Physically Based Simulation of Radar Rainfall Data Using a Space–Time Rainfall Model

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ABSTRACT

A scheme for simulating radar-estimated rainfall fields is described. The scheme uses a two-dimensional stochastic space–time model of rainfall events and a parameterization of drop-size distribution. Based on the statistically generated drop-size distribution, radar observables, namely, radar reflectivity and differential reflectivity, are calculated. The simulated measurable variables are corrupted with random measurement error to account for radar measurement process. Subsequently, radar observables are used in rainfall estimation. Generated fields of the simulated rainfall and the corresponding radar observables are presented. Rainfall estimates from radar simulations are also presented. Use of the described radar-data simulator is envisioned in those applications where the effects of radar rainfall errors are of interest.

1. Introduction

Accurate estimation of rainfall at temporal scales ranging from 1 h to 1 month and spatial scales ranging from 1 to 250 000 km² is of essential importance in hydrometeorological studies. The lower end of both scales is where the interest of operational hydrology lies with its primary objective of flood and flash-flood forecasting. Climate studies are concerned with rainfall at weekly to monthly scales over large areal extents. Meteorological radar offers a convenient way of observing rainfall at all of the above scales. Its ability to provide rainfall estimates from a central location in near-real time makes it a perfect instrument for operational hydrology. Its ability to “see” over large remote areas, including the oceans, makes it the only reasonable calibration tool for satellite-based rainfall-estimation methods. Radar estimation of rainfall, however, is not free of problems. Many aspects of radar accuracy in measuring rainfall have been discussed in the past. Early works of Atlas (1964), Battan (1973), and Wilson and Brandes (1979), as well as recent investigations by Zawadzki (1982) and Austin (1987) and simulation studies by Chandrasekar and Brungi (1987) and Sachidananda and Zrnić (1987), are only a few examples. All of these studies focused on the accuracy of rainfall measurements at the spatial scale of basic sampling (i.e., about 200–400 m in range by 1°–2° in azimuth). The ultimate interest is, however, in the accuracy of rainfall estimates at the scales mentioned before. Also, the effect of the radar rainfall accuracy on the variable of interest in a particular application is perhaps even more important. The studies of these aspects of radar rainfall estimation are less numerous, but works of Chandrasekar et al. (1990a) regarding hydrologic modeling and Rosenfeld et al.’s (1990) analysis of climatic-scale rainfall deserve to be noted.

Motivation for the study described in this paper results from the need for a convenient way to perform hydrologic and climatic investigations in which radar data are involved. Traditionally, hydrologic forecasting models were of the lumped-input, lumped-parameter type. With the advent of new technologies such as remote sensing and geographic information systems (GIS), new approaches to hydrologic modeling have become feasible; however, new technologies also present new challenges and pose new questions. One central question is that of rainfall estimation given various sensors such as radars, satellites, and in situ gauges. Not only do we need to know how to design new observational networks composed of these sensors, but we also need to know what the effect of the errors in rainfall estimates is on the performance of the hydrologic models. Another fundamental question is how to incorporate the remotely sensed information into these models.

In this paper, we propose a physically based generator of space–time radar rainfall observations. The generator is coupled to a stochastic space–time model of rainfall and can be used to study a variety of sampling
and estimation problems. Some of the possible applications of the generator include the multiple-sensor network design problem (Krajewski 1987; Azimi-Zonooz et al. 1989; Seo et al. 1990), effects of rainfall input accuracy on the performance of hydrologic rainfall–runoff models (Krajewski et al. 1991; Chandrasekar et al. 1990a), and satellite rainfall estimation studies.

