Vertical Resolution and Accuracy of Atmospheric Infrared Sounding Spectrometers

HUNG-LUNG HUANG, WILLIAM L. SMITH, AND HAROLD M. WOOLF

Cooperative Institute for Meteorological Satellite Studies, University of Wisconsin, Madison, Wisconsin

(Manuscript received 17 September 1990, in final form 16 July 1991)

ABSTRACT

A theoretical analysis is performed to evaluate the accuracy and vertical resolution of atmospheric profiles obtained with the HIRS/2, GOES I/M, and HIS instruments. In addition, a linear simultaneous retrieval algorithm is used with aircraft observations to validate the theoretical predictions. Both theoretical and observational results clearly indicate that the accuracy and vertical resolution of the retrieval profile would be improved by high spectral resolution and broad spectral coverage of infrared radiance measurements.

The HIS is found to possess the equivalent of 11 pieces of temperature- and 9 pieces of water vapor-independent precise measurements. The characteristics for temperature include a vertical resolution of 1–6 km with an accuracy of 1 K and for water vapor a vertical resolution of 0.5–3.0 km with an accuracy of 3 K in dewpoint temperature. The HIS is a factor of 2–3 times better in vertical resolution and a factor of 2 times better in accuracy than the GOES I/M and HIRS/2 filter radiometers.

1. Introduction

Radiance measurements by satelliteborne multispectral infrared radiometers are used to sense atmospheric temperature and moisture profiles (Kaplan 1959; Wark and Fleming 1966; Smith and Woolf 1976; Susskind et al. 1984). Because the radiances arise from very thick layers of the atmosphere, the poor vertical resolution and associated accuracy of the derived profiles limit their use in numerical weather prediction (NWP) (Bengtsson 1979; Phillips et al. 1979; Smith 1991). The presence of clouds that absorb infrared radiation further complicates the derivation of the sounding profiles from the radiance observations (Chahine 1974, 1977, 1982) and also limits their utility in NWP.

The poor vertical resolution of current operational satellite sounding instruments is attributable, in part, to low spectral resolution. Due to this coarse spectral resolution, the radiometer does not sense radiation between individual absorption lines. Therefore, the emitted radiance reaching the satellite is a mixture of radiation from the high atmosphere due to the strong absorption near line centers and radiation from the lower atmosphere from between the absorption lines. Thus, the relatively poor spectral resolution of current instruments causes a vertical smearing of the atmospheric structure.

Corresponding author address: Dr. Hung-Lung Huang, University of Wisconsin, CIMSS, Space Science and Engineering Center, 1225 West Dayton St, Madison, WI 53706.

To minimize vertical smearing and thereby approach the ultimate vertical sounding resolution and accuracy achievable with passive measurements, a sounding instrument must possess a spectral resolving power (defined by $\lambda/\delta\lambda$, where λ is wavelength) of at least 1000. For example, a spectral resolution of 0.7 cm^{-1} is required in the 600–700-cm^{-1} (15-μm) thermal emission band of CO_2. This spectral resolution can be achieved using a Michelson interferometer (Smith et al. 1979, 1983, 1990) or a large detector array grating spectrometer (Chahine et al. 1990). The High-Resolution Interferometer Sounder (HIS) and Advanced Infrared Sounder (AIRS) are experimental instruments designed to demonstrate improved sounding performance from high spectral resolution and continuous broad spectral coverage. An aircraft model of HIS is a Michelson interferometer with a spectral resolving power of approximately 2000, covering a spectral range of 3.7–16.7 μm (Smith et al. 1983). The AIRS is to achieve measurement characteristics similar to HIS by incorporating new cooled focal plane detector array technology into a grating spectrometer (Chahine et al. 1990).

In this paper, the linear simultaneous retrieval algorithm (Smith et al. 1987, 1991), along with the theoretical vertical resolution (Backus and Gilbert 1970; Conrath 1972; Thompson 1982; Rodgers 1987) and error analysis (Rodgers 1987; Smith et al. 1991), are used to quantify the vertical resolution and accuracy provided by three different sounding instruments: 1) the GOES I/M sounding radiometer; 2) the HIRS/2 aboard the NOAA satellites; and 3) the HIS sounding instrument proposed for future GOES implementation.

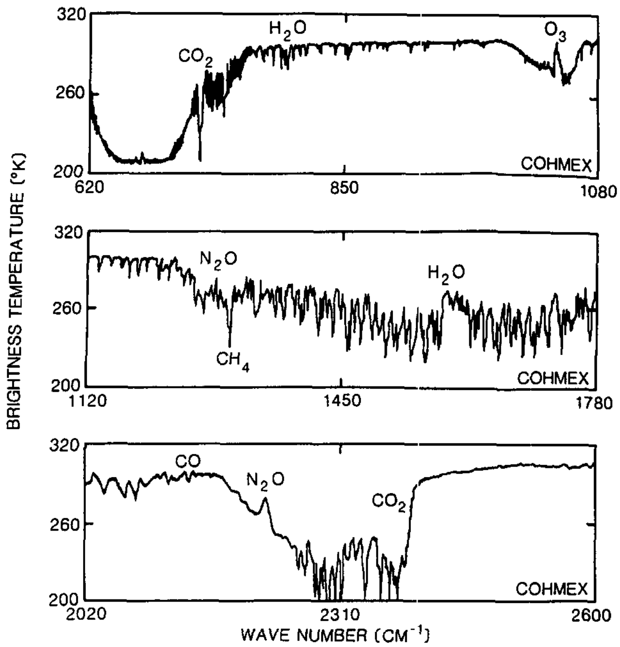


FIG. 1. Brightness temperature spectra measured by HIS from U-2 aircraft over Huntsville, Alabama, on 15 June 1986 of COHMEX.

2. Data

HIS data have been successfully collected from NASA U-2/ER-2 aircraft flights (Smith et al. 1987; Revercomb et al. 1988). With observations at a 6-s sampling intervals from the height of 65 000 ft, an instantaneous ground resolution of 2 km is achieved. Figure 1 shows a clear-sky spectrum of brightness temperature observed during the Cooperative Huntsville Meteorological Experiment (COHMEX), with a spectral resolution of about 0.5 cm^{-1} from 600–1100 cm^{-1} (9.1–16.7 μm) and 1.0-cm^{-1} resolution from 1100–2700 cm^{-1} (3.7–9.1 μm) (Huang 1989). Major absorbing constituents are labeled accordingly; namely, CO_2, H_2O, O_3, N_2O, CH_4, and CO. The selected weighting functions for the three HIS infrared bands are presented in Figs. 2 and 3. These curves express the vertical derivative of atmospheric transmittance with respect to the natural logarithm of pressure and are calculated from a line-by-line model FASCODE II (Clough et al. 1986) using FASCODE standard atmosphere profile conditions. As for the GOES I/M and HIRS/2 data, they are simulated from the HIS radiance spectra by convolution with their spectral response functions.

