Fog (including ground fog) | 78 | 5.1 |
| No fog | 33 | 6.1 |

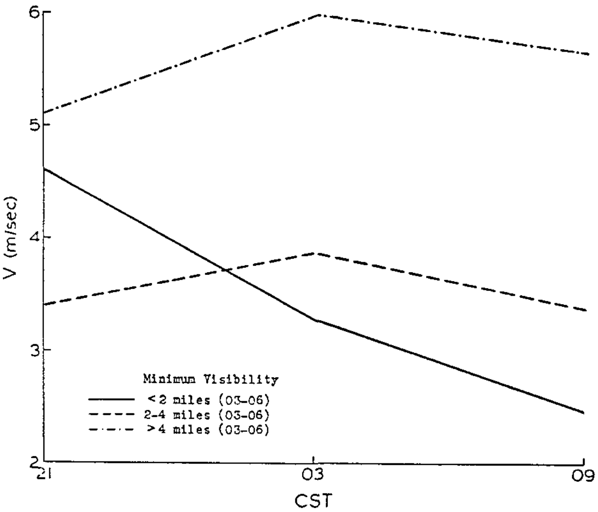


FIG. 7. Wind at 500 m at Montgomery, Ala., related to surface visibility on many clear, humid nights with little surface wind.

TABLE 6. Fog probability and the mean wind speed at 500 m at Norfolk, Va.

| Wind speed, $msec^{-1}$ | Number of cases | Fog, % |
|---|---|---|
| 0– 3.9 | 37 | 74 |
| 4– 7.9 | 57 | 72 |
| 8–13.9 | 17 | 47 |

Again, there is a clear relationship. Although fog is quite probable at all wind speeds below 8 m $sec^{-1}$, it is much less likely at higher wind speeds.

## 5. Summary

A detailed analysis of observations at Brookhaven has shown that when the wind shear is large enough for the Richardson number to drop below 0.25 (and sometimes not quite as low) no strong inversions can be maintained. Since such large wind shears are associated with strong winds some distance above the surface, such winds were related to temperature, dew point, and visibility at the surface.

At Brookhaven and Montgomery, there were clearly larger dew-point depressions with high wind speeds above the surface than with low wind speeds. At Norfolk there was no such relationship. At all three stations, however, the probability of fog on clear nights with light surface wind was highest with weak winds aloft, although the detailed relationship varied among the three places. This suggests, that in future statistical techniques for visibility forecasting, the accuracy of such forecasts can be increased by including among the predictors the upper wind at the lowest level reported.

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