Identification of Vertical Profiles of Radar Reflectivity for Hydrological Applications Using an Inverse Method. Part II: Sensitivity Analysis and Case Study

HERVÉ ANDRIEU

LCPC, Division Eau, Bouguenais, France

GUY DELRIEU AND JEAN DOMINIQUE CREUTIN

Institut de Mécanique, LTHE, Grenoble, France

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ABSTRACT

A method of identification of the vertical profile of reflectivity, formulated by the authors in an accompanying paper, is tested through a sensitivity analysis. Two simulated but realistic profiles (with and without brightband effects) are used. The radar features and the statistical parameters involved in this method are allowed to vary around standard values in order to understand their influence on the results. The main conclusion is that, for a given radar configuration, results of acceptable quality can be obtained with a single adjustment of the method for the two types of profiles, which suggests the approach is operationally applicable.

To complement this theoretical analysis, actual profiles of reflectivities are studied for two rainfall events observed in the Cevennes region of France. The efficiency of the proposed method is appreciated from a hydrological point of view. A comparison is performed at the basin scale between hourly rainfall intensities, measured with a dense network of rain gauges and radar data. The analysis shows that the introduction of the identification and the correction of the influence of vertical profiles of reflectivity improve the accuracy of rainfall estimates from radar data.

1. Introduction

In a companion paper, a method intended to determine the vertical profiles of radar reflectivity (VPR) is proposed (Andrieu and Creutin 1995; hereinafter referred to as Part I). The method is based on the comparison of radar data from two (or more) beam elevation angles. It consists simply of calculating, versus distance, the ratio between radar reflectivities or intensities recorded at different angles of beam elevation. The ratio function obtained depends only on the characteristics of the radar and on the VPR sought. Using an inverse method, it is possible to deduce the VPR from the observed values of this ratio function. This paper deals with the evaluation of the method through a sensitivity analysis and an actual case study.

The sensitivity study is performed on the basis of two characteristic VPRs, VPR1 and VPR2, introduced in Part I; they differ mainly in the way the bright band is included. The objective is to understand the influence of various parameters on the effectiveness of the VPR identification method. These parameters include the conditions of operation of the radar (elevation angles, beamwidth), the shape of the VPR, and the statistical characteristics required by the inversion method (i.e., the first two moments of the ratio function and of the a priori VPR). The sensitivity study is conducted as follows. Standard values are chosen for each parameter of the tested method. The true VPR is known, and the corresponding ratio function is simulated by varying each parameter at one time, while keeping all others at the standard values. The identification algorithm is applied to the simulated data. The true VPR and the estimated VPR are compared using a single simple criterion—the mean absolute error. This sensitivity study makes it possible to issue a number of useful recommendations for the application of the method. However, it must be borne in mind that the VPRs used are assumed to be perfectly homogeneous in space, which is a simplification of reality. This aspect was not dealt with in the sensitivity study, because of a lack of quantitative information about the spatial variability of the VPR. Furthermore, when the sensitivity of the method to one parameter is studied, it is implicitly assumed that the influences of the various parameters are independent, which is not strictly true. In particular, the choice of the standard values used in the sensitivity study is undoubtedly important.

The method is applied to two actual rain situations observed in the south of France in the course of the
radar Cévennes 1986–88 experiment. A 1-h time step was selected in this case study because it corresponds to hydrological needs (governed by the response time of the catchments). In this example, the VPR is determined using the intensity ratio function [Part I, section 4, Eq. (30)]. This situation corresponds to a common case where the radar data have been converted to intensities, using a Z–R relationship, to be totalled over the time step in question. The choice of ratio function type (intensity or reflectivity) does not affect the validity of the VPR identification procedure.

Validating the obtained VPR is a difficult task. In effect, since the true VPR is unknown, the effectiveness of the method can at first be judged only indirectly, by comparing the observed ratio functions to those calculated from the estimated VPR. The evaluation is completed by a hydrological validation. The presence of many recording rain gauges provides a reliable estimate of the rainfall intensities on several catchments; this also makes it possible to compare the performance of the radar, both when the VPR is taken into account and when no correction is made for its influence.

### 2. Sensitivity analysis

The sensitivity study is performed using VPR1 and VPR2, which are hypothetical but realistic examples (Part I, Fig. 2). Other shapes of VPR are possible, but these simply illustrate two very common families of situations: absence of bright band and regular evolution of the VPR with altitude on the one hand, and presence of a bright band and sudden variations of the VPR on the other. In the first step, we consider particularly the link between the ratio function and the VPR. This is to form an idea of the potential effectiveness of the method by considering the data that are the basis of its use. Table 1 groups the standard values taken by the different VPR parameters and intensity ratio function.

<table>
<thead>
<tr>
<th>Radar parameters</th>
<th>2θθ = 1.5°</th>
<th>beamwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1 = 1.0°</td>
<td>low elevation angle</td>
</tr>
<tr>
<td></td>
<td>A2 = 3.0°</td>
<td>high elevation angle</td>
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<tr>
<td>Ratio function</td>
<td>n = 40</td>
<td>number of gates regularly</td>
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<tr>
<td></td>
<td>σR = 0.12 Q + 0.012</td>
<td>spaced between 0 and</td>
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<tr>
<td></td>
<td>Dz = 0</td>
<td>100 km</td>
</tr>
<tr>
<td>A priori vertical profile of reflectivity</td>
<td>Δh = 0.2 km</td>
<td></td>
</tr>
<tr>
<td></td>
<td>h = 5 km maximum</td>
<td>echo-top altitude</td>
</tr>
<tr>
<td></td>
<td>σz = 0.7 dBZ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dz = 0.5 km</td>
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</tbody>
</table>

#### 1) EFFECT OF BEAMWIDTH

Three beamwidths, from 1° to 2°, were taken into account, so as to cover the range of conventional meteorological radars. (The beamwidth is related to the diameter of the antenna d and to the wavelength λ by the approximate relation: 2θθ ≈ 1.2 λ/d.) Figure 1 shows the intensity ratio functions for VPR1 (top) and VPR2 (bottom). It should be noted first that the curves

![Figure 1](image-url)

**Fig. 1.** Ratio curves associated with VPR1 and VPR2 when the beamwidth takes the values 1° (solid line), 1.5° (dotted lines), and 2° (dashed lines), the other parameters being constant.
associated with VPR1 decrease regularly from 1.0 to 0.0 when the upper beam is located completely above the top of the echoes. For VPR2, the intensity ratios exhibit the peak that corresponds to the crossing of the bright band by the upper beam. It therefore appears that the shape of the ratio functions recalls the shape of the VPR (Part I, Fig. 2).