The convenience of synthetic generation of radar rainfall data was recognized by Grayman and Eagleson (1971), who studied the problem of network design for flood forecasting. Greene et al. (1980) proposed the use of a different generator to study the problem of multisensor rainfall estimation from radars and raingage data. Such an approach was also followed by Krajewski (1987), Seo et al. (1990a,b), and Azimi-Zonooz et al. (1989) to compare a number of various stochastic interpolators used for optimally merging radar and raingage data. In the above studies, the statistical generator of radar rainfall data developed by Krajewski and Georgakakos (1985) was used. The generator is capable of producing a radar rainfall error field with prespecified conditions imposed on its statistical structure. The error field can be correlated in space or uncorrelated, biased or unbiased, with high or low variance. Also, the error field is nonhomogeneous by the fact that the errors are a function of rainfall-field magnitude and gradient. The generator, although very attractive from a user point of view, is limited in the

**FIG. 1.** Simulated values of true rainfall versus radar reflectivity. True rainfall was generated from a gamma distribution with the mean of 4 mm h⁻¹. Corresponding drop-size distribution was obtained using the procedure described in section 3. Random measurement error (standard deviation of 1 dBZ) was added. A sample of 1000 points was generated. The Marshall–Palmer line is given for reference only.

**FIG. 2.** (a) Simulated true versus estimated rainfall. The estimated rainfall is based on simulated reflectivity measurements. Random measurement error (standard deviation of 1 dBZ) was added. The mean values (squared points) and the one-standard-deviation lines (small dots) are based on 250 generated samples. No fit of Z–R relation was attempted. Marshall–Palmer coefficient values were used. The 1:1 straight line is shown for reference only. (b) Simulated true versus estimated rainfall. The estimated rainfall is based on simulated differential-reflectivity measurements. Random measurement error (standard deviation of 1 dBZ for reflectivity and 0.1 dBZ for differential reflectivity) was added. The mean values (squared points) and the one-standard-deviation lines (small dots) are based on 250 generated samples. The estimation method of Chandrasekar et al. (1990b) was used [Eq. (10)]. The 1:1 straight line is shown for reference only.
sense that its validity could not be quantitatively demonstrated. Simply, there is not enough evidence from analysis of real data to determine the true statistical structure of radar rainfall error fields. Therefore, the authors feel that a new, physically based approach is necessary in order to give more credibility to the results of various simulation studies that could be performed using synthetic radar rainfall data.

Simulation of radar rainfall observations at a single location in space has been performed by Sachidananda and Zrnić (1987), Chandrasekar and Bringi (1987), and Goldhirsh (1988), among others. In order to simulate space–time radar rainfall observations, however, one needs to account for realistic space–time variability of the rainfall process itself. This could be done either by using a high-quality radar field as a starting point for simulation, as was done by Krajewski (1987), who used GATE data (Hudlow and Patterson 1979), or by using a space–time model of rainfall. In the past decade, significant research efforts have been devoted to the development of such models. Statistical space–time models by Waymire et al. (1984), Smith and Karr (1985), Bell (1987), and Smith and Krajewski (1987) could be given as examples. In this paper, we describe development of a radar-data generator based on a model by Rodríguez-Iturbe and Eagleson (1987) and modified by Krajewski and Rodríguez-Iturbe (unpublished report).

Fig. 3. True rainfall fields $R_m$ (mm h$^{-1}$) generated by a stochastic space–time model. The panels are at 30, 120, 180, and 330 min reading across from upper left to lower right.
TABLE 1. Parameters used in the space-time rainfall model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Intensity of cluster occurrences in space</td>
<td>0.004 clusters per square kilometer</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Intensity of cell occurrences in a cluster</td>
<td>4.0 cells per cluster</td>
</tr>
<tr>
<td>$E(\omega)$</td>
<td>Mean rainfall intensity at the center of a cell</td>
<td>1.0 mm min$^{-1}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Parameter controlling the birth process in time</td>
<td>0.0066 cells per minute</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Decay parameter of the spread</td>
<td>0.025 min$^{-1}$</td>
</tr>
<tr>
<td>$\text{cov}$</td>
<td>A 2 x 2 variance covariance matrix controlling</td>
<td>29.92, 0.0, 29.92</td>
</tr>
<tr>
<td></td>
<td>the birth of cells around a cluster center</td>
<td></td>
</tr>
<tr>
<td>$xdom$</td>
<td>$x$ dimension of the large mesoscale area (LMSA)</td>
<td>100.0 km</td>
</tr>
<tr>
<td>$ydom$</td>
<td>$y$ dimension of the large mesoscale area (LMSA)</td>
<td>100.0 km</td>
</tr>
<tr>
<td>$D$</td>
<td>Cell radius</td>
<td>2.1 km</td>
</tr>
<tr>
<td>$u_1$, $u_2$</td>
<td>Velocity vector components</td>
<td>20.0, 20.0 km h$^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{\text{jitter}}$</td>
<td>Standard deviation of the lognormal jitter process</td>
<td>1.0</td>
</tr>
<tr>
<td>$nx$, $ny$</td>
<td>Number of the computational grid cells</td>
<td>100 x 100</td>
</tr>
<tr>
<td>$dx$, $dy$</td>
<td>Mesh size of the grid</td>
<td>1.0 km x 1.0 km</td>
</tr>
</tbody>
</table>