3. Retrieval algorithm, vertical resolution, and error analysis methods

a. *Linear simultaneous retrieval algorithm*

A computationally efficient retrieval methodology has been developed by Smith et al. (1991), which is a

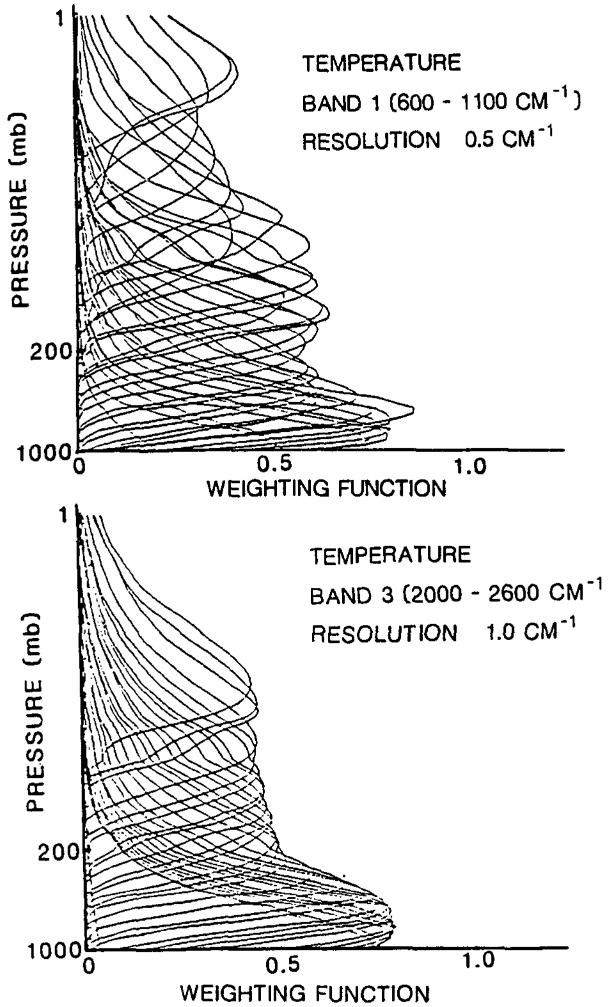


FIG. 2. Temperature component Planck radiance weighting functions for a small set of HIS band 1 and band 3 channels.

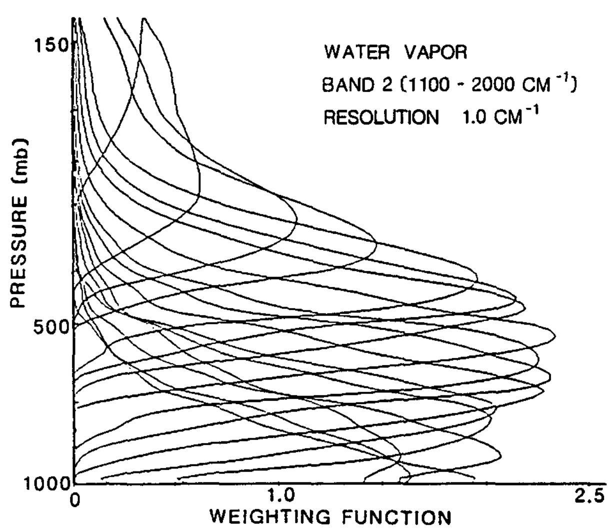


FIG. 3. Water vapor component Planck radiance weighting functions for a small set of HIS band 2 channels.

unique linear algorithm that simultaneously retrieves temperature and absorbing constituent profiles (i.e., water vapor, ozone, and methane) from observations of spectral radiances. The radiative transfer equation (RTE) linearization results from a definition for the deviation of the true gas concentration profiles from an initial state in terms of the deviation of their *effective* temperature profiles from the true atmospheric temperature profile. The *effective* temperature profile for any absorbing constituent is defined as the temperature profile that satisfies the observed radiance spectra under the assumption that the initial absorber concentration profile is correct. Differences between the effective temperature, derived for each absorbing constituent, and the true atmospheric temperature are proportional to the error in the initial state of the gas concentration profiles. The gas concentration profiles are thus determined after inversion of the linearized RTE from the retrieved effective temperature profiles assuming that one of the assumed concentration profiles is known (i.e., CO_2).

Following the algorithm of Smith et al. (1991), the linearized RTE is

$$\delta T_{B\nu} = \beta_\nu{}^0(p_s)\tau_\nu{}^0(p_s)\delta T_s - \sum_{i=1}^{N}\int_0^{p_s} \beta_\nu{}^0(p)\delta T_i(p)\tau_\nu{}^0(p)\, d\ln\tau_{\nu i}^0(p), \quad (1)$$

where

$\beta_\nu{}^0(p) = [\partial B_\nu(T^0)/\partial T][\partial B_\nu(T_B{}^0)/\partial T]^{-1}$;

N = the number of optically active atmospheric constituents [$N = 2$ when only the uniformly mixed gases (mainly CO_2) and water vapor are considered];

p_s = pressure with subscript s denoting the surface;

$\tau_\nu(p)$ = the total transmittance of the atmosphere above atmospheric pressure level p;

$\delta(\ )$ = the difference between the true quantity and the initial state denoted by a superscript 0;

$\Sigma(\ )_i$ = the summation of i quantities;

$\tau_{\nu i}$ = the transmittance of the atmosphere for the ith absorbing constituent;

$T_{B\nu}$ = the brightness temperature at spectral frequency ν;

$T(p)$ = the true atmospheric temperature profile;

$T_i(p)$ = the effective temperature of the ith absorbing constituent;

$U_i(p)$ = pathlength profile of the ith absorbing constituent

$$U_i(p) = \frac{1}{g}\int_0^p q_i(p')dp';$$

g = acceleration due to gravity;

$q_i(p)$ = mixing ratio of the ith absorbing constituent; and

$\delta T_i(p) = T_i(p) - T^0(p)$.

In matrix form, again following Smith et al. (1991), the generalized statistical–physical solution of Eq. (1) is

$$t_r = \mathbf{C}t_b \quad (2a)$$

and the retrieval coefficient matrix **C** is

$$\mathbf{C} = (\mathbf{A}^T\mathbf{E}^{-1}\mathbf{A} + \mathbf{S}^{-1})^{-1}\mathbf{A}^T\mathbf{E}^{-1}, \quad (2b)$$

where subscript r denotes retrieved quantity. The elements of the weighting function matrix **A** are

$$A_{\nu,0} = \beta_\nu{}^0(p_s)\tau_\nu{}^0(p_s)$$

$$A_{\nu,j} = -\beta_\nu{}^0(p_{ij})\tau_\nu{}^0(p_{ij})\, d\ln\tau_{\nu i}^0(p_i \cdots j), \quad j = 1, 2, \ldots, NM_i,$$

where N is the number of constituents; M_i is the number of quadrature pressure levels denoted to each constituent; **S** is the sample statistical covariance matrix; **E** is the covariance of the brightness-temperature error; $(\ )^T$ denotes the matrix transpose; and $(\ )^{-1}$ denotes the matrix inverse.