As for the beamwidth, the curves associated with VPR1 are practically unaffected by this parameter. In the presence of a bright band, the difference appears primarily in the reflectivity peak, which becomes smoother as the beamwidth increases.

2) Influence of Elevation Angles

Since two parameters ($A_1$ and $A_2$) must be considered simultaneously, the sensitivity is more difficult to judge, and two tests have been performed. The first consists of increasing the upper elevation from $2^\circ$ to $8^\circ$, while the lower elevation remains equal to $1^\circ$ (see Fig. 2). In the second, the lower angle decreases from $8^\circ$ to $1^\circ$, while the higher is kept at $9^\circ$ (Fig. 3).

Figure 2 clearly illustrates the importance of a higher elevation angle, very different from the lower elevation angle, in ascertaining the level of the top of the echoes or recognizing a bright band. It is clear from these curves that the quantity of information on the VPR contained in the intensity ratios decreases as the upper elevation angle decreases for reasons of smoothing. The limit is encountered when the two beams are the same ($A_1 = A_2$) and the intensity ratio is constant and equal to 1.0. Thus, to remain close to the VPR, it is best to have a high upper elevation. However, increasing this angle reduces the useful spatial coverage of the radar beam. In effect, for the VPRs in question and with $A_1 = 8.0^\circ$, sampling is limited to the 30 km closest to the radar. When observations at only two elevation angles exist, as is the case in this work, an upper elevation angle close to $3^\circ$ is a good compromise. As the number of available elevation angles increases, it becomes easier to satisfy both constraints: sufficient spatial coverage and adequate vertical sampling of the atmosphere.

Figure 3 confirms the role of the lowest beam elevation in the VPR identification: it constitutes the horizontal reference. As was noted in Part I (section 3b, Eqs. (23) and (24)), the ratios are very close to the apparent VPR if it is assumed that the beam remains very narrow and horizontal or nearly so. These theoretical conditions are never met, and Fig. 3 confirms that when the lower elevation angle increases, the relation between the ratios and VPR is no longer linear. This finding is particularly obvious for VPR2, in the presence of a bright band, where the shape of the ratio function clearly differs from the shape of the VPR. Thus, it seems important to preserve an observation at a low beam elevation angle in order to minimize the impact of nonlinearity on the VPR identification procedure.

![Fig. 2. Ratio curves associated with VPR1 and VPR2 when the upper elevation angle takes four different values between $2^\circ$ (solid line) and $8^\circ$ (dashed line), while the lower one is kept constant and equal to $1^\circ$. The beamwidth is $1.5^\circ$.](image)

3) Influence of the VPR

Without modifying the shape of the theoretical VPR, the sensitivity of the ratio function can be studied by considering only the variations of the two key altitudes: the altitude for which the reflectivity becomes equal to zero (hereafter called the echo-top altitude), and the bright band. For this purpose, the thickness of the lower part of both VPRs, which remains equal to 1, has been modified to vary the echo-top altitude between 3 and 5 km above the radar. For VPR1 and VPR2, Fig. 4 confirms, in particular, the influence of the echo-top level on the ratio function. According to a reviewer comment, for increasing distances, the ratio values decrease and are prone to sensitive fluctuations. Thus, a correct sampling of the higher part of the VPR becomes more difficult. This element will be confirmed by the case study in section 3.
1) The VPR (VPR1 or VPR2) is converted into intensity ratios simulating an observed ratio function. For the sake of simplicity, it is assumed that observed values of the ratios are not noisy, even if an error variance exists. The case where a random error affects the observed value of the ratios is dealt with further below.

2) After the parameter to be evaluated is selected, the identification algorithm is applied to the observed intensity ratio data, yielding an estimated VPR, denoted by $z^*$. 

3) The quality of the result is judged by evaluating the absolute deviation, denoted MAE (mean absolute error), between the determined VPR and the true initial VPR, $z$:

$$\text{MAE} = \frac{1}{n_z} \sum_{i=1}^{i=n_z} |z_i^* - z_i|,$$

Fig. 3. Ratio curves associated with VPR1 and VPR2 when the lower elevation angle rises from 1° (solid line) to 8.5° (dashed line), while the high elevation angle is kept equal to 9°. The beamwidth is 1.5°.

This first set of sensitivity tests concerns only the model connecting the VPR and the ratio function. The standard parameters used (Table 1) seem appropriate to the contradictory constraints to be satisfied: moderate nonlinearity of the model, adequate spatial coverage, and correct vertical sampling. The results of these tests show clearly the potential utility of matching the radar operating parameters as closely as possible to each meteorological situation. However, having radar observations from more than two elevation angles alleviates these constraints.

b. Sensitivity study of the VPR identification method

The second group of sensitivity tests concerns the effectiveness of the identification method of VPR and are performed as follows:

Fig. 4. Ratio curves associated with VPR1 and VPR2 different echo-top altitudes above the radar ranging from 3.5 (solid line) to 5 km (dashed lines) when standard radar features are adopted.
where \( n_z \) is the number of vertical elements \( z_i \) and \( z_i^* \) of the discretized VPRs.

The sensitivity tests concern both the influence of radar parameters on the quality of the estimated VPR, that of the statistical parameters characterizing the intensity ratios, and the a priori VPR. The standard parameter values are grouped in Table 1. The estimated VPR1 and VPR2 using these standard values are illustrated by Fig. 5. It shows an example where the method tested gives correct results; the MAEs corresponding to this figure are 0.01 for VPR1 and 0.09 for VPR2. With reference to section 5 of Part I, it is recalled that the result depends in particular on the confidence granted to the observed values (covariance of errors of the ratios and of the a priori VPR) and on the confidence accorded to the latter (covariance of the components of this VPR). Let us now examine the incidence of these three factors on the quality of reconstitution of VPR.

