

# An Analysis of Wind Direction and Horizontal Wind Component Fluctuations over Complex Terrain

K. H. PAPADOPOULOS AND C. G. HELMIS

*Laboratory of Meteorology, Department of Applied Physics, University of Athens, Athens, Greece*

G. T. AMANATIDIS

*National Center for Scientific Research "Demokritos" Institute of Nuclear Technology and Radiation Protection, Athens, Greece*

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## ABSTRACT

Based on an extensive wind dataset over complex terrain, the commonly used small-angle approximation  $\sigma_\theta \approx \sigma_\theta V$  is studied and found to overestimate over all wind speeds and  $\sigma_\theta$  values observed. This should be anticipated due to the assumptions necessary to derive the approximation. Overestimation (of 10%–30%) is also observed in the small  $\sigma_\theta$  range. The three parameters involved are further discussed to gain better understanding of the behavior of the approximation under different conditions. The standard deviation of wind direction  $\sigma_\theta$  is shown to vary inversely with wind speed not only under stable, but also under convective conditions, reaching a site-dependent constant value at high wind speeds. The dependence of the ratio of the mean longitudinal wind component to the scalar mean wind speed on wind speed and  $\sigma_\theta$  is examined, as well as that of the relevant standard deviations ( $\sigma_u, \sigma_{V_x}$ ). While the former obtains small values in the high- $\sigma_\theta$ , or low-wind range, or both, estimated values of the latter justify equivalence of  $\sigma_u, \sigma_{V_x}$  under most conditions. Finally, the effects of wind speed and  $\sigma_\theta$  on  $\sigma_u$  are examined.

## 1. Introduction

The wind-direction standard deviation  $\sigma_\theta$  constitutes one of the principal estimators of the horizontal diffusing ability of the atmospheric boundary layer. During the usually unstable daytime conditions a maximum is attained. Toward the evening, lower values are measured, while nighttime  $\sigma_\theta$  is influenced by wave activity, mesoscale eddies imposed by the terrain features and meandering. Under stable conditions  $\sigma_\theta$  varies inversely with wind speed over both flat (Joffe and Laurila 1988; JL hereafter) and complex terrain (Hanna 1980; Etling 1990; Leahey et al. 1988). Additionally, JL found the same behavior also to be valid for unstable conditions. Computation of the wind-direction standard deviation is not straightforward as for other scalar quantities, because of the circular distribution of wind direction values and crossover effect. Therefore, several algorithms incorporating certain assumptions have been presented for handling with the problem (Verrall and Williams 1982; Ackermann 1983; Nelson 1984; Yamartino 1984; Turner 1986).

The standard estimators for the mean value of the wind vector's magnitude are the scalar mean and the

vector mean wind speed, to be defined below. Let  $V_i$  and  $D_i$  be the magnitude and direction, respectively, of the instantaneous wind vector and define the  $x$  axis to be along the east–west component of the wind and the  $y$  axis along the south–north. Then, if  $n$  observations of  $V_i, D_i$  are obtained over some selected period of time (the sampling duration) (Mori 1986), the following are defined:

(i) scalar-mean wind speed

$$V_a = \frac{1}{n} \sum V_i \quad \text{and} \quad (1)$$

(ii) vector-mean wind speed

$$V_v = (V_x^2 + V_y^2)^{1/2}, \quad (2)$$

where

$$V_x = \frac{1}{n} \sum V_{xi} = \frac{1}{n} \sum V_i \sin D_i \quad (3)$$

$$V_y = \frac{1}{n} \sum V_{yi} = \frac{1}{n} \sum V_i \cos D_i. \quad (4)$$

The vector-mean wind direction  $D_v$  is defined as

$$D_v = \tan^{-1}(V_x/V_y). \quad (5)$$

The instantaneous wind components parallel and perpendicular to the mean wind direction are  $u_i$  (lon-

*Corresponding author address:* Dr. C. G. Helmis, University of Athens, Laboratory of Meteorology, Department of Applied Physics, 33 Ippokratous, 10680 Athens, Greece.

gitudinal),  $v_i$  (lateral), respectively. The mean value of  $u_i$  is the mean longitudinal wind component  $u$ , which equals the vector mean wind speed, provided  $D_v$  is used as the estimator of mean direction.

Similarity theory provides relations for the standard deviations of the longitudinal, lateral, and vertical wind components ( $\sigma_u$ ,  $\sigma_v$ , and  $\sigma_w$ , respectively). Quite often  $u$  and  $V_a$ , as well as their standard deviations, are used interchangeably in application of similarity formulas without distinction between them.

Assuming small values of  $\sigma_\theta$ , the standard deviation of the lateral wind component  $\sigma_v$  can be approximated by the product  $\sigma_\theta V$ , where  $V$  is either of the estimators for mean wind speed. This approximation is widely used for estimation of  $\sigma_\theta$  or  $\sigma_v$ , depending on which of these is readily available. Yamartino (1984), using a limited dataset created with a Gaussian random number generator, discussed the overpredictions of  $\sigma_\theta$  estimated by  $\sigma_v/V_v$  for  $\sigma_\theta > 40^\circ$ .

In this study  $\sigma_\theta$  is computed by Yamartino's estimator, while the vector-mean direction is used for estimating mean direction (therefore,  $u$ ,  $V_v$  are equal). The validity of  $\sigma_v \approx \sigma_\theta V$  over complex terrain is tested for a wide range of wind speeds and  $\sigma_\theta$  values, particularly in the small  $\sigma_\theta$  range. The dependence of  $\sigma_\theta$  on wind speed under daytime, nighttime, and transition conditions is presented. The differences between  $u$  and  $V_a$  and  $\sigma_u$  and  $\sigma_{V_a}$  in relation to wind speed and  $\sigma_\theta$  are examined. Dependence of  $\sigma_v$  on wind speed and  $\sigma_\theta$  is also provided.