b. Analysis of retrieval vertical resolution

The vertical resolution of the various sounding instruments is analyzed using the concepts of Backus and Gilbert (1970), Conrath (1972), Thompson (1982), and Rodgers (1987). The retrieved temperature can be expressed as a smooth version of the true temperature profile,

$$T_r(p_i) = \sum_j R(p_i, p_j)T(p_j), \quad (3)$$

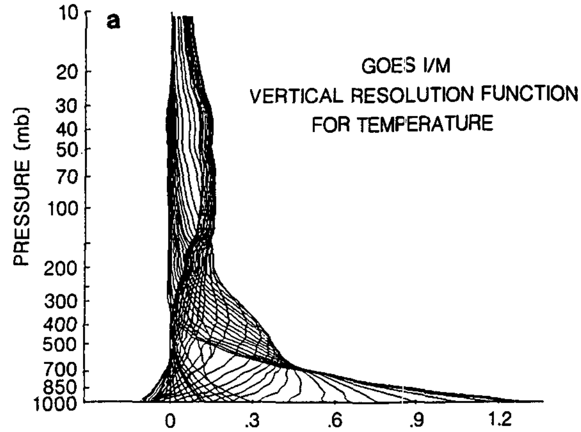
where $T_r(p_i)$ is the retrieved temperature at pressure level i, $T(p_j)$ is the true temperature profile, and $R(p_i, p_j)$ is the vertical resolution function or averaging kernel for level p_i. Equation (3) describes the retrieved temperature at any level p_i as a vertical average of the true temperature profile weighted by the vertical resolution function R for the level p_i. In terms of the sounding retrieval algorithm given by Eq. (2) $t_r = \mathbf{C}t_b$ and the RTE (1), written in matrix notation as $t_b = \mathbf{A}t$, can be rewritten as

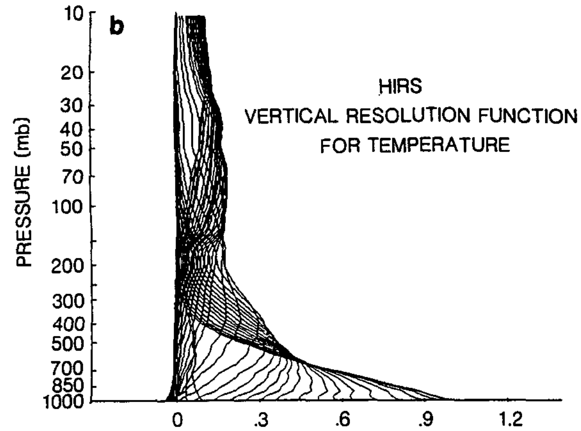
$$t_r = \mathbf{CA}t = \mathbf{R}t,$$

with

$$\mathbf{R} = (\mathbf{A}^T\mathbf{E}^{-1}\mathbf{A} + \mathbf{S}^{-1})^{-1}\mathbf{A}^T\mathbf{E}^{-1}\mathbf{A}. \quad (4)$$

It has been found that the concept of resolution is vague and has many possible definitions (Thompson 1982; Rodgers 1987). Backus and Gilbert (1970) defined the "spread" as a measure of the resolution and used it to quantify the vertical resolution of a retrieval system using satellite radiance measurements. However, Newman (1979) and Thompson (1982) have pointed out that the definition of spread suffers from several mathematical deficiencies that are caused by the oscillatory sidelobes of resolution functions, which





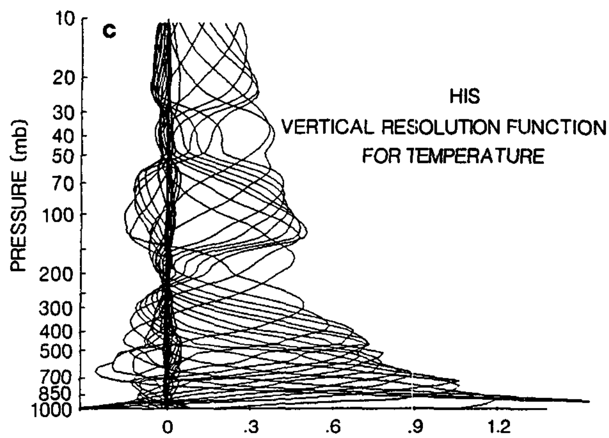


FIG. 4. (a) Temperature vertical resolution functions of GOES I/M sounder. (b) Temperature vertical resolution functions of HIRS/2. (c) Temperature vertical resolution functions of HIS.

results in an unreliable estimation of the resolution. The mathematical difficulties become increasingly severe with increasing spectral resolution and measurement precision. The computations of spread of HIS, HIRS/2, and GOES I/M (not shown) also confirm these findings.

It is these difficulties that motivate the development of an alternative technique to define an effective vertical resolution, which would directly apply to any satellite retrieval system. The Appendix gives an abbreviated derivation of this technique (Purser, personal communication), which will be discussed in complete detail in a subsequent article. If one lets J be the trace of matrix $\mathbf{R}$, then J can be thought of as the total effective degrees of constraint (independent precise measurement) imposed by the satellite data. The measure of the local effective data density is

$$\rho_i = \sum_j F_{i,j} R_{j,j},$$

where

$$F_{i,j} = R_{j,i}^2 \left(\sum_k R_{j,k}^2 \delta Z_k\right)^{-1},$$

δZ_k is the height increment at the level k, and the vertical resolution defined in terms of data density is simply

$$W_i = \rho_i^{-1}. \tag{5}$$

c. *Analysis of retrieval error*

Within the framework of the linear simultaneous inversion theory, a retrieval error analysis algorithm is developed following the concepts of Rodgers (1987).

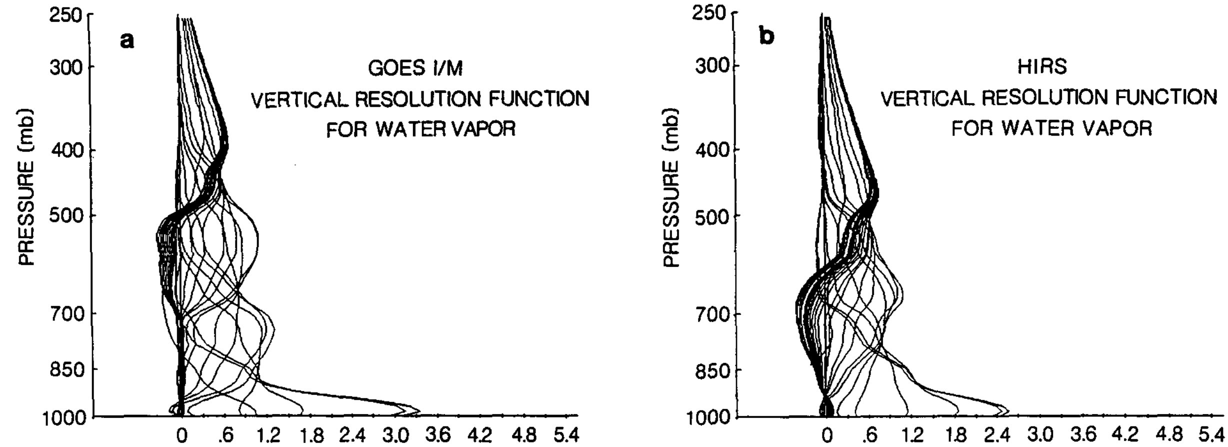
The covariance of the retrieval temperature error G, $G = \langle (t - t_r)(t - t_r)^{\mathrm{T}} \rangle$, can be obtained from