2. Radar rainfall errors

An extensive discussion of the various sources of radar rainfall error is given in Zawadzki (1982, 1984). Error structure of radar and surface measurements have been studied in detail through simulation by Chandrasekar and Bringi (1988). A full list of error sources would be very long, with as many as 20 different causes. Here only the most significant error sources are repeated—those that result in either systematic or random error of magnitude greater than 0.5 dBZ.

Natural variability of drop-size distribution and lack of unique correspondence between the observed reflectivity and estimated rainfall rate are perhaps the most often quoted causes of radar rainfall errors. Other errors include inappropriate $Z$–$R$ relationships; radar reflectivity measurement errors, including (random) estimation error due to finite sampling and radar system miscalibration (systematic); ground clutter; subcloud droplet evaporation; advection-caused displacement of radar echoes; the bright band; and anomalous propagation. Other interrelated errors are those due to the range effect, which amplifies the problems of reflectivity gradients across the sampling volume, and partial beam filling.

3. Radar rainfall simulation

In order to simulate, in a physically plausible manner, radar reflectivity measurements, one needs to know the parameters describing raindrop-size distribution. Methods that would allow space–time simulation of the evolution of a raindrop-size distribution (or its parameters) on a scale of a radar umbrella are impractical. On a somewhat smaller scale (20 km x 20 km) such simulations could be performed using cloud thermodynamical and microphysical models of the type described by Clark (1973), Tripoli (1982), or Kogan (1991). Due to the very high computational expense of these models, however, the usefulness of simulating the radar observations of rainfall generated by these models would be very limited. As an alternative we propose to use stochastic space–time rainfall models (e.g., Waymire et al. 1984; Rodriguez-Iturbe and Eagleson 1987; Bell 1987) because they are relatively inexpensive in large-scale computer simulations. These models have been recently used by Bell et al. (1990) and Valdés et al. (1990) to study the satellite sampling problem for the planned Tropical Rainfall Measuring Mission (TRMM) (Simpson et al. 1988).

The above models simulate rainfall patterns and rainfall intensity described as random fields. Thus, if one begins the simulation from a rainfall model, which gives rainfall (specifically, rainfall rate) values but not the drop-size distribution, one faces the problem of a consistent conversion of rainfall rate $R$ to drop-size distribution $N(D)$. It is obvious that from a single relationship

$$R = \frac{\pi}{6} \int_0^{D_{\text{max}}} D^3 v(D) N(D) dD,$$

(1)