1) Influence of the a Priori VPR

Two different a priori VPRs were used: their shape is shown by Fig. 6. The first is very similar to VPR1; in other words, it remains equal to 1.0 (0.0 when expressed in decibels, Fig. 6) up to half the assumed altitude of the top of the echoes (htop/2) and decreases linearly above this. The second has a reflectivity peak at htop/2, assuming the presence of a bright band. The effectiveness of identification of the VPR versus the a priori altitude of the echo top is studied. The second

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**Fig. 5.** Identification of two examples of VPR through the proposed method when the standard parameters of Table 1 are used. For each example both the VPRs (upper graphs) and the ratio curves (bottom graphs) are displayed in three modes: the theoretical one (solid line), the a priori one (dotted line), and the identified one (dashed line). The left two figures are for VPR1, and the right two figures are for VPR2.
shape of initial VPR is used only for VPR2, when the shape of the observed ratios suggests that there is a bright band. In all cases (VPR1 and VPR2 with or without bright band), Fig. 7 shows that an underestimate of the top of the echoes causes more error than an overestimate. This finding should be considered in connection with the constraint imposed on the variance of the a priori VPR (Part I, section 5), consisting of a reduction above the a priori altitude of the top of the echoes. The introduction of this constraint is intended to eliminate the oscillations that sometimes occur in the upper part of the VPR due to the underdetermination of the VPR in the upper part of this function (Part I, section 5). Concerning VPR2, the a priori introduction of a reflectivity peak is useful only when the altitude of the bright band is correctly known and the improvement is still limited.

A simple initial VPR shape therefore seems to be the most effective choice, even for VPR2, but care must be taken not to underestimate the altitude of the top of the echoes.

2) Influence of the Covariance of the VPR

The Gaussian model proposed in Part I to describe the covariance of the VPR $z$ depends on two parameters. The first is the standard deviation of error $\sigma_z$. It is assumed constant at all altitudes, except at the altitude of the radar, so as to force the first vertical component of the VPR not to depart from 1. It is also constant above the a priori altitude of the top of the echoes, according to the explanation given above. The second parameter is the decorrelation distance $D_z$ between the values of the VPR at different altitudes. Figure 8 shows the MAE criterion for values of the couple $(\sigma_z, D_z)$ varying in a realistic way. The conclusions are different for VPR1 and VPR2. When the variations of the VPR with altitude are regular (VPR1), the effectiveness of the algorithm is moderately sensitive to the two parameters tested, and average values give correct results ($\sigma_z$ close to 0.7 dBZ and $D_z$ close to 0.5 km). In the presence of more complex VPR shapes, a clearer optimum appears at the same value of $\sigma_z$ but with a smaller decorrelation distance. When errors are introduced in observed ratios, it appears to be preferable to increase slightly the value of $D_z$ in order to filter random variations of the ratios. In the end, while the value of $\sigma_z$ is easy to choose, the appropriate decorrelation distance is variable. A value $D_z = 0.6$ km seems be a good compromise avoiding excessively rapid fluctuations of the VPR determined.

3) Influence of Covariance on Intensity Ratios

It is assumed that the standard deviation of error on the intensity ratios can be expressed linearly versus the corresponding mean value of the ratio according to the simple formula of Paper 1:

$$\sigma_q(x) = \alpha q(x) + \gamma.$$  

The main function of the term $\gamma$ is to avoid the standard deviation of error associated with zero values of the ratios equaling 0, which would make the covariance matrix singular; a value of $\gamma = 0.1 \alpha$ is used. The slope $\alpha$ represents the coefficient of variation of the observed ratios. It is allowed to vary from 0.1 to 0.5, with its increase reflecting less confidence in the observations. The second parameter considered is the decorrelation distance $D_q$. The influence of these two parameters on

![Fig. 7. MAE associated to a priori echo-top altitudes htop ranging from 2 to 6 km above the radar altitude for VPR1 (solid line) and VPR2 with and without brightband peak on the a priori VPR (dotted and dashed lines, respectively).]
2) The optimal value for $\alpha$ is 0.1, that is, when the observations are very reliable. These results concern the case where the observations match the true theoretical values.

The same sensitivity analysis was repeated with the addition of Gaussian random noise to the observed intensity ratios data, which are then written

$$ q_i^* = q_i + \epsilon_i, \quad (3) $$

where $\epsilon_i$, the observation error, is taken from the normal distribution $N(0, \sigma_i^2)$. The standard deviation of observation error $\sigma_i^*$ depends on the value of the ratio and is also written

$$ \sigma_i^* = \text{constant}. $$

the quality of reconstitution of the VPR is shown by Fig. 9. It shows that for the two examples in question, VPR1 and VPR2,

1) The quality of the identification is not sensitive to the decorrelation distance $D_q$ that can be chosen equal to 0. This finding is explained by the fact that there is a good continuity in the observation of the intensity ratios.

Fig. 8. MAE associated with different covariances of VPR1 and VPR2. The point standard deviation is represented by the $x$ axis, and the decorrelation distance is on the $y$ axis.

Fig. 9. MAE associated with different covariances of the ratio curve associated with VPR1 and VPR2. The slope of the mean-standard deviation relationship is represented by the $x$ axis and the decorrelation distance is on the $y$ axis.
\[ \sigma_i^* = \alpha^* q_i + \gamma^*, \]

where \( \alpha^* \) and \( \gamma^* \) are two parameters. The a priori, or assumed, observation errors are characterized by \( \alpha^* \) and \( \gamma^* \), while \( \alpha \) and \( \gamma \) characterize the true observation errors. Nearby errors are independent and the following approach is used. For a given VPR, the curve of the exact intensity ratios is calculated, the observed values of the ratios are deduced from it by introducing an error of observation defined by Eq. (2), and the VPR is estimated from the noisy observations. To apply the method of VPR identification, one must define the error on the ratios (\( \alpha \) and \( \gamma \)) without knowing them exactly. So the a priori covariance, characterized by \( \alpha^* \) and \( \gamma^* \), may differ from the true covariance. About 100 simulations were performed in each case, although the detailed results do not appear in this article. The main conclusion from this aspect of the sensitivity study concerns the standard deviation of error of the ratios. When the a priori standard deviation is smaller than the true standard deviation, the method of VPR identification fails to screen out measurement errors correctly, whatever the shape of the VPR. On the contrary, an initial overestimate of the standard deviation of true error is much better accepted and only complex shapes of VPR are penalized.