## 2. Observation site—instrumentation

A three-component four-bladed anemometer (distance constant about 2 m, threshold of  $0.15 \text{ m s}^{-1}$ ) is used for the current analysis. The anemometer is operated on the 42-m level of an 84-m meteorological tower (DEMERT) in Athens, Greece (Amanatidis et al. 1990). The tower lies on the foot of the 1024-m-high Hymettos Mountain, which traverses the eastern edge of the Athens urban area along the north-south direction. The base of the tower is 283 m above the mean sea level. The ground around the tower is stony with bushes 50 cm high. There are some trees on the periphery. The roughness length  $z_0$  has been computed

from neutral wind profiles to be 1.1–1.6 m, independent of season.

The three outputs of the anemometer, corresponding to the three components of the wind  $V_{yi}$ ,  $V_{xi}$ , and  $w_i$  are sampled at a 1-Hz frequency and converted to the longitudinal  $u_i$ , lateral  $v_i$ , and vertical  $w_i$  wind components. Mean values and second-order moments are stored every 15 min. Hourly values are also directly calculated. Standard deviation of wind direction is calculated according to the Yamartino algorithm, which is more accurate than other estimators of  $\sigma_\theta$  (Yamartino 1984; Turner 1986; Mori 1986), especially at great  $\sigma_\theta$  values (up to  $104^\circ$ , according to Yamartino). This is important for our analysis since great  $\sigma_\theta$  are expected over complex terrain (Hanna 1981).

The data analyzed cover the period between July 1990 and November 1990, giving a total number of 9712 samples of 15-min. duration each. Measurements from a narrow region of  $30^\circ$  were excluded from subsequent analysis to avoid possible interference with the tower structure (Amanatidis et al. 1991).

## 3. Data analysis and results

### a. Dependence of $\sigma_\theta$ on wind speed

According to the results presented by Smith and Abbott (1961), Hanna (1981), and Leahey et al. (1988) for stable conditions over rough terrain,  $\sigma_\theta$  varies inversely with wind speed. The same behavior was reported by JL over flat terrain for both stable and unstable conditions. The best-fit lines reported in these papers are summarized in Table 1, together with the relevant type of measuring site, atmospheric conditions, measuring height, and sampling duration. The second formula of Table 1 was computed by Etling (1990) based on the measurements of Hanna (1981) and Leahey et al. (1988). Such a behavior, according to  $\sigma_v \approx \sigma_\theta V$ , implies constancy of  $\sigma_v$  during nighttime conditions, as stated by Hanna (1983), Etling (1990), and others. Figure 1 presents a plot of  $\sigma_\theta$  versus  $V_a$  for the 5-month period analyzed. Due to the bulk of data, the 15-min  $\sigma_\theta$  and mean wind speed were grouped in subsets of  $V_a$  (each one covering a  $0.5 \text{ m s}^{-1}$  wide region), and median values of  $\sigma_\theta$  were computed for each one

TABLE 1. Best-fit lines for relations between  $\sigma_\theta$  and wind speed.

Equation ( $\sigma_\theta$ in degrees)	Height (m AGL)	Type of surface	Conditions	Averaging time	Reference
$\sigma_\theta = 57/V$	10	rough	nighttime (21–06)	1 h	Hanna (1981)
$\sigma_\theta = 45/V$	10	slightly irregular	low speed, stable	1 h	Etling (1990)
$\sigma_\theta = 34/V$	10	foothills	all	1-h median values	Leahey et al. (1988)
$\sigma_\theta = 18/V$ for $V < 5 \text{ m s}^{-1}$	10–15	flat	all	1 h	Joffe and Laurila (1988)
$\sigma_\theta = 4$ for $V > 5 \text{ m s}^{-1}$					

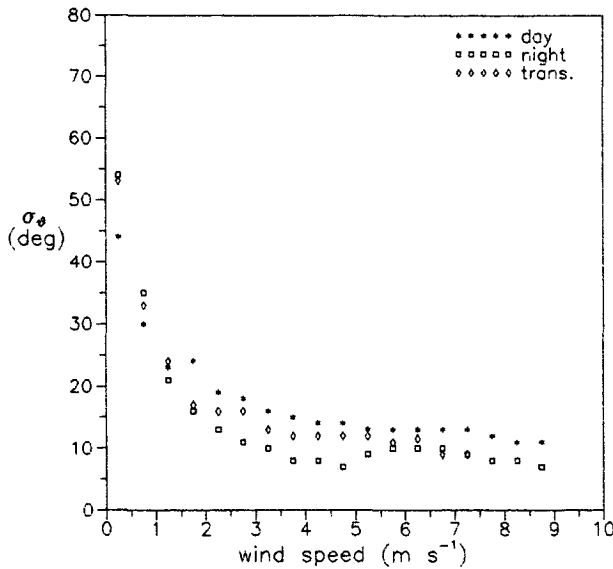


FIG. 1. Median values of 15-min samples of  $\sigma_\theta$  values in terms of wind speed for day (stars, 1000–1800 LST), night (squares, 2100–0600 LST), and transition periods (rhomboids, remaining hours).