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>$R_m$ (mm h$^{-1}$)</th>
<th>$\dot{R}_m$ (mm h$^{-1}$)</th>
<th>$\dot{R}_\text{ZaK}$ (mm h$^{-1}$)</th>
<th>$\dot{R}_Z$ (mm h$^{-1}$)</th>
<th>$\dot{R}_\text{ZaKN}$ (mm h$^{-1}$)</th>
<th>$Z$ (dBZ)</th>
<th>$Z_{\text{DB}}$ (dB)</th>
<th>$R_m - \dot{R}_Z$ (mm h$^{-1}$)</th>
<th>$R_m - \dot{R}_\text{ZaKN}$ (mm h$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0</td>
<td>12.6</td>
<td>15.5</td>
<td>13.9</td>
<td>18.4</td>
<td>128.8</td>
<td>42.1</td>
<td>2.1</td>
<td>7.5</td>
<td>7.6</td>
</tr>
<tr>
<td>120.0</td>
<td>12.3</td>
<td>15.6</td>
<td>13.3</td>
<td>19.5</td>
<td>155.4</td>
<td>42.1</td>
<td>1.9</td>
<td>5.7</td>
<td>7.2</td>
</tr>
<tr>
<td>180.0</td>
<td>11.2</td>
<td>16.3</td>
<td>13.2</td>
<td>15.3</td>
<td>2981.7</td>
<td>42.4</td>
<td>2.1</td>
<td>5.2</td>
<td>5.7</td>
</tr>
<tr>
<td>330.0</td>
<td>3.3</td>
<td>4.6</td>
<td>2.7</td>
<td>4.6</td>
<td>344.1</td>
<td>33.6</td>
<td>1.4</td>
<td>1.3</td>
<td>2.1</td>
</tr>
</tbody>
</table>
where \( R \) is in millimeters per hour, the terminal velocity \( v ( m \ s^{-1}) \) of a drop with diameter \( D \) (mm) is given by Atlas and Ulbrich (1977) as

\[
v(D) = 17.67D^{0.67}
\]

(2)

and the drop-size distribution is described by a gamma function

\[
N(D) = N_0 D^m \exp(-\Delta D).
\]

(3)

There is no unique correspondence between rainfall \( R \) and \( m \) the parameters describing the distribution. To circumvent this problem, one could use an exponential distribution in which the shape parameter \( \Lambda \) is related to rainfall rate as in the Marshall–Palmer relationship (Marshall and Palmer 1948) and \( N_0 \) is determined by solving (1). We propose a different solution for generating the drop-size distribution parameters that also gives the required rainfall rate. It is based on the three-parameter gamma distribution of drop size given by (3).

First, generate the time evolution of the (dimensionless) parameter \( m \) from a uniform distribution over the range \( m_{\text{min}}(t) \) to \( m_{\text{max}}(t) \):

\[
m = U\{m_{\text{min}}(t), m_{\text{max}}(t)\}.
\]

(4)

The symbol \( U\{a, b\} \) denotes a uniform probability distribution defined over an interval \( (a, b) \). Studies by Chandrasekar and Bringi (1987) support the assump-

Fig. 4. Reflectivity fields \( Z \) (dBZ) generated by a stochastic space–time model. The panels are at the same times as in Fig. 3.
tion of uniform distribution for \( m \) and demonstrate that a good choice of the values for the limits is \( m_{\text{min}} = -1 \) and \( m_{\text{max}} = 4 \) for a duration spanning the life cycle of a storm. The values of \( m \) are decorrelated in both space and time.

The next step is the generation of the parameter \( N_0 \) \([\text{mm}^{-1-m} \text{ m}^{-3}]\), where the value of \((-1 - m)\) is obtained from (4):

\[
N_0 = U\{N_{0\text{min}}, N_{0\text{max}}\}. \tag{5}
\]

A lower and upper bound of \(10^{4.2-1-m}e^{2.8m} \) and \(10^{5.5-1-m}e^{3.57m} \), respectively, have been used in this study based on the findings of Ulbrich (1983).