$$t_r = ct_b = \mathbf{C}(t'_b + e) = \mathbf{C}(\mathbf{At} + \mathbf{e}) = \mathbf{CAt} + \mathbf{Ce},$$

where the angle bracket denotes the expectation operator, and $\mathbf{e}$ is assumed to be the brightness-temperature measurement error,

$$\begin{aligned}\mathbf{G} &= (\mathbf{t} - \mathbf{CAt} - \mathbf{C}e)(\mathbf{t} - \mathbf{CAt} - \mathbf{C}e)^{\mathrm{T}} \\ &= [(\mathbf{I} - \mathbf{CA})\mathbf{t} - \mathbf{C}e][(\mathbf{I} - \mathbf{CA})\mathbf{t} - \mathbf{C}e]^{\mathrm{T}},\end{aligned}$$

where $\mathbf{I}$ is the identity matrix. By assuming that temperature $\mathbf{t}$ and random brightness-temperature error $\mathbf{e}$ are uncorrelated, then



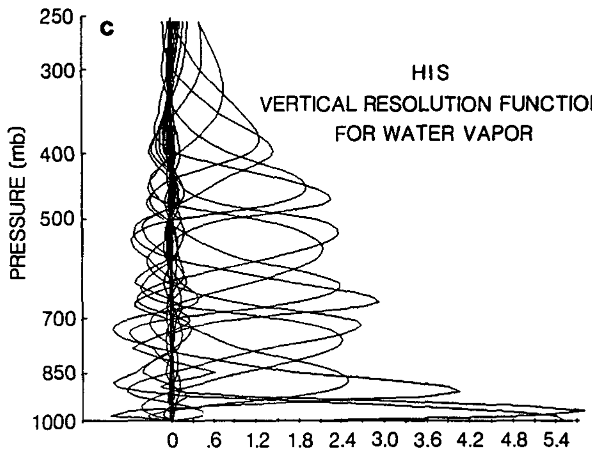


FIG. 5. (a) Water vapor vertical-resolution functions of GOES I/M sounder. (b) Water vapor vertical-resolution functions of HIRS/2. (c) Water vapor vertical-resolution functions of HIS.

$$\mathbf{G} = (\mathbf{I} - \mathbf{CA})\mathbf{tt}^{\mathrm{T}}(\mathbf{I} - \mathbf{CA})^{\mathrm{T}} + \mathbf{Cee}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}$$
$$= (\mathbf{I} - \mathbf{CA})\mathbf{S}(\mathbf{I} - \mathbf{CA})^{\mathrm{T}} + \mathbf{CEC}^{\mathrm{T}} = \mathbf{V} + \mathbf{M}.$$

Here

$$\mathbf{V} = (\mathbf{I} - \mathbf{CA})\mathbf{S}(\mathbf{I} - \mathbf{CA})^{\mathrm{T}} \quad (6)$$
$$\mathbf{M} = \mathbf{CEC}^{\mathrm{T}} \quad (7)$$

and $\mathbf{S} = \mathbf{tt}^{\mathrm{T}}$ and $\mathbf{E} = \mathbf{ee}^{\mathrm{T}}$. Terms $\mathbf{V}$ and $\mathbf{M}$ can be defined as the vertical resolution component error and the measurement noise component error, respectively (Rodgers 1987). The total root-mean-square (rms) error is the square root of the diagonal elements of matrix $\mathbf{G}$, if interlevel correlations are ignored, and can be written as

$$\mathbf{G} = (\mathbf{A}^{\mathrm{T}}\mathbf{E}^{-1}\mathbf{A} + \mathbf{S}^{-1})^{-1}. \quad (8)$$

Equation (8) is obtained by using the matrix identity

$$\mathbf{I} - (\mathbf{Y} + \mathbf{Y})^{-1}\mathbf{X} = (\mathbf{X} + \mathbf{Y})^{-1}\mathbf{Y}.$$

The vertical resolution error results from those components of the profile that cannot be recovered by the retrieval process. In other words, the accuracy of the retrieved profiles is limited by the vertical resolution of observations. This vertical-resolution–related error is the square root of the diagonal elements of the matrix $\mathbf{V}$ (Rodgers 1987). The contribution to the retrieval error due to radiance measurement error is given by the square root of the diagonal elements of the matrix $\mathbf{M}$ (Rodgers 1987). It should be noted that the systematic retrieval error due to the imperfection of the linearized RTE and inverse model is not included in $\mathbf{G}$.

4. Results

a. *Results of retrieval vertical resolution and error analysis*

Retrieval vertical resolution and error are determined for HIS, GOES I/M, and HIRS/2 sounding instruments. A radiance error equivalent to a bright-

TABLE 1. The total effective degrees of constraint (precise independent measurements) for temperature and water vapor retrievals obtained from GOES I/M, HIRS/2, and HIS.