of the subsets. Points plotted represent these median values for day (1000–1800 LST), night (2100–0600 LST), and transition periods (the remaining hours). The inverse relation between  $\sigma_\theta$  and  $V_a$  exists for both day and night, suggesting that  $\sigma_\theta$  is strongly related to wind speed under both convective and stable conditions. Transition periods exhibit the same pattern. Great variability of  $\sigma_\theta$  is observed (not shown here) for low-speed cases with extreme values of  $103^\circ$ . Hanna (1983) also noted a variation of  $\sigma_\theta$  between  $20^\circ$  and  $100^\circ$  at wind speeds of  $1 \text{ m s}^{-1}$ . A leveling of  $\sigma_\theta$  accompanied by suppression of variability of  $\sigma_\theta$  values is attained for  $V_a$  roughly greater than  $4 \text{ m s}^{-1}$  in accordance with JL's results over flat terrain. Another feature of Fig. 1 is that below  $1 \text{ m s}^{-1}$  nocturnal values are greater than daytime ones, while the reverse is observed for greater wind speeds. Low wind speeds during the daytime are expected to lead to unstable conditions, while at night they would permit meandering of wind direction. Both these imply big  $\sigma_\theta$ , but the reason that nocturnal values exceed the daytime ones is not obvious. For  $1 \text{ m s}^{-1} < V_a < 6 \text{ m s}^{-1}$ , daytime  $\sigma_\theta$  are greater than nocturnal ones, probably because of enhancement of turbulence due to instability. For  $V_a > 6 \text{ m s}^{-1}$ , as neutral conditions (where turbulence levels are determined by mechanical production) should be approached, nocturnal and daytime  $\sigma_\theta$  values converge. Results for transition periods rank in between those for convective and stable conditions. In the 2.5–5  $\text{m s}^{-1}$  range, during the night only, lower  $\sigma_\theta$  values than for  $V_a > 5 \text{ m s}^{-1}$  are observed. These low values may be attributed to a significantly enhanced frequency of occurrence of winds blowing parallel to the Hymettos

Mountain slope. These wind directions have  $\sigma_\theta$  lower than other directions (for all wind speeds) possibly because of the presence of the slope that limits wind direction fluctuations.

As  $\sigma_\theta$  is known to be affected by the sampling duration selected, in order to be justified in making a comparison with the results of Table 1, the same analysis in Fig. 1 was followed, but using 1-h samples of  $\sigma_\theta$  and  $V_a$  (Fig. 2). Qualitatively, Figs. 1 and 2 are equivalent, with hourly values being greater than 15-min values. This is expected, as the greater the sampling duration the bigger the scales of turbulence allowed to contribute to the variance. Results, if only nighttime conditions are considered, are also included. The three lines represent the best-fit lines of Table 1 [with the exception of Etling's (1990), which is almost identical to that of Leahey et al. (1988)]. There is general agreement between the present results and those of Hanna (1981) and Leahey et al. (1988) referring to rough terrain and stable conditions. Disagreement with JL's line for flat terrain is expected as  $\sigma_\theta$  is enhanced over complex terrain. Their values are  $5^\circ$ – $10^\circ$  smaller for high wind speeds (neutral conditions, when turbulence is determined by surface roughness and wind shear). The reader should keep in mind that our measuring height is significantly greater than those of Table 1. However, similarity theory predicts (Hanna 1981) that during neutral conditions  $\sigma_\theta$  is given by the formula

$$\sigma_\theta = \frac{\sigma_v}{V} = \frac{0.8}{\ln(z/z_0)}, \quad (6)$$

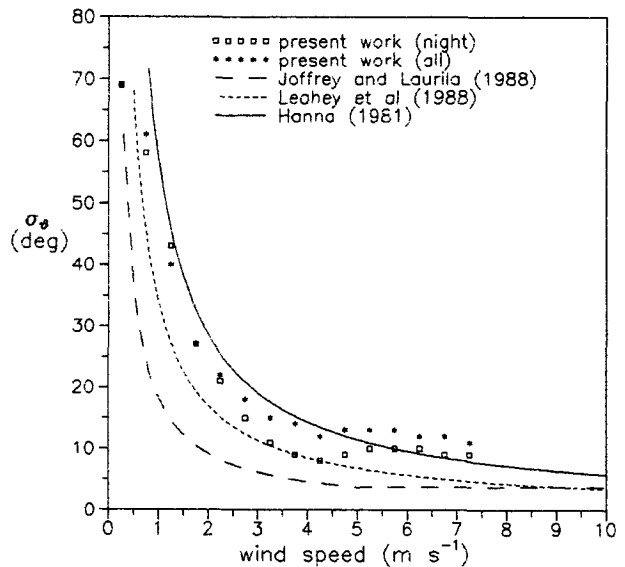


FIG. 2. Median values of 1-h samples of  $\sigma_\theta$  values in terms of wind speed under both stable and convective conditions (stars) and under stable conditions only (squares, 2100–0600 LST). The lines represent best-fit lines proposed by Hanna (1981; solid), Joffrey and Laurila (1988; long dashed), and Leahey et al. (1988; short dashed). Best-fit lines are summarized in Table 1.

( $V$  is any of the vector- or scalar-mean wind speed) based on the assumption of horizontal homogeneity.

Application of (6) with  $z = 42$  m and  $z_0 = 1.1$  or 1.6 yields an estimated  $\sigma_\theta$  value of  $13^\circ$  and  $14^\circ$ , respectively. The mean value of  $\sigma_\theta$  for wind speeds exceeding  $6 \text{ m s}^{-1}$  was computed to be  $12^\circ$  with a standard deviation of  $3^\circ$ , in agreement with (6).

*b. Comparison of  $V_a$  and  $u$  and of relevant standard deviations  $\sigma_{V_a}$  and  $\sigma_u$*

In estimating turbulence intensities and other relevant parameters, the standard deviations of longitudinal ( $\sigma_u$ ), lateral ( $\sigma_v$ ) wind components, and that of the wind speed ( $\sigma_{V_a}$ ) are required.