It has already been noted that the characteristics and conditions of operation of the radar are likely to affect the quality of the result. In the following sections, the influence of two factors is considered: the beamwidth and the upper elevation angle.

4) EFFECT OF BEAMWIDTH

Figure 10 confirms the need to have a beamwidth of not more than 2°, particularly when there is a bright band (VPR2). However, beamwidths of less than 1.5° do not seem to yield any significant advantage for the two shapes of VPR considered. This conclusion is valid for short and medium ranges (not exceeding 100 km) adapted for hydrological applications.

5) EFFECT OF THE HIGH ELEVATION ANGLE

The VPR identification was performed while varying the upper elevation angle of the radar beam. The MAEs obtained (see Fig. 11) show that it is preferable to increase the upper elevation angle. This result agrees with the earlier discussion in section 2, since a higher upper elevation angle provides a better perception of the true shape of the VPR (see Fig. 2). However, the need to sample the rain field as completely as possible involves a compromise.

Selection of the vertical discretization step of the VPR, \( \Delta h \), has not been precisely addressed. Its choice results from a concern to preserve a correct description of the vertical variations of the VPR without increasing the number of parameters to be determined. It has been observed that \( \Delta h = 0.2 \text{ km} \) and \( \Delta h = 0.3 \text{ km} \) yield very similar results and the former was used in this work.

In spite of its simplicity, the sensitivity study leads to a triple conclusion:

1) First, the radar operating parameters must be matched to the region of VPR identification and the meteorological situation. From this viewpoint, the presence of data from a number of elevations angles greater than 2 is desirable. Nevertheless, to work with only two elevations angles is convenient to better illustrate the main points of the use of the proposed method.
2) The choice of the various factors or parameters—
a priori VPR and a priori covariance of the ratios and
of the VPR—is more important when the shape of the
vertical profile is much more elaborate. Acceptable re-
results obtained in all cases, with the standard values of
the statistical characteristics (Table 1), indicate that a
robust average adjustment of the method is possible.
This property, confirmed by the following case study,
could encourage operational application.

3) An interesting way to both adapt the method to
the meteorological context and to choose a correct a
priori VPR could be to initialize VPR identification
using a climatological VPR (Joss and Waldvogel 1990).
The aim of the VPR identification is then to modify
the climatological VPR and account for observed ra-
tios; the method appears to be complementary to a
classical approach.

3. Case study

a. Presentation of data

The data used to test the proposed method were col-
clected during a 3-yr experiment, Cévennes 1986–88,
named for the mountainous region in the south of
France where it was conducted. This region is subjected
regularly to intense and extended rains associated with
the blocking of cyclonic perturbations. These rain
events occur frequently in the autumn season and cause

Fig. 12. Maps of study area (Cévennes region). (a) The
radar site, rain gauges (black dots), and the three catchments
are plotted as well as the 100° sector in which the VPR were
identified. (b) Relief map of the Cévennes region.
flash floods (see Billaut and Tourasse 1980). During the experiment, a dual polarization S-band radar (see Messaud and Pointin 1990; Pointin et al. 1988) was installed at an altitude of 1030 m, covering three catchments subject to this problem. In this geographical region, ground level ranges from 0 to 1700 m. A mountainous region lies along the western edge of the area and runs approximately north—south (Fig. 12). The radar data were recorded at the frequency of one plan position indicator (PPI) (combining two rotations of the antenna) every 4 min, with the radar elevation angle being alternately 1.1° and 3.1°. The usual problems caused by the mountainous relief—ground echoes, partial blocking—were solved using simple and conventional methods and a numerical terrain model (Andrieu et al. 1989). Five significant rain events were observed during the experimental period (see Table 2), grouping quite varied rain situations: warm and low orographic clouds (for example, from 13 to 15 November 1986) and highly developed convective cells. There are, furthermore, 48 recording rain gauges in the study zone, with two-thirds telemetering the data every hour. Very high rainfall intensities were recorded, and the maximum rainfall accumulation exceeded 220 mm in 2 h in the eastern part of the region. Figure 12 shows the study zone and indicates the position of the radar and the recording rain gauges.

This case study concerns particularly two rain events: 13–15 November 1986 and 4–6 October 1987. As a first step, a sequence of eight rainy hours (in the course of the night of 4–5 October 1987) is selected to illustrate operation at the 1-h time step. During this sequence, an extended convective cell, coming from the Mediterranean, was blocked along the mountains. Figure 13 represents the hourly rainfall accumulation maps measured by the radar during these 8 h. The data shown are taken from the radar observations at low elevation, with ground echoes suppressed and beam blockage corrected.

The VPR in the sector of the image corresponding to the catchments is determined every hour. The aim is to re-create a situation close to the operational context in which it is not possible to wait for the end of the rain event to determine a mean VPR. In addition, this approach makes it possible to take variations of the VPR in the course of time into account.

The validation of the method is undertaken by comparing the radar measurements to pluviographic data. It completes a first work (Andrieu and Creutin 1991) that has shown good agreement between the theoretical errors due to the mean VPR of these two rain events and the differences found between the rainfall amounts measured by the radar and by the rain gauges.

b. Use of the VPR identification method

1) EXPERIMENTAL INTENSITY RATIO

In Part I, it was noted that the algorithm for identification of the vertical profiles of reflectivity applies either to the reflectivities or to the equivalent rain intensities, after selection of a Z–R relationship. Given the time step in question (1 h) and the planned application, hydrological prediction, it was decided to take into account hourly radar intensities. These images of hourly radar intensities are obtained by averaging the images belonging to the considered time interval; the relationship Z = 200R^{1.6} is used for this purpose. Then, the ratios function used to determine the 1-h VPR is the ratio of the intensities measured by the radar at the two available elevation angles. It could be transformed very simply into an equivalent function of reflectivity ratios: q = q^{1/1.6}.