The last parameter of the drop-size distribution to be determined is the equivolumetric spherical diameter \( D_0 \) (mm) which is simply related to \( \lambda \). Since it is now the only unknown in (1), relating the (given) rainfall rate to the drop-size parameters, it can be easily obtained by solving the equation as

\[
D_0 = \left[ \frac{1}{7.12 \times 10^{-3}} \frac{(3.67 + m)^{4.67+m}}{\Gamma(4.67 + m)} \frac{R}{N_0} \right]^{(4.67+m)^{-1}}. \tag{6}
\]

In (6), \( D_0 \) is in millimeters, \( N_0 \) has the units of \((\text{mm}^{-1-m} \text{ m}^{-3})\), and \( R \) is in millimeters per hour. In this study, an upper bound of 2.5 mm has been imposed on \( D_0 \). If the upper limit in (1) is a \( D_{\text{max}} \neq \infty \),

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**FIG. 5.** Differential reflectivity fields \( Z_{DR} \) (dB) generated by a stochastic space–time model. The panels are at the same times as in Fig. 3.
the solution for $D_0$ can be obtained numerically. Also, if both $m$ and $D_0$ are known, one can obtain the parameter $\Lambda$ in (3) as $\Lambda = 3.67 + m/D_0$ (Chandrasekar and Bringi 1987).

Given the drop-size distribution parameters, one can calculate the radar reflectivity as

$$Z = \int_0^{D_{\text{max}}} D^6 N(D) dD. \quad (7)$$

In this study, a $D_{\text{max}} = 8$ mm has been used. Equation (7) is valid only for the range of radar wavelength for which the Rayleigh approximation holds.

In addition to radar reflectivity, one can also calculate the differential reflectivity $Z_{DR}$, which allows for simulation of the multiparameter radar techniques. The equilibrium shape of a raindrop falling at its terminal fall speed is the shape for which forces due to surface tension, hydrostatic pressure, and aerodynamic pressure (due to airflow around the drop) are in balance (Pruppacher and Pitter 1971). For drops greater than 1 mm in diameter, the shapes are nonspherical and can be approximated by oblate spheroids. Pruppacher and Pitter (1971) show that the approximate axis ratio $b/a$ of a drop and the diameter $D_e$ (of an equivalent spherical raindrop) are related by

$$\frac{b}{a} = r = 1.03 - 0.062 D_e, \quad (8)$$

Fig. 6. Estimated rainfall fields $R_2$ (mm h$^{-1}$) generated by a stochastic space–time model. The panels are at 30, 120, 180, and 330 min reading across from upper left to lower right.
where $D_e$ is in millimeters, and $a$ and $b$ are the major and minor axes of the drop, respectively. The differential reflectivity technique exploits this fact by measuring the polarization-dependent backscattered power from the raindrops at two polarization states that nominally coincide with the horizontal and vertical directions. The differential reflectivity $Z_{DR}$ is defined in terms of the drop-size distribution as (Seliga and Bringi 1976)

$$Z_{DR} = \frac{\int_0^\infty \sigma_H(D)N(D)dD}{\int_0^\infty \sigma_V(D)N(D)dD}, \quad (9)$$

where $\sigma_H$ and $\sigma_V$ are the radar cross section of the oblate raindrops at horizontal (H) and vertical (V) polarizations, respectively. Differential reflectivity $Z_{DR}$ can then be computed knowing $m$, $N_0$, and $D_0$.

The radar observable quantities, radar reflectivity $Z$, and differential reflectivity $Z_{DR}$ computed using (7) and (9) should be considered as the theoretical values. Their measurement by radars is associated with a measurement error. This measurement error results from the properties of the signal (averaging of finite pulses) and radar system noise. In order to simulate these effects, a measurement error must be added. Chandrasekar and Bringi (1987) described a procedure that simulates the measurement error as a function of radar system parameters. Figure 1 illustrates the effects of

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**Fig. 7.** Estimated rainfall fields $\bar{R}_{zdr}$ (mm h$^{-1}$) generated by a stochastic space–time model. The panels are at the same times as in Fig. 6.
simulated measurement error for an S-band radar operating with pulse repetition time (PRT) of 1 ms. The variability in Fig. 1 is due to both the proposed parameterization of the drop-size distribution and the added measurement error.