Mori (1986) showed that  $V_v$  and  $V_a$  are close to each other, their difference growing with  $\sigma_\theta$  and turbulence intensity, and presented several relations between the ratio  $V_v/V_a$  and  $\sigma_\theta$ . One of the relations (based on Yamartino's work) that was found to match our results better is given here,

$$\frac{u}{V_a} = \exp\left(-\frac{1}{2} \sigma_\theta^2\right), \quad (7)$$

and is valid when fluctuations in wind speed and wind direction are uncorrelated and  $D_i$  are normally distributed.

Joffe and Laurila (1988) claim equality of  $V_a$ ,  $u$ , provided that lateral turbulence intensity  $I_v = \sigma_v/u$  is small compared to unity.

Figure 3 shows the dependence of the ratio  $u/V_a$  on wind speed and  $\sigma_\theta$  based on the 15-min samples. Points represent median ratios for bands of  $V_a$  values  $0.5 \text{ m s}^{-1}$  wide (squares) or for bands of  $\sigma_\theta$  values  $5^\circ$  wide (stars). The solid line is (7). As theoretically expected,  $u$  is

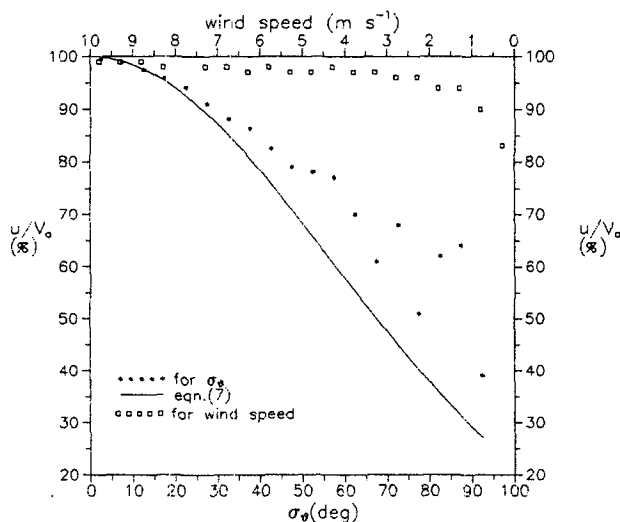


FIG. 3. Relation between ratio of mean longitudinal wind component  $u$  to scalar-mean wind speed and wind speed (squares) and  $\sigma_\theta$  (stars). The solid line is (7).

lower than  $V_a$  through all the range of  $\sigma_\theta$  and  $V_a$  encountered in this study. For  $\sigma_\theta < 10^\circ$ , the mean longitudinal component is almost equal to the scalar-mean wind speed. According to Fig. 3, differences greater than 10% are to be expected for  $\sigma_\theta > 30^\circ$ . Despite the scatter of points above  $50^\circ$  (where the amount of data is rather limited), a tendency for reduction of the  $u/V_a$  ratio is indicated. Big  $\sigma_\theta$  values are encountered at high turbulence intensities where JL's requirement for equivalence of  $u$  with  $V_a$  is violated. Indeed, Fig. 3 shows that at these conditions  $u$  may well be less than  $V_a/2$ . The present results follow (7) up to  $\sigma_\theta = 30^\circ$ . In the high  $\sigma_\theta$  region, the assumptions for (7) to be valid are possibly improper, leading to the deviations from these experimental results. If only wind speed is considered, it is seen that for  $V_a > 2 \text{ m s}^{-1}$   $u$  is less than 5% smaller than the scalar mean wind speed. At strong winds, mechanical production of turbulence prevails and  $I_v$  attains a minimum. Bigger values of  $I_v$  are produced under light-wind convective conditions, while under stable conditions the standard deviations increase again, becoming extremely variable (Panofski and Dutton 1984). For  $V_a < 2 \text{ m s}^{-1}$ , where the majority of big  $\sigma_\theta$  occur, deviations grow up to 20%.

In view of these results one could expect equivalence of  $V_a$  and  $u$  (except, perhaps, close to the rough surface where high turbulence levels are expected) for wind speeds exceeding  $2 \text{ m s}^{-1}$ . However, if  $\sigma_\theta$  exceeds the limit of  $20^\circ$ – $30^\circ$ ,  $u$  is much lower than  $V_a$ , while at  $\sigma_\theta < 20^\circ$   $u$  is 1%–5% smaller than  $V_a$ . It could also be expected that when strong winds ( $>10 \text{ m s}^{-1}$ ) prevail  $u$  and  $V_a$  should be equal.

In several studies there does not seem to exist a distinction between  $\sigma_u$  and  $\sigma_{V_a}$  or the relevant turbulence intensities  $I_u$  and  $I_{V_a}$ . Figure 4 is the analog of Fig. 3 for the standard deviations. There is a tendency for  $\sigma_u$  to be slightly greater than  $\sigma_{V_a}$ , but on the average agreement is excellent, the differences being less than 5%. In contrast to Fig. 3 the effect of  $\sigma_\theta$  and of  $V_a$  on the ratio of the standard deviations is small. For wind speeds greater than  $1 \text{ m s}^{-1}$   $\sigma_u$  is within 1% of  $\sigma_{V_a}$ , while below that limit the ratio drops to 96%. The reason why  $\sigma_u$  is smaller than  $\sigma_{V_a}$  is not obvious. Referring to Fig. 3, it is seen that in the same region the deviation of  $u$  from  $V_a$  is maximum. When  $\sigma_\theta < 15^\circ$   $\sigma_u$ ,  $\sigma_{V_a}$  are equivalent on the average. For bigger  $\sigma_\theta$  values, deviations smaller than 5% occur, apparently growing in magnitude as  $\sigma_\theta$  increases. Scattering of points for  $\sigma_\theta > 80^\circ$  is attributed to the limited number of samples in this region.