The observed intensity ratio functions are calculated in the 100° sector (between azimuth 60° and 160°) mentioned in Fig. 12. At each of the n_{0} = 40 gates at regular intervals from 5 to 60 km, the mean and standard deviation of the ratios are determined using the n_{g}(x) = 50 pairs of intensity measurements located at the same distance x from the radar. The estimated ratio at distance x is given by

\[ q^{\text{est}}(x) = \frac{1}{n_{g}(x)} \sum_{i=1}^{n_{g}(x)} q_{i}(A_{1}, A_{2}, x) \]  \hspace{1cm} (5)

and

\[ q_{i}(A_{1}, A_{2}, x) = \frac{R_{i}(A_{2}, x)}{R_{i}(A_{1}, x)}, \]

in which \( q^{\text{est}} \) is the mean ratio of the intensities at distance x when the lower and upper elevations are \( A_{1} \) and \( A_{2} \). The radar measurements of the rainfall intensity at the i\text{th} point at the same distance for the same elevation angles are \( R_{i}(A_{1}, x) \) and \( R_{i}(A_{2}, x) \), and \( q_{i} \) is the corresponding ratio of intensities. The standard deviation of the intensity ratios at distance x is directly calculated by

\[ \sigma_{q}^{\text{est}}(x) = \left\{ \frac{1}{n_{g}(x)} \sum_{i=1}^{n_{g}(x)} \right\}^{1/2} \]

\[ \times \left[ q_{i}(A_{1}, A_{2}, x) - q^{\text{est}}(x) \right]^{2} \right\}^{1/2}. \]  \hspace{1cm} (6)

<table>
<thead>
<tr>
<th>Date of the event</th>
<th>Maximum rain gauge total for the event</th>
<th>Maximum 1-h gauge rainfall</th>
<th>Duration of the radar record</th>
</tr>
</thead>
<tbody>
<tr>
<td>13–15 Oct 1986</td>
<td>240 mm</td>
<td>26 mm</td>
<td>18 h</td>
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<tr>
<td>12–15 Nov 1986</td>
<td>512 mm</td>
<td>33 mm</td>
<td>48 h</td>
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<td>(in 5 days)</td>
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<tr>
<td>4–6 Oct 1987</td>
<td>370 mm</td>
<td>60 mm</td>
<td>40 h</td>
</tr>
<tr>
<td>11 Oct 1988</td>
<td>337 mm</td>
<td>225 mm</td>
<td>20 h</td>
</tr>
<tr>
<td>(in 2 h)</td>
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<td>20 Oct 1988</td>
<td>164 mm</td>
<td>41 mm</td>
<td>12 h</td>
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TABLE 2. General features of the five major events observed during the Cevennes 1986–88 experiment.
This experimental quantity is directly useful in specifying the confidence to be accorded to the values of the ratios and infer the second-order moment of the ratio function required by the method of VPR identification. The representativeness of \( \sigma_q^{\text{est}} \) depends on the assumption of uniformity of the VPR and therefore on the sampling provided by the \( n_a(x) \) points located at distance \( x \). Strictly, the number of points \( n_a(x) \) should be replaced by \( n_a(x) \), which is less than the foregoing value and represents the number of independent values. The difference between these two quantities depends on the spatial correlation of the errors, which is still unknown. It is, however, permitted to think that \( n_a(x) \) converges on \( n_a(x) \) as the distance from the radar increases, since the area covered by the same number of points is larger. Analysis of the observed ratio curves and of their associated standard deviations (Fig. 14) suggests four remarks:

1) All the observations reach the value zero. The sector selected is sufficient to ensure correct vertical sampling of the VPR. Out of all of the data, only two hours exhibit a ratio function that does not reach zero; they correspond to a very large vertical extent and to intensities on the ground reaching 225 mm in 2 h. The homogeneity of the VPR in the region is no longer correct for these two hours.

2) The level of variability of the ratios varies considerably from one hour to another. When the mean of the standard deviation on the observations at all distances is selected as index of variability (\( \sigma_q^{\text{est}} \)),

\[
\sigma_q^{\text{est}} = \frac{1}{n_a} \sum_{i=n_q}^{i=n_a} \sigma_q^{\text{est}}(x_i),
\]

it is found that this quantity goes from 0.2 (at 0100 LT in this example) to 0.8 (e.g., at 2300 LT), where LT is local time. The existence of a spatial heterogeneity of the VPR certainly explains these deviations, together with the large variability of the intensities in the sector studied.

3) The intermittency of the rain fields also influences the standard deviation of the intensity ratios. At the rain–no-rain limit, the variability of the ratios increases considerably, certainly at the same time as the fluctuations of the VPR in this transition zone. An illustration is provided by the curves of standard deviation of the ratios at 0300 or 0400 LT of Fig. 14 that show a maximum at approximately 40 km, the distance corresponding to the limit of the rainy zone confined to the mountainous part of the region (see Figs. 12 and 13).

4) To verify the linearity between the mean ratio and its associated standard deviation, the coefficient of correlation between observed curves of the two parameters is calculated for each hour. The mean value of the obtained coefficients of correlation is 0.73, varying from 0.34 to 0.94. Figure 15 groups all the coefficients of correlation, classified according to increasing values.

The assumption of a linear relationship between mean ratios and associated standard deviations appears to be realistic, and Eq. (2) is justified.

The VPR will be determined by distinguishing two tests:

1) To simplify the use of the method, the “RCA” test consists of keeping the value \( \alpha = 0.35 \) constant for all hours. This value corresponds approximately to the mean of the coefficients of variation \( \sigma_q^{\text{est}} / q_{\text{est}} \) of the different hours. It means that the user assumes that the reliability of the data does not change.

2) The second test will be called “RCB”; it attempts to match the standard deviation of the ratio error to each hour’s observations. Three values of \( \alpha \) are distinguished according to the value of the ratio \( \sigma_q^{\text{est}} / q_{\text{est}} \); they are summarized in Table 3. It means that the user can have more or less confidence in the observed ratio. In both cases, \( \gamma = 0.1 \alpha \) and \( D_q = 0.0 \) will be used.