	GOES I/M	HIRS/2	HIS
Temperature	3.8	4.8	11.0
Water vapor	3.8	3.0	9.2

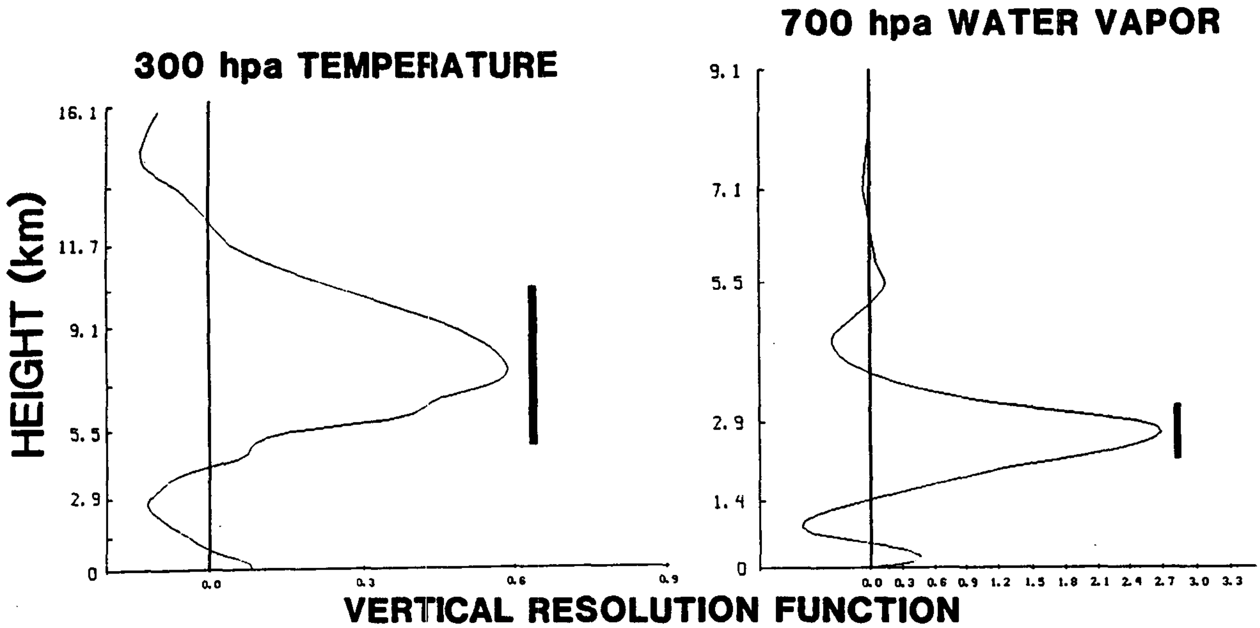


FIG. 6. Vertical-resolution functions and their vertical resolution (the fine vertical bars) of HIS 300-hPa temperature and 700-hPa water vapor retrievals.

ness-temperature measurement error (or noise) of 0.25 K at a scene temperature of 260 K is assumed for all measurement situations. The sample covariance matrix **S** is derived from 400 midlatitude global climatological atmospheric profiles (Smith et al. 1974). The global sample mean profile and FASCODE II (Clough et al. 1986) are used to calculate the spectrum of atmospheric transmittance profiles for all three instruments considered, and to compute all vertical resolution and error analyses. The uniformly mixed constituents (CO_2, N_2O, CH_4, SO_2, and CO) are combined to form a single transmittance profile, so that together with water vapor the number of atmospheric transmittance components was two (ozone absorption regions are excluded in this analysis). Forty pressure levels between 0.1 and 1000 hPa are considered for each of the two constituents.

Results of the retrieval vertical resolution functions R for temperature and water vapor are shown in Figs. 4 and 5, respectively. For temperature vertical resolution, one can see little difference in these functions (except near the surface) between HIRS/2 and GOES I/M sounding conditions. In general, both HIRS/2 and GOES I/M sounders only are able to resolve very broad vertical layers of the temperature profile. However, examination of HIS temperature vertical resolution functions (Fig. 4c) reveals that HIS possesses many more fine layers of temperature profile information. Similar conclusions can be made for water vapor vertical resolution functions of HIRS/2, GOES I/M, and HIS. Nevertheless, the GOES I/M sounder seems to be able to obtain a little more vertical water vapor information than the HIRS/2 because of its additional tropospheric moisture channel (7.43 μm).

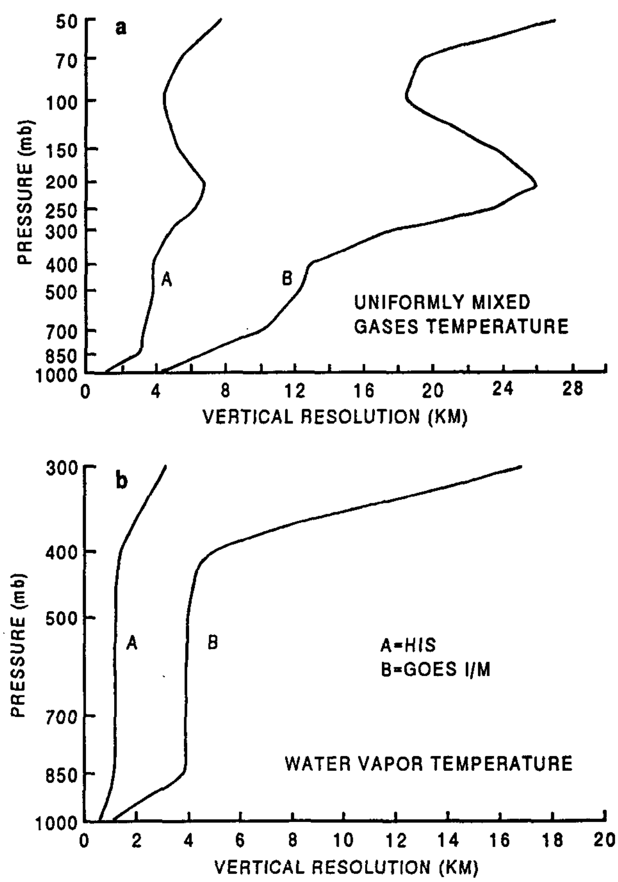


FIG. 7. (a) Temperature vertical resolution (km) of HIS and GOES I/M sounder. (b) Water vapor vertical resolution (km) of HIS and GOES I/M sounder.

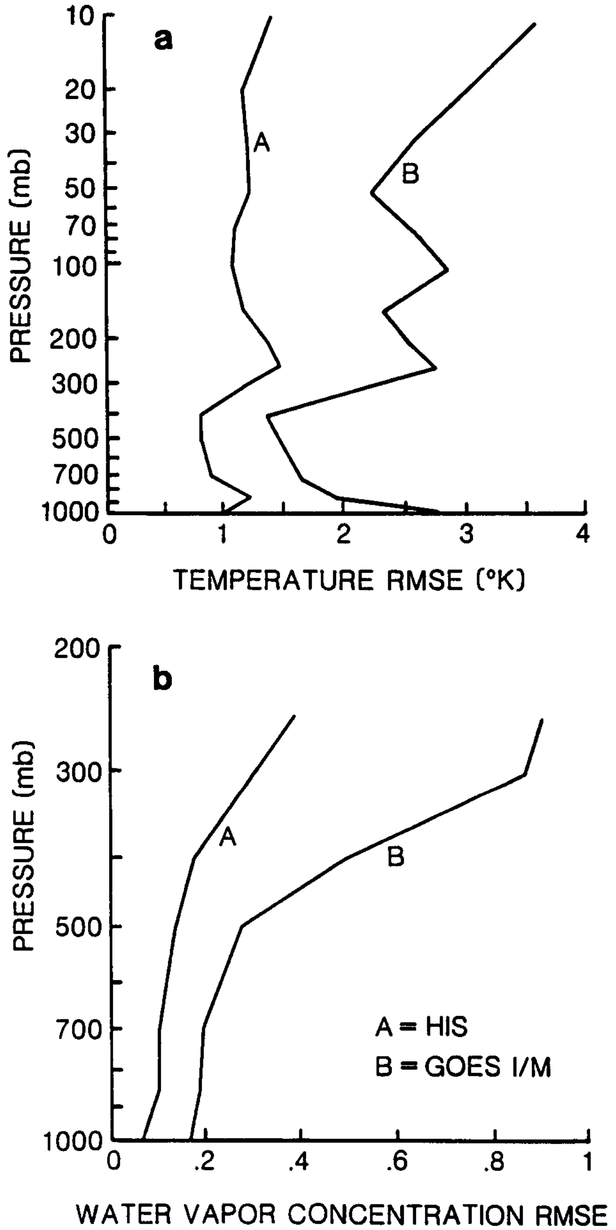


FIG. 8. Root-mean-square temperature and water vapor concentration retrieval errors of HIS and GOES I/M sounder.