*c. Evaluation of the approximation  $\sigma_v \approx \sigma_\theta V_a$  (or  $\sigma_v \approx \sigma_\theta u$ )*

It would be of interest to locate the assumptions necessary to derive the approximation  $\sigma_v \approx \sigma_\theta V_a$  or  $\sigma_v \approx \sigma_\theta u$ .

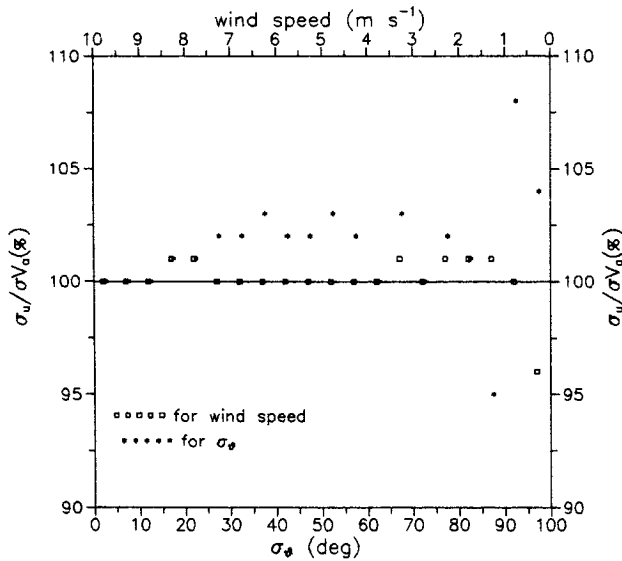


FIG. 4. As for Fig. 3 for the standard deviations.

In Fig. 5,  $V_i$  is the magnitude of the instantaneous wind vector that forms an angle  $\theta'_i$ , with the mean direction derived for the sampling duration under consideration;  $u_i$  is the instantaneous longitudinal component, and  $v_i$  is the lateral one. Clearly, the rms value of  $\theta'_i$  is the standard deviation of wind direction. Assuming that (i)  $\theta'_i$  values are sufficiently small and (ii)  $V_i$  is steady in magnitude ( $V_i = V_a$ ) through the sampling duration and/or fluctuations of direction and wind speed are noncorrelated (horizontal homogeneity is satisfied), then

$$\theta'_i \approx \tan \theta'_i = v_i / u_i \tag{8a}$$

$$\theta'_i \approx \sin \theta'_i = v_i / V_i \quad (\theta'_i \text{ in radians}). \tag{8b}$$

Solving (8b) for  $v_i$  and applying assumptions (i) and (ii),

$$\begin{aligned} \theta'_i V_i \approx v_i \rightarrow \text{var}(\theta'_i V_i) &\approx \text{var}(v_i) \\ &\rightarrow V_a^2 \text{var}(\theta) \approx \text{var}(v) \rightarrow \\ \sigma_\theta V_a &\approx \sigma_v. \end{aligned} \tag{9}$$

Similarly, if the tangent approximation is preferred,

$$\sigma_\theta u \approx \sigma_v. \tag{10}$$

Mathematically, the sine approximation is more precise. Equation (9) is widely used for estimation of either  $\sigma_\theta$  or  $\sigma_v$ , depending on which of the two is readily available.

Over complex terrain, all the prementioned assumptions are questionable and most possibly invalid.

Using 15-min samples of  $\sigma_\theta$ ,  $\sigma_v$ ,  $V_a$ , and  $u$  and averaging the  $\sigma_\theta u$ , and  $\sigma_\theta V_a$  values in  $0.1 \text{ m s}^{-1}$  wide regions of measured  $\sigma_v$ , Fig. 6 was constructed. Equation (9) leads to overestimation of  $\sigma_v$ . Overestimation

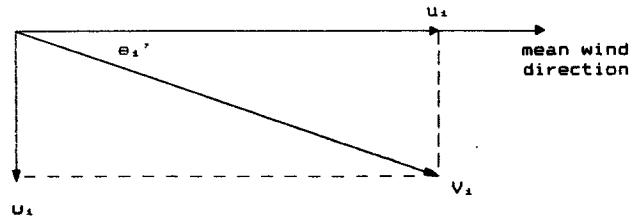


FIG. 5. Geometric representation of instantaneous wind vector, lateral and longitudinal wind components, and mean direction.

for (10) appears smaller; however, at high  $\sigma_v$  greater scatter is observed.

Table 2 includes the mean-bias error (MBE), the root-mean-square error (rmse), and the linear correlation coefficient  $r$  for the approximations expressed by (9) and (10) for three groups of  $V_a$  and totality. Equation (9) has a lower rmse and a higher MBE than (10). Both are positively biased. A remarkable reduction of MBE and rmse is observed as wind speed increases. According to Figs. 1 and 2, high wind-speed conditions are characterized by low  $\sigma_\theta$  values and the small angle approximation used for extraction of (9) and (10) is more justified. Additionally, (9), which as stated is mathematically more precise, has a considerably greater correlation coefficient, .88 against .68 for (10). It is again seen that as wind speed increases,  $r$  increases (note that the data in the high wind speed range are less than those in the low and medium range). As (9) is used more at various studies, in the following we will concentrate on this. In Table 2 the linear correlation coefficient  $r_h$  between  $\sigma_v$  and  $\sigma_\theta V_a$  is also com-