2) A priori VPR

The result of the VPR identification is a compromise of two points: (a) to obtain a VPR whose ratio curve fits the observed ratios very well, and (b) to stay in the neighborhood of the a priori VPR. If the user is confident about the data, the priority is to satisfy (a). But if he is able to carefully initialize the method, the second aspect (b) becomes more important. For the shape of the a priori VPR, two cases were considered:

1) The first, called “VPRA,” uses the same initial VPR for each hour. A rather neutral shape with no bright band, where the altitude of the echotop level is 4 km above the radar, equivalent to that of VPR1, was selected. It means that the user does not give a great importance to the a priori information. This represents case (a).

2) The second test, called “VPRB,” consists of initializing the algorithm for a given hour with the VPR found as the result for the previous hour. This procedure is equivalent to centering the identification on the variations of the VPR from one hour to another. In this situation, the a priori information has a more important role in the identification procedure; it corresponds to case (b).

c. Indirect validation of the VPR identification

The indirect validation concerns an 8-h sequence. The goal is to define a robust way of initializing and applying the identification method. This is done in the following way which justifies the term “indirect validation.” The a priori information about data (RCA or RCB) and parameters (VPRA or VPRB) is selected and the VPR identification is performed. The accuracy of resulting VPR is appreciated by comparing the observed ratio curve and the ratio curve attached to the identified VPR. It is assumed that the closer the two
Fig. 13. Radar pictures corresponding to 8-h rain totals recorded between 2300 LT 4 October 1987 and 0600 LT 5 October 1987. These pictures result from the combination of 4-min PPI at two elevation angles.
Fig. 14. Rain-rate ratio curves (solid line) and associated standard deviation (dotted line) concerning the eight example hourly rain fields.
ratio curves, the more accurate the obtained result. The experimental intensity ratio curves and the VPR determined are grouped in Fig. 16. Intensity ratios are on the left-hand side of the figure, and the corresponding VPR on the right-hand side. The dashed lines represent the observed ratios; they have no corresponding VPR. The dotted lines show the starting point of the algorithm (a priori VPR) and the attached ratio curves. The identified VPR and the corresponding ratios are represented by the two continuous lines. These results were obtained with the RCA option (identical statistical characteristics for all ratio curves) and VPRA option (unchanged a priori VPR). The mean absolute error between the observed ratios function and the one that corresponds to the determined VPR is then calculated. The values of this criterion are grouped in Table 4, which takes three options into account: RCA + VPRA, RCB + VPAB, and RCA + VPAB. The error between the observed ratios and the ratios attached to the a priori VPR occupies the last column of the table.

When the method operates in its simplest form (VPRA + RCA), the case study confirms the results of the sensitivity study. The VPR determined yields an improvement with respect to the a priori VPR (comparison of columns 1 and 4 of the sensitivity study). But the most regular profiles are better recognized than those having a bright band: the best results are observed after 0400 LT, when the bright band disappears. From a meteorological viewpoint, there is nothing abnormal; the bright band 2 km above the radar (i.e., at an altitude of 3 km) is consistent with the RHIs (range–height indicators) recorded on the night of 4 October 1987 and with an atmospheric sounding at Nîmes at 0100 LT the same night. The regular decrease of the altitude of the echo-top level (from 3 to 1 km above the radar) is correlated properly with the reduction of rainfall intensities.

When the standard deviation of error on the ratios depends on the experimental observations (RCB + VPRA), the performance of the method changes little (columns 1 and 2 of Table 4). This, therefore, shows the moderate sensitivity of the results to this parameter and is in agreement with the sensitivity study only when the VPR exhibits regular variations.

The initialization of the identification algorithm with the result of the previous time step (RCA + VPRA) leads to findings in agreement with the sensitivity study. In the presence of a bright band at an approximately constant altitude (during the first three hours of the sequence), the performance is improved since the a priori VPR is closer to the VPR to be found. But a significant degradation might occur when the peak of the bright band is kept in the a priori VPR and propagated from one hour to another.

In conclusion to this set of tests, it appears that the simple ways of using the method of identification of the VPRs seem robust, which is a positive point in favor of an operational application.

d. Hydrological validation

A different means of checking the effectiveness of the proposed method consists of checking whether it allows improvement of the measurement of rain by radar. This check was undertaken on the scale of the three catchments shown in Fig. 12. The object of interest will be the rainfall accumulation at the 1-h time step on these catchments, the areas of which range from 300 to 500 km². This choice corresponds to the need for hydrological modeling of these catchments, performed using lumped models of which the mean rainfall on the catchment is the input (Obled 1991; Duband et al. 1993). It also answers to the concern to obtain a reliable estimate of the hourly rain accumulation that can be used as reference for validation of the radar measurement. But an isolated comparison between pluviographic measurements and the radar data on the grid pixel above it is affected in particular by the pluviographic measurement errors and the difference of representativeness between the two devices.

1) Determination of the reference rainfall

The presence of a dense network of recording rain gauges provides an opportunity to determine reliably
FIG. 16. For the eight selected rain fields, the measured (dashed lines), a priori (dotted line), and identified (solid line) ratio curves are on the left-hand side. The corresponding VPR (right-hand side of the figure) are the a priori VPR (dotted line) and identified VPR (solid line).
05/10/87 - 03 LT

05/10/87 - 04 LT

05/10/87 - 05 LT

05/10/87 - 06 LT

Fig. 16. (Continued)
Table 4. Mean absolute error on the ratio curves when the method
is performed with the options RCA + VPRA (first column), RCB +
VPRA (second column), and RCA + VRPB (third column). As
reference, the mean absolute difference between the a priori ratio
curve and the measured one is also given (fourth column).