Once radar measurables are obtained, one can use them to estimate the rainfall rate. Conventional Z–R relations can be used to estimate rainfall rate from the radar reflectivity. For the dual-parameter (Z, Z_{DR}) estimates of rainfall rate, a nonlinear regression equation of the form (Chandrasekar et al. 1990b)

\[ R = 1.96 \times 10^{-3} Z^{0.981} Z_{DR}^{1.06} \]  

is used. In (10), Z is in standard units (mm$^6$ m$^{-3}$) and Z_{DR} is in decibels.

To account for the effect of a wrong Z–R relationship, one can use arbitrary values for the coefficients (e.g., Marshall–Palmer). Figure 2a shows the results of the above-described procedure applied to rainfall rate ranging from 0 to 200 mm h$^{-1}$. The mean and standard deviation band are shown based on 250 realizations generated for each value of the rainfall rate. Figure 2b demonstrates similar results obtained for rainfall estimated using the differential reflectivity measurements (10). It is clear that the Z–R relationship used (Marshall–Palmer) introduces a significant bias. For the differential-reflectivity method, there is virtually no bias. To eliminate the effect of wrong Z–R relationship, the Z–R coefficients could be calibrated to the particular rainfall regime generated by the stochastic

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**Fig. 8.** Estimated rainfall fields $\hat{R}_{20}$ (mm h$^{-1}$) generated by a stochastic space–time model. The panels are at the same times as in Fig. 6.
model. Since the objective of the proposed simulator is to simulate errors in radar rainfall estimates, however, the biases should be removed only if the effects of random errors are of main interest in a particular application of the simulator.

The results presented in Figs. 1 and 2 demonstrate a reasonable performance of the scheme in that they resemble the visual appearance of the data analysis shown by Richards and Crozier (1981).

4. Space–time rainfall model

The criterion used in selecting a space–time model of rainfall for this study was the model’s ability to generate the space–time evolution of instantaneous rainfall rate. An event-type model by Rodriguez-Iturbe and Eagleson (1987) was chosen. It was implemented into a computerized form by Frank et al. (1990) and used to study various hydrologic problems (Chandrasekhar et al. 1990a).

Here only a brief description of the model will be presented. An interested reader is referred to the original papers for details.

The model conceptualizes mesoscale rainfall fields as clusters of cells (rainbands) having certain attributes. The attributes are cell life span, cell velocity, and rainfall intensity, as well as the time and place of its birth. The number of rain cells constitutes a spatial Poisson process around the location of the cell cluster. By proper accounting for the individual-cell rainfall con-

![Estimated rainfall fields](image)

**Fig. 9.** Estimated rainfall fields $\hat{R}_{Z_{a,n}}$ (mm h$^{-1}$) generated by a stochastic space–time model. The panels are at the same times as in Fig. 6.
tributions, one can construct rainfall fields with very realistic appearances and statistics in the form of a space–time covariance function behaving as that of the observed rainfall. By controlling the parameters of the model, one can generate rainfall fields appropriate for distinctly different climates (see Valdés et al. 1985). The most important feature of the model, as far as our study is concerned, is that based on the parameters of the model, one can calculate the space–time mean and covariance of the resulting rainfall fields and thus characterize the results associated with the generated rainfall.

For the Rodriguez-Iturbe and Eagleson (1987) model, the mean and the variance of rainfall $\xi$ at time $t$ and location $\mathbf{u} = (u_x, u_y)$ are the following functions of the model parameters:

$$E\{\xi(t, \mathbf{u})\} = 2\pi D^2 E\{\nu\} \beta E\{i_0\} \times (\alpha - \beta)^{-1} (e^{-\beta t} - e^{-\alpha t})$$  \hspace{1cm} (11)

and

$$\text{var}\{\xi(t, \mathbf{u})\} = \pi D^2 E\{\nu\} \beta E\{i_0^2\}(2\alpha - \beta)^{-1} \times (e^{-\beta t} - e^{-2\alpha t}) + \pi D^4 E\{\nu(\nu - 1)\} \beta^2 E^2\{i_0\} \times (\alpha - \beta)^{-2} (D^2 + \sigma^2)^{-1} (e^{-\beta t} - e^{-\alpha t}),$$  \hspace{1cm} (12)

where $D$ is the cell diameter, $\nu$ is the number of cells associated with each cluster and is a Poisson distributed random variable, $\rho$ is the spatial intensity parameter of the Poisson process controlling the occurrence of clusters in space, $\beta$ and $\sigma$ are the parameters controlling distribution of the cells around a cluster center (could