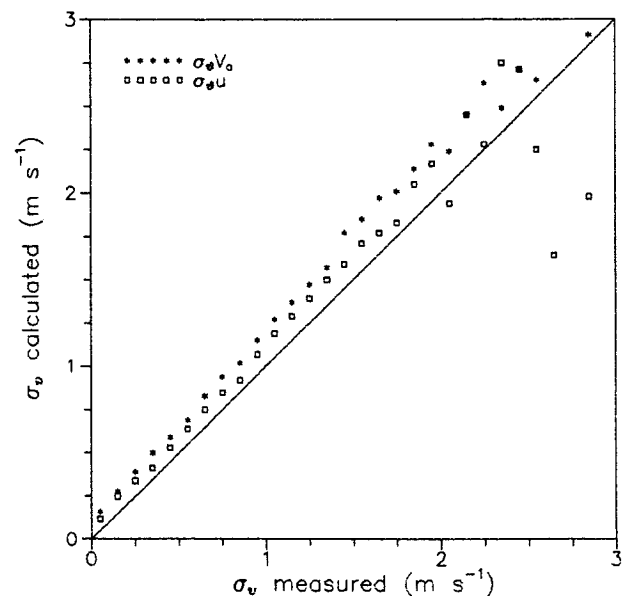


FIG. 6. Mean calculated values of  $\sigma_v$  according to (4) and (5) (stars and squares, respectively) versus measured  $\sigma_v$  values.

TABLE 2. Statistics for the approximations (9) and (10). MBE—mean-bias error, rmse—root-mean-square error,  $r$ —correlation coefficient for 15-min samples,  $r_h$ —as  $r$  for 1-h samples, and  $N_{15}$ —number of 15-min samples.

	$\sigma_\theta V_a$				$\sigma_\theta u$		
	MBE	rmse	$r$	$r_h$	MBE	rmse	$N_{15}$
	( $\text{m s}^{-1}$ )				( $\text{m s}^{-1}$ )		
$V_a \leq 3 \text{ m s}^{-1}$	0.21	0.37	.82	.71	0.14	0.47	4634
$3 \text{ m s}^{-1} < V_a \leq 6 \text{ m s}^{-1}$	0.15	0.28	.90	.84	0.08	0.35	4164
$V_a > 6 \text{ m s}^{-1}$	0.14	0.19	.96	.98	0.03	0.23	914
All	0.18	0.32	.88	.71	0.10	0.40	9712

puted for hourly samples. The hourly dataset is obviously one-quarter of the 15-min dataset. Again, a remarkable value (.98) of correlation is noticed for high wind speeds.

Overestimation of  $\sigma_v$  by use of the approximation (9) could be in principle expected: an assumption made was that fluctuations of wind speed and wind direction are independent and that wind speed is steady in magnitude ( $V_i = V_a$  through the sampling duration). Then,

$$\text{var}(\theta'_i V_i) = V_a^2 \text{var}\theta. \quad (11)$$

But if these fluctuations are not completely independent, as would be expected over a nonhomogeneous complex terrain, then the aforementioned variance should be reduced by an amount analogous to the covariance of wind direction and wind speed fluctuations. Ignoring the covariance provides a potential source of overestimation, as evident in Fig. 6.

Use of  $\tan(\sigma_\theta) = \sigma_u/u$ , as proposed by Hanna (1983), would blow up for  $\sigma_\theta$  approaching  $90^\circ$ , a situation quite usual for low wind speeds (Figs. 1 and 2).

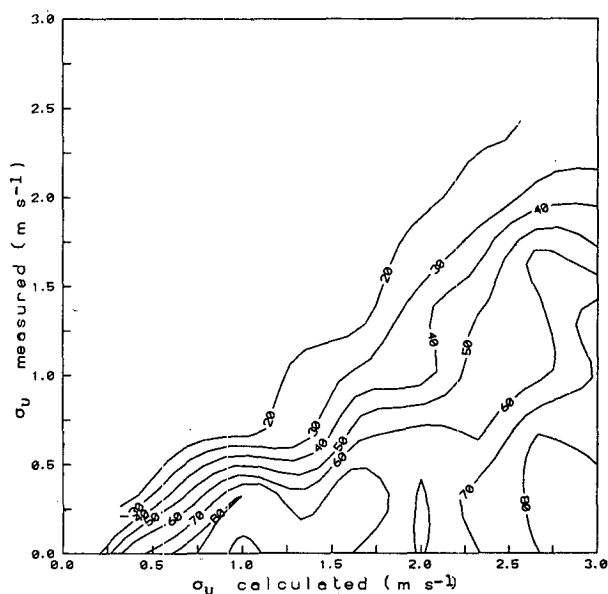


FIG. 7. Contours of  $\sigma_\theta$  (deg) values in a ( $\sigma_v$ , calculated,  $\sigma_v$ , measured) space.

Figure 7 is a more detailed representation of the results of Fig. 6 as it helps clarify the behavior of the approximation in terms of observed  $\sigma_\theta$ . For construction of this figure, a grid of  $\sigma_\theta$  values in terms of  $\sigma_v$  measured ( $y$  axis) and  $\sigma_v$  calculated ( $= \sigma_\theta V_a$ ,  $x$  axis) was computed by taking averages of  $\sigma_\theta$  for all data points ( $x_i, y_i$ ) in a properly selected radius around each specified grid point. Size of grid cells and of averaging radius was determined by the requirement to produce a smooth pattern of contours.