Table 1 presents the total effective degrees of constraint (precise independent measurements) for temperature and water vapor retrievals. It is found that both HIRS/2 and GOES I/M are only able to obtain approximately three-to-five pieces each of temperature and water vapor retrievals. In particular, GOES I/M seems to possess less temperature information than HIRS/2 (3.8 versus 4.8). In contrast to temperature, GOES I/M possesses more water vapor information than HIRS/2 (3.8 versus 3.0). The total effective independent measurement possessed by HIS, however, has 11 of temperature and 9 of water vapor. This indicates a factor of 2–3 improvement of the HIS over that of GOES I/M and HIRS/2.

Vertical resolution functions of 300-hPa temperature and 700-hPa water vapor, together with their vertical resolution W obtained from Eq. (5), are shown in Fig. 6. The lengths of the fine vertical bars are the vertical resolution W. The examples presented in Fig. 6 demonstrate the reliable estimation of the vertical resolution that their function has oscillatory sidelobes. Figure 7 shows the vertical resolution computed from Eq. (5), which is defined in terms of data density and is in the unit of kilometers. It can be seen that a major im-

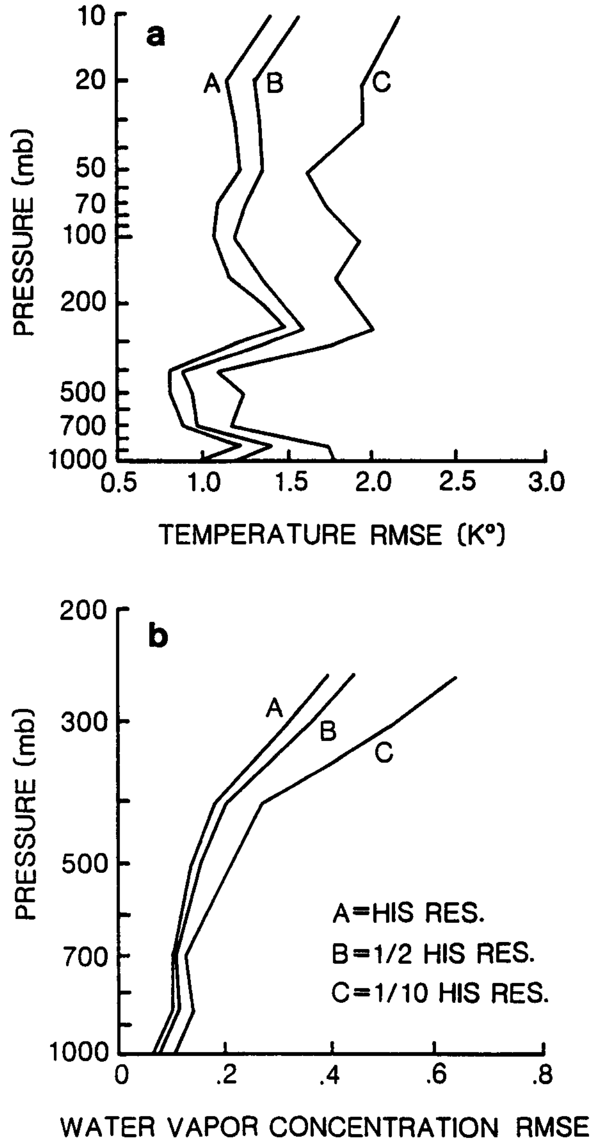


FIG. 9. Temperature and water vapor concentration rms errors of HIS in full resolution, one-half of the HIS resolution, and one-tenth of the HIS resolution.

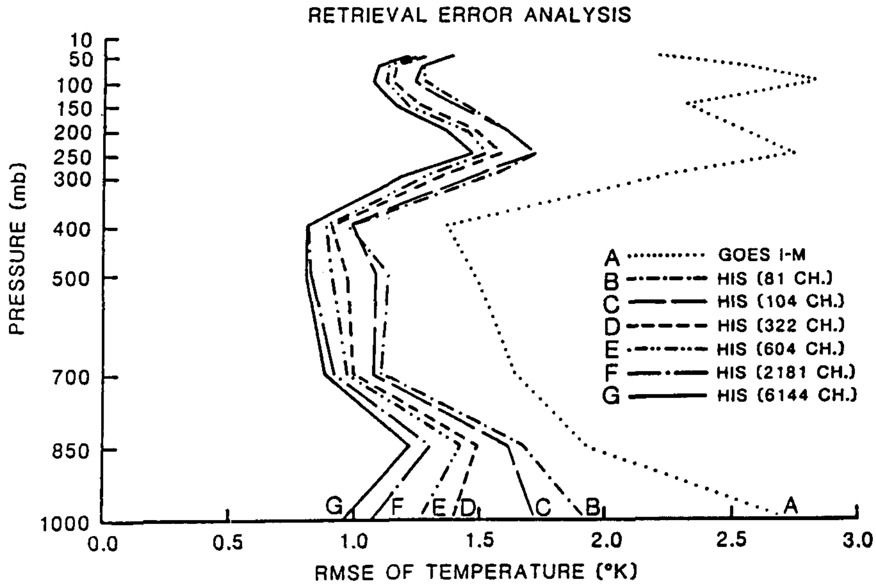


FIG. 10. Temperature rms errors for the use of different optional selections of HIS channels and GOES I/M sounding channels.

provement in the vertical resolution of temperature and water vapor retrieval profiles is demonstrated by the HIS instrument approach.

Figure 8 shows the total rms error of HIS and GOES I/M temperature and water vapor concentration profiles. This result suggests that, consistent with the vertical resolution analysis shown earlier, sounding accuracy is improved by a factor of 2–3 over that of current radiometers by using measurements with high spectral resolution and broad spectral coverage.

Figure 9 demonstrates the resulting total random rms error of temperature and water vapor profiles for different spectral resolutions of HIS measurements. These three spectral resolutions are the full HIS, one-half of HIS, and one-tenth of HIS spectral resolution, each with the same noise (0.25 K) and spectral coverage (600–2700 cm^{-1}). The sensitivity of the retrieval error to spectral resolution is large, with the higher spectral resolution providing the most accurate sounding product.