**FIG. 10.** Residual rainfall fields $R_m - R_{2N}$ generated by a stochastic space–time model. The panels are at 30, 120, 180, and 330 min reading across from upper left to lower right. The viewing angle of the figure was chosen to highlight positive and negative values of the residuals. The minimum and maximum values are shown to assist in interpreting the residual values in the diagram.
be elliptical), \( \alpha \) is the time decay parameter for rainfall intensity resulting from a particular cell, and \( i_0 \) is a random variable denoting rainfall intensity at the center of a cell.

The covariance of the rainfall intensity processes is given by

\[
\text{cov}[\xi(t_1, u_1), \xi(t_2, u_2)] = \pi D^2 E\{\nu\} \rho \beta E\{i_0\} \\
\times (2\alpha - \beta)^{-1} e^{-\alpha t_1} [e^{(\alpha - \beta) t_2} - e^{-\alpha t_2}] e^{-d^2/(4D^2)} \\
+ \pi D^2 E\{\nu(\nu - 1)\} \rho \beta^2 E^2\{i_0\} (\alpha - \beta)^{-2} \\
\times (D^2 + \sigma^2)^{-1} (e^{-\beta t_1} - e^{-\alpha t_1}) \\
\times (e^{-\beta t_2} - e^{-\alpha t_2}) e^{-d/[4(D^2 + \sigma^2)]}. \quad (13)
\]

Knowledge of the statistical structure (second-order moments) of the underlying rainfall rate is important to characterize to results of studies in which the simulator can be used. For example, in the next section, the statistics of the errors in the simulated radar rainfall estimation process are presented. The errors exhibit spatially correlated behavior that cannot be fully understood without a prior knowledge of the correlation in the original (simulated) rainfall fields.

Figure 3 shows the rainfall fields generated by the model. The space–time rainfall model, henceforth referred to as the STM, generates a snapshot of rainfall over a 100-km \( \times \) 100-km area every 15 min. For this study, the simulated events were limited to a duration of 500 min. Figure 3 shows the rainfall fields generated at 30, 120, 180, and 330 min into the simulation. The values of the model parameters used in the simulation are shown in Table 1. Table 2 shows the peak rain rate

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**Fig. 11.** Residual rainfall fields \( R_m - \tilde{R}_{zm} \) generated by a stochastic space–time model. The panels are at the same times as in Fig. 10. The viewing angle of the figure was chosen to highlight positive and negative values of the residuals. The minimum and maximum values are shown to assist in interpreting the residual values in the diagram.
for the above events. The peak rain rate decreases as the event progresses in time. There is also a corresponding increase in the spatial spread, with early fields resembling convective storm activity and the latter fields representing stratiform rainfall in an mesoscale convective complex (MCC) system. The rainfall fields generated by the model look quite realistic. It should be emphasized that there was no attempt to fit the model parameters to a particular rainfall regime. The parameters were determined to obtain a qualitative agreement of the resultant patterns with those observed at radar scopes.

5. Simulation results

The capabilities of the model to generate realistic radar rainfall error fields can only be demonstrated qualitatively. A quantitative analysis requires calibration of the stochastic model to a particular rainfall regime and availability of extensive database of simultaneous radar and rainage data. Such an analysis is beyond the scope of this paper. Since neither ground clutter nor the anomalous propagation echoes are generated by the described method, the patterns of rainfall and the radar observable quantities of reflectivity and differential reflectivity should appear quite similar. In this paper no application of the developed simulator is included.