A clear ordering of contours is noted: for  $\sigma_\theta > 20^\circ$  there seems to be a severe overestimation of  $\sigma_v$ . It is also revealed that for low  $\sigma_\theta$ , both  $\sigma_v$  and  $\sigma_\theta V_a$  grow along the same contours, which lie close to the  $y = x$  line, meaning a linear interdependence. For greater  $\sigma_\theta$  values,  $\sigma_v$  seems invariable for values smaller than  $0.5 \text{ m s}^{-1}$ , while in this region the approximation yields unrealistically big values: overestimations on the order of 100% and more are common. Considerable overestimation was also reported over a plant canopy by Maitani (1983), but for the  $\sigma_w \approx \sigma_\theta V_a$  approximation that is in principle the exact analog of  $\sigma_w \approx \sigma_\theta V_a$ . In Table 3 we present the statistical properties for (9) in the small  $\sigma_\theta$  range, where it is supposed to be valid. For clarity, MBE and rmse are now expressed as percentage errors of observed  $\sigma_v$ . Even in this range the error grows with  $\sigma_\theta$  and is not trivial. Relative MBE is 10%–20% (overestimation), and relative rmse is 20%–30%. As shown in the discussion following (11), the overestimation is expected and inherent in the approximation (9) [or (10)]. Therefore, fluctuations of wind speed are always correlated to those of wind direction over complex terrain, as is the site of the present study. The

TABLE 3. Statistics for (9) in the small- $\sigma_\theta$  range. MBE—mean-bias error, rmse—root-mean-square error,  $r$ —correlation coefficient (15-min samples), and  $N$ —number of 15-min samples.

$\sigma_\theta$ (deg)	MBE (%)	rmse (%)	$r$	$N$
0–5	12	20	.97	316
5–10	13	22	.98	1843
10–15	15	29	.98	2641
15–20	20	29	.98	1885
0–20	16	27	.98	6685

same was concluded in Fig. 3 for the big  $\sigma_\theta$  range. The correlation coefficient in Table 3 is around .98 through the whole small  $\sigma_\theta$  range.

The contours of Fig. 8 reveal that the best performance of (9) occurs for wind speeds greater than 3–4  $\text{m s}^{-1}$ , while for low-wind conditions (where  $\sigma_\theta$  attains great values, Figs. 1 and 2) the performance seems awkward (see Table 2).

It would be interesting to show the corresponding dependence of  $\sigma_v$  on wind speed and standard deviation of wind direction. Two distinct regions with an intermediate region in between are seen in Fig. 9, which is like Fig. 2 with contours of measured  $\sigma_v$  added. At low wind speeds ( $< 2 \text{ m s}^{-1}$ )  $\sigma_v$  attains generally low values ( $< 1 \text{ m s}^{-1}$ ) that increase as  $V_a$  increases. However, at the same region (I), contours are vertical showing that for constant wind speed  $\sigma_v$  is independent of  $\sigma_\theta$ . In this region,  $\sigma_\theta V_a$  would grow with  $\sigma_\theta$  at constant wind speed, leading to the significant errors already seen in Table 2. Note that for  $V_a < 3 \text{ m s}^{-1}$   $\text{rmse} = 0.37 \text{ m s}^{-1}$ , while  $\sigma_v$ , in region I of Fig. 8, is in the range of 0.2–0.8  $\text{m s}^{-1}$ , resulting to errors on the order of 50%–180%.

On the lower right corner of the diagram, for  $V_a > 6 \text{ m s}^{-1}$  (region III), the behavior of  $\sigma_v$  is reversed: contours become horizontal, and for constant  $\sigma_\theta$ ,  $\sigma_v$  is independent of wind speed. Here  $\sigma_v$  values are sensitive to  $\sigma_\theta$  and their range is between 0.4 and 2.2  $\text{m s}^{-1}$ . Referring to Table 2 for  $V_a > 6 \text{ m s}^{-1}$ ,  $\text{rmse} = 0.19 \text{ m s}^{-1}$ , thus giving errors on the order of 10%–50%. The slight trend of contours to move slightly downward as wind speed increases in region III permits the product  $\sigma_\theta V_a$  to follow satisfactorily the observed values of  $\sigma_v$ .

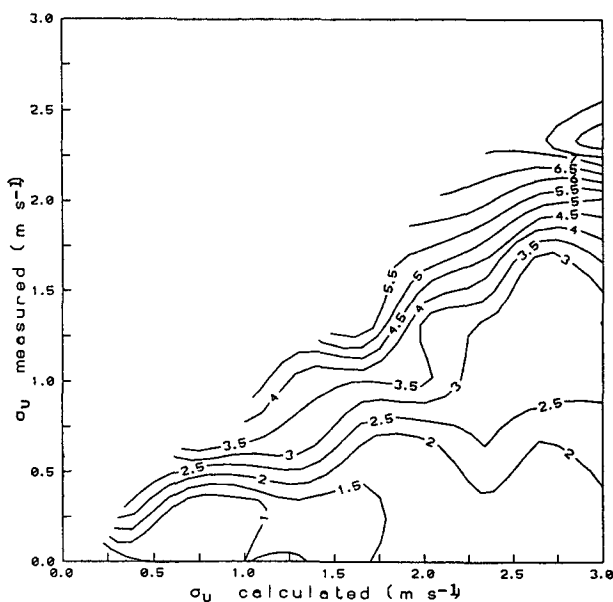


FIG. 8. As for Fig. 8 for wind speed  $V_a$  ( $\text{m s}^{-1}$ ).