Different "optimal" sets of HIS channels have been selected to investigate retrieval error sensitivity to the total number of spectral channels used. For each optimal case, those channels whose weighting functions have the greatest sensitivity to each of the 40 sounding levels between 0.1 and 1000 hPa were selected for long-wavelength (15 μm), midwavelength (6.3 μm), and short-wavelength (4.3 μm) bands, respectively (Huang 1989). Figure 10 presents rms error of temperature for different sets of optimal channels. Curve A in this figure is the result obtained by using 19 GOES I/M channels and is presented as a reference. Curves B–G are based on optimally selected HIS channel sets of 81, 104, 322, 604, 2181, and 6144 (complete set). As shown in this figure, the addition of more optimally chosen channels

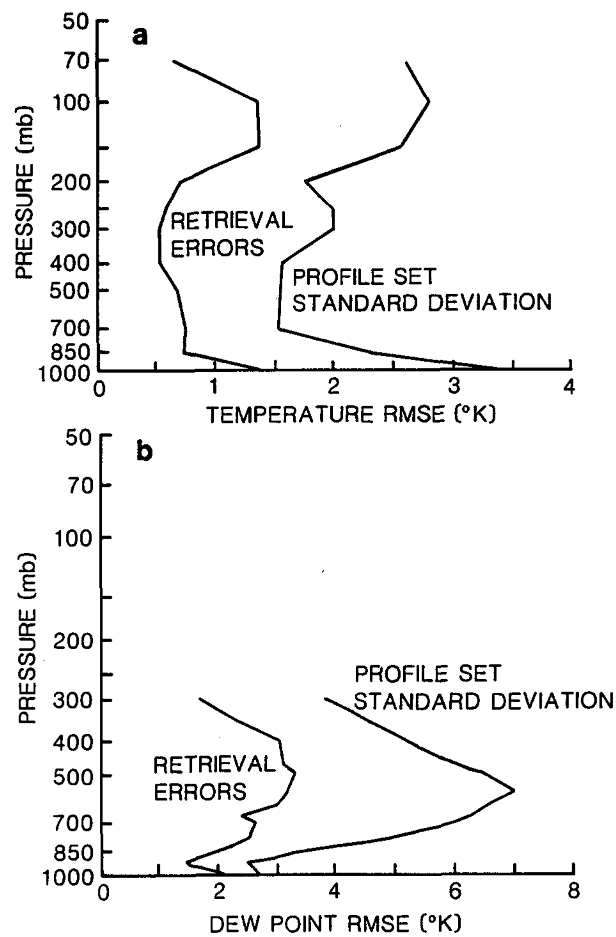


FIG. 11. Simulated temperature and dewpoint temperature rms errors and standard deviation of statistical profile set from the mean profile.

does provide more independent retrieval information and results in a smaller retrieval error.

b. Results of synthetic and aircraft observation retrievals

Figure 11 shows the rms error of sounding retrievals from HIS radiance spectra synthesized from an independent set of 38 special radiosondes during COHMEX. The temperature and dewpoint soundings were retrieved using the solution of Eq. (2) based on a dependent statistical set of 153 different COHMEX radiosondes. The guess profile used for retrieval is the mean of these 153 soundings. Figure 12 shows an estimate of the rms error of retrievals from actual HIS measurements from the NASA ER-2 aircraft for the two COHMEX days, 15 and 19 June 1986. The results in Fig. 12 were obtained by Bradshaw and Fuelberg (1990) using statistical structure function analysis. As can be seen by comparing Figs. 11 and 12 for the COHMEX region, observational results confirm the theoretically expected accuracies close to 1 K for temperature and 3 K for dewpoint temperature.

5. Conclusions

The theoretical retrieval vertical resolution and profile accuracy of GOES I/M, HIRS/2, and HIS are quantified and compared by using a linear simultaneous retrieval algorithm and vertical resolution and error analysis techniques. These observed and simulated retrievals indicate that high spectral resolution and broad spectral coverage are indeed the key to both the improved vertical resolution and the profile accuracy that results.

In conclusion, the passive remote atmospheric sounding of HIS is found to possess the equivalent of 11 pieces of temperature- and 9 pieces of water vapor–independent precise measurements. The characteristics for temperature include a vertical resolution of 1–6 km with an accuracy of 1 K and for water vapor a vertical resolution of 0.5–3.0 km with an accuracy of 3 K in dewpoint temperature. The HIS is a factor of 2–3 times better in vertical resolution and a factor of 2 times better in accuracy than the GOES I/M and HIRS/2 filter radiometers.

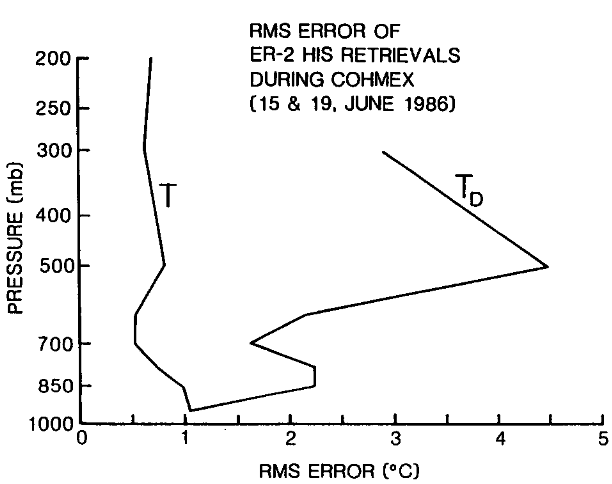


FIG. 12. Temperature and dewpoint temperature rms error of ER-2 HIS retrievals during COHMEX (15 and 19 June 1986).

Acknowledgments. The authors are grateful for stimulating discussions with professor Owen Thompson. The authors are indebted to R. J. Purser, CIMSS/UW for developing the theory of effective resolution provided in the Appendix to this paper. This research was supported by NASA Grant NAS8-36169 and NAGW-1831.

APPENDIX

A Theory of Effective Resolution

Let vertical resolution functions matrix $\mathbf{R}$ be defined as in Eq. (4). From the fact that the precision matrices $(\mathbf{A}^T\mathbf{E}^{-1}\mathbf{A})$ and $\mathbf{S}^{-1}$ are nonnegative symmetric matrices, it follows that each eigenvalue λ of $\mathbf{R}$ is bounded within the interval [0, 1]. Since the sum of nonvanishing eigenvalues is simply the trace of the matrix,

$$J = \mathrm{Tr}(\mathbf{R}), \tag{A1}$$

it can be found that

$$0 \leqslant J \leqslant M, \tag{A2}$$

where M is the number of independent measurements. The case $J = 0$ occurs in the singular limit of vanishing measurement precision, $E^{-1} \rightarrow 0$, and implies no effect of the satellite measurements on the retrieval; while the case $J = M$ occurs only in the case of infinite measurement precision, $E \rightarrow 0$, and implies a retrieval that always fits all measurements exactly. Thus, it seems at least intuitively reasonable to consider J to be a measure of the total effective degrees of constraint imposed by the satellite data used in the retrieval.