In the following discussion and figures, a notation convention is used: $R_m$ is rainfall-generated model; $Z$ is radar reflectivity (dBZ); $Z_{DR}$ is differential reflectivity (dB); $\hat{R}_Z$ is rainfall estimated based on reflectivity measurements; $\hat{R}_{ZDR}$ is rainfall estimated based on differential reflectivity measurements; $\hat{R}_{ZN}$ is rainfall estimated based on measurement error–corrupted reflectivity measurements; $\hat{R}_{ZDRN}$ is rainfall estimated based on...
based on measurement error–corrupted differential reflectivity measurements. In Figs. 10–13 there is no scale indicated on the plots. For information regarding the scaling, one should refer to Table 2 for the maximum values of the plotted fields.

Figures 4 and 5 show the reflectivity and the differential reflectivity fields, respectively, obtained from the (given) STM rainfall and the drop-size parameters, following the procedure outlined in (4)–(7) and (9). Table 2 lists the peak values corresponding to the Z and Z_{DR} fields for the snapshots at the same times as the rainfall fields presented in Fig. 3. The figures show well-defined areas of constant Z and Z_{DR} that are very similar to Z and Z_{DR} displays on the radar scope during real-time monitoring of storms.

The corresponding fields of estimated rainfall obtained using (10) and (11) are shown in Figs. 6 and 7. The estimated peak values for the fields considered are listed in Table 2. Figures 8 and 9 show similar quantities except that the measurements of reflectivities used in the estimation were corrupted by random measurement error. These errors were generated from Gaussian distribution with standard deviation of 1 dBZ for reflectivity measurements and 0.4 dB for differential reflectivity measurements. The procedure allows evaluation for the effect of the radar measurement error in a particular application of the generator in isolation from other error sources—something clearly impossible when dealing with real data.

Finally, Figs. 10–13 show the plots of residuals between the true rainfall (R_m) and the estimated rainfall, (\tilde{R}_Z) and (\tilde{R}_{Z_{DR}}) as well as the spatial correlation function of the residuals. It can be noticed that the patterns of the residuals are quite different for the two compared methods. This observation is confirmed by the analysis of the spatial correlation function of the

![Fig. 13. Spatial autocorrelation functions for the residuals R_m - \tilde{R}_{Z_{DR}}. The panels are at the same times as in Fig. 12.](image-url)
residuals. For the reflectivity-based method, the residuals display strong spatial correlation with the correlation distance (defined in terms of exponential covariance) being about 10–12 km, while the $Z_{DR}$-based method displays virtually uncorrelated residuals. The correlation of the residuals in the reflectivity-based method can be easily explained, especially if there is bias in the $Z-R$ relationship. Such a bias, if experienced over a significant rain-intensity range, combined with correlation in the rainfall field itself, causes correlated residuals. On the other hand, in the studies performed by Krajewski (1987) and Seo et al. (1990a,b), knowledge of the correct correlation of the estimation errors was demonstrated to be important for optimal merging of radar and raingage observations.

6. Closing remarks

A simple model for efficient generation of radar observables useful in studies of the effects of radar rainfall estimation errors has been developed. The model is used together with a space–time stochastic model of rainfall that generates the true rainfall fields. A parametrization of the stochastic model-generated rainfall rate into a corresponding drop-size distribution is proposed. Based on the drop-size distribution, radar observables such as reflectivity factor and differential reflectivity are calculated. From these, rainfall can be estimated using the standard methods and compared with the original rainfall generated by the stochastic model. The model was tested in a qualitative sense. The two-dimensional displays of the generated fields, as well as the statistics and patterns of the underlying errors, look reasonable.

The current version of the model is valid for the Rayleigh regime, that is, it is not suitable for simulation of the effect of attenuation or hail contamination; however, there are no conceptual obstacles why this could not be done. Also, the model could be extended to account for the errors due to horizontal gradient of reflectivity across a sampling volume.

It is hoped that the model will be an attractive tool to study the performance of hydrologic parameter-distributed models using radar rainfall input and hydrologic network-design studies where the choice of the radar system parameters, as well as the density and configuration of the accompanying raingage network, are of interest.

The computer programs for the stochastic model and the radar rainfall simulator are available from the authors upon request.

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