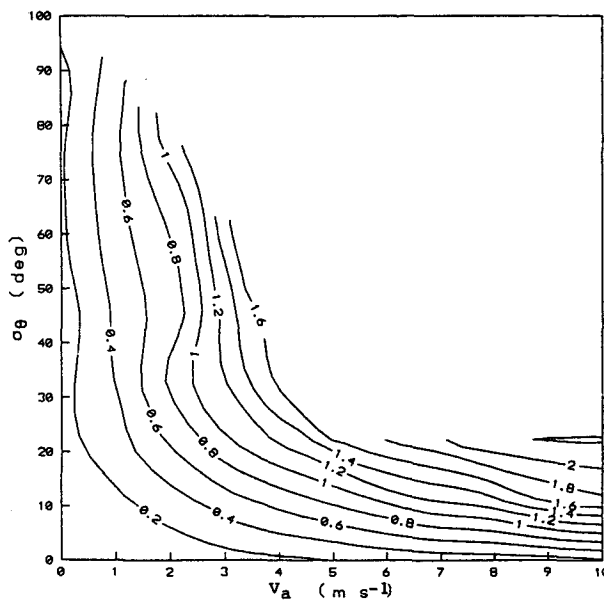


FIG. 9. Contours of measured  $\sigma_v$  ( $\text{m s}^{-1}$ ) values in a  $(\sigma_\theta, V_a)$  space.

In region II the behavior of the approximation is intermediate, with errors ranging from 20% to 140%.

In view of these, it is suggested that computation of  $\sigma_v$  is straightforward when possible; otherwise, use of the approximation  $\sigma_\theta V_a \approx \sigma_v$  can lead to significant errors up to 180%, especially at low wind speeds and great  $\sigma_\theta$ . We note again that low wind-speed conditions are linked to pollution episodes, and an overestimation of  $\sigma_v$  by use of the approximation could lead to great underestimation of pollutant concentrations. However, even in the small  $\sigma_\theta$  range, use of (9) would lead to overestimations on the order of 10%–30% that in certain applications might be considered significant.

In order to further test the constancy of nighttime  $\sigma_v$  (Hanna 1983) using 15-min samples, median values of  $\sigma_v$  were calculated in 18 wind-speed regions  $0.5 \text{ m s}^{-1}$  wide each (Fig. 10). Generally, no constancy is observed. Standard deviation of the lateral wind component increases with wind speed. For wind speeds 2–4  $\text{m s}^{-1}$  and mostly for  $V_a > 6 \text{ m s}^{-1}$ ,  $\sigma_v$  is constant with values of 0.5 and 1.15  $\text{m s}^{-1}$ , respectively.

#### 4. Conclusions

Standard deviation of wind direction shows strong dependence on wind speed, varying inversely to it for day, night, and transition periods independently of sampling duration. Big  $\sigma_\theta$  values with considerable scatter are observed under low-speed conditions, while for higher speeds  $\sigma_\theta$  attains a value determined by surface roughness. A value of about  $10^\circ$ – $15^\circ$  (night and day, respectively) corresponds to our observation site and height in accordance with Hanna's (1980) and Leahey et al.'s (1988) results over complex terrain.

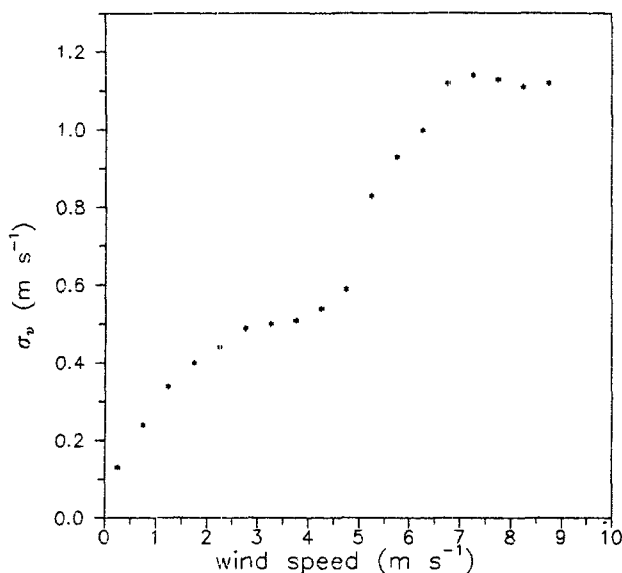


FIG. 10. Median values of  $\sigma_v$  in terms of wind speed for nighttime conditions (for 15-min samples).

Hanna's (1983) proposed best-fit line proves fair during the night, but does not imply constancy of  $\sigma_v$  as stated by the same author. During the night  $\sigma_v$  increases with wind speed, varying between 0.15 and 1.15  $\text{m s}^{-1}$ . Additionally,  $\sigma_v$  is shown here to vary with speed at low speed and high  $\sigma_\theta$  and to vary with  $\sigma_\theta$  at high speeds.

The widely used approximation  $\sigma_v \approx \sigma_\theta V_a$  overestimates measured  $\sigma_v$  by 10%–30% for  $\sigma_\theta < 20^\circ$  but leads to severe overestimation reaching 180% at greater  $\sigma_\theta$ , or low wind speeds, or both. Overestimation is attributed to the assumptions inherent in the approximation. Therefore, direct measurements of both  $\sigma_v$  and  $\sigma_\theta$  are required for proper description of turbulence, as the requirement for low  $\sigma_\theta$  is not met even over flat terrain when speeds lower than 3–4  $\text{m s}^{-1}$  are observed.

Correlation between calculated and measured  $\sigma_v$  becomes higher as wind speed increases, reaching .96. In the small  $\sigma_\theta$  range the correlation is .98.

Finally, on the average, standard deviations of the longitudinal component and of wind speed are equivalent. On the other hand, mean longitudinal wind component may be as small as 50% of the scalar-mean

wind speed (at least for 15-min samples) when  $\sigma_\theta$  exceeds  $70^\circ$  (usually not a very common situation). In the small- $\sigma_\theta$  range,  $u/V_a$  approaches unity and the same probably holds for wind speeds greater than 10  $\text{m s}^{-1}$ . For lower winds the ratio is 80%–95%.

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