It would be useful to be able to extrapolate from the measure J of the effective total quantity of satellite data to a measure of the local effective retrieval density. The simplest choice that integrates to J would be

$$\rho'_i = R_{i,i}/\delta Z_i, \tag{A3}$$

where δZ_i is the height increment of the level i. In theory, there is no guarantee that some components $R_{i,i}$ are not negative and in practice the distribution ρ' is invariably noisy. However, it is observed (Purser, personal communication) that, averaged locally over a scale comparable with that implied by the structure of the corresponding rows and columns of $\mathbf{R}$, the typical magnitudes of ρ' genuinely reflect the local density of distinct measurements when these measurements are precise and are themselves localized. A serviceable measure of local effective data density ρ is therefore given by applying a unimodular nonnegative smoothing operator F to ρ', that is,

$$\rho_i = \sum_j F_{i,j} R_{j,j} \quad \text{(A4)}$$

with

$$\sum_i F_{i,j} \delta Z_i = 1. \quad \text{(A5)}$$

For the univariate data, one need look no further than **R** itself to find the material from which to construct a smoother with the desired qualities. Of the two natural definitions,

$$F^{(a)}_{i,j} = R^2_{i,j} / \sum_k R^2_{k,j} \delta Z_k \quad \text{and} \quad \text{(A6)}$$

$$F^{(b)}_{i,j} = R^2_{j,i} / \sum_k R^2_{j,k} \delta Z_k, \quad \text{(A7)}$$

the first is more universal in the sense that it continues to apply when the retrieval formalism is generalized to an optimal interpolation (Daley 1991) that incorporates pointlike measurements (such as radiosonde temperature observations). The second method has been found to yield superior estimates when, as in the present study, attention is confined exclusively to remotely sensed data whose weighting functions (the rows of **A**) are mainly much smoother than the covariance profiles (rows of **S**). It is the second form, (A7), that is therefore adopted here.

REFERENCES

Backus, G., and F. Gilbert, 1970: Uniqueness in the inversion of inaccurate gross earth data. *Phil. Trans. Roy. Soc. London,* **A266,** 123–192.

Bengtsson, L., 1979: Problems of using satellite information in numerical weather prediction. *Proc. Technical Conf., in its use of data from Meteor. Sat.,* Paris, European Space Agency, 87–100.

Bradshaw, J. T., and H. E. Fuelberg, 1990: HIS analysis of mesoscale phenomena. Preprints, *Fifth Conf. on Satellite Meteorology and Oceanography,* London, Amer. Meteor. Soc., 329–333.

Chahine, M. T., 1974: Remote sounding of cloudy atmospheres. I. The single cloud layer. *J. Atmos. Sci.,* **31,** 233–243.

——, 1977: Remote sounding of cloudy atmospheres. II. Multiple cloud formations. *J. Atmos. Sci.,* **34,** 744–757.

——, 1982: Remote sensing of cloud parameters. *J. Atmos. Sci.,* **39,** 159–170.

——, F. G. O'Callaghan, H. H. Aumann, R. W. Capps, R. D. Haskins, R. J. Pagano, and R. A. Schindler, 1990: Atmospheric Infrared Sounder (AIRS)—Science and Measurement Requirements. NASA-TM-104977, 36 pp.

Clough, S. A., F. X. Kneizys, E. P. Shettle, and G. P. Anderson, 1986: Atmospheric radiance and transmittance: FASCODE 2. Preprints, *Sixth Conf. on Atmospheric Radiation,* Williamsburg, VA, Amer. Meteor. Soc., 141–144.

Conrath, B. J., 1972: Vertical resolution of temperature profiles obtained from remote radiation measurements. *J. Atmos. Sci.,* **29,** 1262–1272.

Daley, R., 1991: *Atmospheric Data Analysis.* Cambridge University Press, 457 pp.

Huang, H.-L., 1989: An analysis of the characteristics of atmospheric profiles obtained with the High-Resolution Interferometer Sounder (HIS). Ph.D. dissertation, University of Wisconsin-Madison, 145 pp.

Kaplan, L. D., 1959: Inference of atmospheric structure from remote radiation measurements. *J. Opt. Soc. Am.,* **49,** 1004–1007.

Newman, W. I., 1979: The application of generalized inverse theory to the recovery of temperature profiles. *J. Atmos. Sci.,* **36,** 559–565.

Phillips, N. A., L. M. McMillin, D. Wark, and A. Gruber, 1979: An evaluation of early operational temperature soundings from *TIROS-N. Bull. Amer. Meteor. Soc.,* **60,** 1188–1197.

Revercomb, H. E., H. Buijs, H. B. Howell, D. D. Laporte, W. L. Smith, and L. A. Sromovsky, 1988: Radiometric calibration of IR Fourier transform spectrometers: Solution to a problem with the High-Resolution Interferometer Sounder. *Appl. Opt.,* **27,** 3210–3218.

Rodgers, C. D., 1987: A general error analysis for profile retrieval. *RSRM 87, Advances in Remote Sensing Retrieval Methods,* Deepak, 285–297.

Smith, W. L., 1991: Atmospheric soundings from satellites—false expectation or the key to improved weather prediction? *Quart. J. Roy. Met. Soc.,* **117,** 267–297.

——, and H. M. Woolf, 1976: The use of eigenvectors of statistical covariance matrices for interpreting satellite sounding radiometer observations. *J. Atmos. Sci.,* **33,** 1127–1140.

——, ——, P. G. Abel, C. M. Hayden, M. Chalfant, and N. Grody, 1974: *Nimbus-5* sounder data processing system. Part I: Measurement characteristics and data reduction procedures. NOAA Tech. Memo. NESS 57.

——, H. B. Howell, and H. M. Woolf, 1979: The use of interferometric radiance measurements for sounding the atmosphere. *J. Atmos. Sci.,* **36,** 566–575.

——, H. E. Revercomb, H. B. Howell, and H. M. Woolf, 1983: HIS—A satellite instrument to observe temperature and moisture profiles with high-vertical resolution. Preprints, *Fifth Conf. on Atmospheric Radiation,* Baltimore, Amer. Meteor. Soc., 9 pp.

——, H. M. Woolf, H. B. Howell, H.-L. Huang, and H. E. Revercomb, 1987: The simultaneous retrieval of atmospheric temperature and water vapor profiles—applications to measurements with the High spectral resolution Interferometer Sounder (HIS). *RSRM 87, Advances in Remote Sensing Retrieval Methods,* Deepak, 189–202.

——, H. E. Revercomb, D. D. Laporte, L. A. Sromovsky, S. Silverman, H. M. Woolf, H. B. Howell, R. O. Knuteson, and H.-L. Huang, 1990: GHIS—the GOES High-Resolution Interferometer Sounder. *J. Appl. Meteor.,* **29,** 1189–1204.

——, H. M. Woolf, and H. E. Revercomb, 1991: A linear simultaneous solution for temperature and absorbing constituent profiles from radiance spectra. *Appl. Opt.,* **30,** 1117–1123.

Susskind, J., J. Rosenfield, D. Reuter, and M. T. Chahine, 1984: Remote sensing of weather and climate parameters from HIRS2/MSU on *TIROS-N. J. Geophys. Res.,* **890,** 4677–4697.

Thompson, O. E., 1982: HIRS-AMTS satellite sounding system test—theoretical and empirical vertical resolving power. *J. Appl. Meteor.,* **21,** 1550–1561.

Wark, D. Q., and H. E. Fleming, 1966: Indirect measurements of atmospheric temperature profiles from satellite: I. Introduction. *Mon. Wea. Rev.,* **94,** 351–362.