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (LT)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
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<tr>
<td>4 Oct 1987</td>
<td>2300</td>
<td>0.18</td>
<td>0.17</td>
<td>0.15</td>
<td>0.40</td>
</tr>
<tr>
<td>5 Oct 1987</td>
<td>0000</td>
<td>0.15</td>
<td>0.15</td>
<td>0.09</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0100</td>
<td>0.09</td>
<td>0.08</td>
<td>0.05</td>
<td>0.18</td>
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<td></td>
<td>0200</td>
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<td>0.10</td>
<td>0.013</td>
<td>0.23</td>
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<tr>
<td></td>
<td>0300</td>
<td>0.086</td>
<td>0.085</td>
<td>0.086</td>
<td>0.20</td>
</tr>
<tr>
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<td>0400</td>
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<td>0.030</td>
<td>0.04</td>
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</tr>
<tr>
<td></td>
<td>0500</td>
<td>0.05</td>
<td>0.055</td>
<td>0.05</td>
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<td></td>
<td>0600</td>
<td>0.045</td>
<td>0.050</td>
<td>0.05</td>
<td>0.67</td>
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</tbody>
</table>

the hourly rainfall on a catchment. This value is ob-
tained by kriging the isolated pluviographic data. This
method of interpolation, based on a geostatistical ap-
proach, will be not described in this article. Interested
readers may refer to Creutin and Obled (1982) and
Bras and Rodriguez-Iturbe (1985), which contain de-
tailed descriptions and examples of applications to rain
fields. One of the benefits of kriging lies in the fact that
it provides the variance of estimate associated with the
calculated mean intensity and thus an indication about
the reliability of the reference. In the present case, par-
ticular care was taken to check that the conditions of
validity of this approach were satisfied and that the variance of the estimate of the rain intensity is credible
(Faure 1993). One must, however, bear in mind the dif-
ference between the altitude of the radar (1030 m)
and the mean altitude of the catchments (close to 600
m). Because of this, the radar measurement does not
perfectly represent the rain at ground level and this
factor can explain some of the discrepancies between
the two measurement methods. This point also affects
the influence of the VPR. The correction is done at
the radar altitude, which is the reference level in this
particular case. Then it cannot take into account event-
ual changes affecting the VPR between the radar al-
titude and the mean catchments altitude.

2) COMPARISON OF GROUND RAINFALL
ESTIMATE AND RADAR OBSERVATIONS

The comparison of radar and rain gauge observations
is performed for the two rain events of 13–15 November
1986 and 3–5 October 1987, which total 70 h of
data. The mean distance between the radar and the
different catchments varies from 18 to 35.5 km, and
equivalently, the beam altitude above the catchment
varies from 400 to 800 m (Table 5). It can be seen
that all the catchments are close to the radar, at dis-
tances little contaminated by the incidence of the VPR.
To introduce more diversified examples, the rainfall
intensities measured by the radar at high elevation are
included in the comparison. This is equivalent to in-
cluding hypothetical catchments in the comparison for
which the axis of the beam is farther from the ground
(see Table 5) or for which the radar conditions of op-
eration are less favorable. For instance, the mean al-
titude of the higher beam for Anduze catchment is 1050
m above the radar. This value is to be paralleled with
the low beam (mean altitude is 400 m), which never
intersects the melting layer. It is then interesting to
consider high-elevation radar data, more contaminated
by the VPR influence, in order to test the efficiency of
a correction procedure. Finally, the six different ex-
amples (two radar observations for the three catch-
ments) should not be affected in the same way by the
errors due to the VPR, which increase with increasing
distance from the radar (Table 5). The 1-h VPR was

Table 5. Evaluation of the VPR identification and correction efficiency.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Beam axis altitude (m)</th>
<th>RE (%)</th>
<th>MAE (%)</th>
<th>COR (%)</th>
<th>PC 80 (%)</th>
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<tr>
<td></td>
<td></td>
<td>Case A</td>
<td>Case B</td>
<td>Case C</td>
<td>Case A</td>
</tr>
<tr>
<td>Anduze LE</td>
<td>400</td>
<td>-0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.21</td>
</tr>
<tr>
<td>Ales LE</td>
<td>630</td>
<td>-0.02</td>
<td>0.05</td>
<td>0.05</td>
<td>0.29</td>
</tr>
<tr>
<td>Ceze LE</td>
<td>790</td>
<td>-0.21</td>
<td>-0.09</td>
<td>-0.15</td>
<td>0.32</td>
</tr>
<tr>
<td>Anduze HE</td>
<td>1050</td>
<td>-0.13</td>
<td>0.03</td>
<td>-0.05</td>
<td>0.28</td>
</tr>
<tr>
<td>Ales HE</td>
<td>1650</td>
<td>-0.25</td>
<td>0.03</td>
<td>-0.15</td>
<td>0.40</td>
</tr>
<tr>
<td>Ceze HE</td>
<td>2000</td>
<td>-0.50</td>
<td>-0.12</td>
<td>-0.35</td>
<td>0.58</td>
</tr>
<tr>
<td>Total</td>
<td>-0.19</td>
<td>-0.02</td>
<td>-0.12</td>
<td>-0.12</td>
<td>0.35</td>
</tr>
</tbody>
</table>

a RE—relative error
b MAE—mean absolute error
c COR—coefficient of correlation
d PC80: percentage of radar data belonging to the 80% confidence interval
e LE—low elevation
f HE—high elevation
determined using the RCB + VPRA protocol defined previously; that is, the confidence accorded to the intensity ratios was adjusted hour by hour in accordance with Table 3. The echo-top level of the a priori VPR was fixed at 2.5 km above the radar for the rain of November 1986 and at 3.5 km for the rain event of October 1987. The vertical discretization step of the VPR is kept to 0.2 km.

The ground measurements and radar data were compared using four criteria:

1) The coefficient of correlation between the two series of values:

\[
COR_k = \frac{\sum_{i=1}^{i=n_h} (P_{ik} - \bar{P}_k)(R_{ik} - \bar{R}_k)}{\left[ \sum_{i=1}^{i=n_h} (P_{ik} - \bar{P}_k)^2 \right]^{1/2}\left[ \sum_{i=1}^{i=n_h} (R_{ik} - \bar{R}_k)^2 \right]^{1/2}},
\]

where \(P_{ik}\) and \(R_{ik}\) are, respectively, rain gauge and radar rainfall for the \(i\)th time step and catchment number \(k\), \(\bar{P}_k\) and \(\bar{R}_k\) the means of the ground and radar intensities on all time steps, \(n_h\) the number of time steps, and \(COR_k\) the resulting correlation coefficient. The radar estimate is the mean value of all the radar pixels located in the given catchment. The rain gauge–based estimate is obtained by the way described in the previous section.

2) The mean error (ME) between radar and ground observations indicates the bias affecting radar data:

\[
ME_k = \frac{\sum_{i=1}^{i=n_h} (R_{ik} - P_{ik})}{\sum_{i=1}^{i=n_h} P_{ik}}.
\]

3) The MAE between the ground and radar intensities:

\[
MAE_k = \frac{\sum_{i=1}^{i=n_h} |P_{ik} - R_{ik}|}{\sum_{i=1}^{i=n_h} P_{ik}}.
\]

where \(k\) indicates the number of the catchment basin.

4) The percentage of radar measurements included in the 80% confidence interval of the pluvigraphic reference, called "PC80_k," for the catchment number \(k\). This indicator represents the proportion of radar measurements that do not differ significantly from the reference value at the fixed threshold.

3) RESULTS

The effectiveness of the correction of the VPR is evaluated by considering three different cases in order to compare the radar measurements to the reference values.

- Case A: The radar measurements are not corrected from the influence of the VPR.
- Case B: The VPR identification is performed every hour, and the correction of radar data is done according to this hourly VPR. Figure 17 groups all of the hourly VPR for both of the studied rain events.
- Case C: A mean VPR is considered for each of the two rain events. All the radar observations of a given event are corrected using this mean VPR. Figure 18 shows the mean VPR of the two rain events.

It would have been interesting to introduce a climatological VPR in order to compare the proposed method to a climatological correction (Joss and Waldvogel 1990). The observed rain events being the only available data, it was not possible to determine a climatological VPR. It should be noted that case C is different from a climatological approach. The two mean VPRs are significantly different and none of them can be assumed to represent a climatological VPR. Moreover, case C is not appropriate for operational purposes: using the mean VPR of the rain event to correct radar data means that this correction cannot be performed before the end of the event.

All of the obtained results are grouped in Table 5. The abbreviation LE or HE added to the name of the catchment means that the results concerns, respectively, low-elevation or high-elevation radar data. One notes first the existence of a good agreement between the radar measurements and the results deduced from the recording rain gauges. This is illustrated in particular by the high correlation coefficients obtained and the low values of the MAE. The deterioration due to an increase of the altitude of the beam axis appears clearly; it is observed between the different catchments for low- or high-elevation data and between low- and high-elevation data. However, the agreement between the two types of measurement is not perfect, since the 80% confidence interval of the reference contains less than 80% of the radar data. The favorable position of the radar, with respect to the catchments, explains the good general agreement. As for the correction of the VPRs, several comments are suggested.

(i) Efficiency of the correction based on hourly VPR.

It appears, first of all, that identification of the VPR and correction of its influence improves the agreement between the ground and radar data. In effect, the 19% underestimate of the rainfall by the radar completely disappears after correction of the VPRs. The mean absolute error decreases from 35% to 27% and the percentage of the radar measurements included in the 80%
confidence interval of the ground data increases from 46% to 58%. The gain is also found in the correlation, from 0.76 to 0.84.

The improvement is even clearer when the axis of the radar beam is at a high altitude. This result, therefore, confirms the decisive role of the VPR in errors of measurement of rainfall intensities by radar. It also shows that it is possible to correct this source of error. It is found, however, that the proposed method also yields a smaller but significant improvement for the Alès and La Cèze basins, favorably located with respect to the radar.

The effectiveness of the correction is not perfect. The measurement at high elevation (HE), corrected for the error due to the VPR, is still of poorer quality than the measurement at low elevation corrected for the same effect. The difference is even greater if the altitude of the beam is high and so the basic radar measurement degraded. The example of La Cèze HE and Alès HE with respect to Anduze HE bears this out. This means, in particular, that the correction method does not substitute for an improvement of the quality of the radar measurement but complements it.

(ii) Efficiency of the correction based on mean VPR.

The results obtained with mean VPRs are halfway between the efficiency of hourly VPR and raw radar data. This statement is shown clearly by the last line of Table 5. All criteria are improved compared to raw radar data but less than the correction using hourly VPR. It appears that the mean VPR is less efficient in correcting the relative error, which shows an underestimation of radar data for farther catchments—for example, for Alès HE and La Cèze HE. The underestimation could be explained by the fact that the mean VPR, keeping a constant echo-top level, cannot take into account the temporal variations of the echo-top altitude.

4. Concluding remarks and future perspectives

This article is a first attempt to qualify the proposed method of identifying the vertical profiles of radar reflectivity. The sensitivity study served to define the conditions of application and to prepare properly for the case study. The validation of the method for hydrological applications was then undertaken through examination of two rain events. The results obtained show an improvement of the accuracy of the radar data when the VPR is determined regularly and corrected. The potential value of the proposed method is thus illustrated. Its practical use still depends on the strong assumption of spatial homogeneity of the VPR and on the absence of intermittence of the rain fields in the region studied. These assumptions are certainly met in the two cases studied but could be erroneous—for example, when convective storms occur far from the radar.

The obtained results are encouraging, but in order to evaluate this method of VPR identification rigorously, it is clear that additional data are necessary. There are two ways in which a more robust evaluation
could be performed. The first consists of applying the method to a long series of data, representative of very different meteorological conditions and of testing its efficiency by the hydrological validation described in the previous section. This approach could also be used to check the influence of all the parameters involved and eventually to simplify the conditions of use of the method. The presence of a vertically pointing radar, directly providing the true VPR (Fabry and Austin 1991), is the second means of validating the identified VPR. These data would also contribute valuable information about the time variability of the VPR.

This method can be extended to cases where more than two elevation-angle radar images exist. An increase in the number of radar images at different elevation angles corresponds to a better sample of the VPR. One could expect to identify local VPRs and satisfy more easily the assumption of spatial homogeneity of the VPR. But the useful information provided by radar images at additional elevation angles depends on the distance. This information is important close to the radar where two elevation angles are sufficient or nearly sufficient. The value of additional elevation angles is certainly weaker at increasing ranges where the problem becomes underdetermined (see Fig. 7, Part I). A sensitivity study and actual case studies are required to determine the gain in efficiency associated with additional elevation angles versus the distance to the radar.

The proposed method is to be considered as a complement to classical approaches, such as the use of climatological VPR. In effect, it seems evident that a climatological VPR could be an appropriate choice for the a priori VPR, a starting point of the identification procedure. The goal of the inverse method is to make the a priori (climatological) VPR consistent with available observations. In this respect, the VPR identification method is an extension of previous works in this field.

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REFERENCES


