NOTES AND CORRESPONDENCE

Wind Energy Resource Estimation of the Upper Atmosphere over Southern Africa

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26 July 1993 and 19 March 1995

ABSTRACT

On the basis of daily ECMWF data over the period 1982–89, the mean seasonal and mean annual wind energy resource fields on the isobaric surfaces 1000, 850, 700, 500, 300, 200, and 100 hPa, within the latitude-longitudinal zone 0°–50°S and 0°–45°E, have been calculated and analyzed. Generally, maximum wind energy is recorded on the 300-hPa surface, and the intensities are greater by about 25%–30% in winter than in summer. The greatest continental wind energy resources are experienced over the southern tip of the southern African subcontinent, where it was established that the free atmospheric resources exceed those near the surface by at least an order of magnitude. The two-dimensional Gaussian distribution of the wind vector has been used to provide an indirect estimate of the wind energy resource. Good agreement between this measure and the direct estimate of wind energy was obtained, demonstrating the potential usefulness of the two-dimensional Gaussian distribution in estimating upper-air wind energy resources.

1. Introduction

The wind energy resource in the atmospheric boundary layer has been investigated in detail in many countries, including South Africa (Diab 1985, 1988; Jury and Diab 1989; Viljoen 1991). In search of powerful wind streams, an investigation of the wind energy potential of the upper atmosphere (where the most wind energy is concentrated) is worthy of attention (Inglis 1978). Similar investigations have been conducted elsewhere (Fletcher and Roberts 1979; Bryukhan 1989), in which, interalia, questions concerning the design of wind energy plants suspended on tethered balloons or aerodynamic platforms at altitudes of 5–6 km and more were discussed (Fletcher and Roberts 1979; Furuya and Maekawa 1984). A wind energy resource estimation of the upper atmosphere is also relevant for energy balance studies.

Taking into account the prospects for the development of wind energy in Southern Africa, it is considered useful to estimate the potential wind energy resource of the free atmosphere over this region and to locate areas of maximum wind energy. Such an investigation will contribute toward a fuller picture of the availability of renewable energy resources.

Coupled with the obvious practical and theoretical interest in the free atmospheric wind energy resource is an investigation of the feasibility of an alternative method of calculating the wind energy resource. Such a calculation relies on the approximation of the empirical wind speed distribution by a statistical distribution model. It does not require the use of a time series of wind observations and as such considerably reduces the volume of data required. Each of these topics will be addressed in this paper.

2. Data

In this paper, daily (1200 UTC) ECMWF data (European Centre for Medium-Range Weather Forecasts), which consists of numerically analyzed, gridded meteorological parameters on isobaric surfaces, for the period 1982–89 were used. The domain extended from 0° to 50°S and from 0° to 45°E, with a grid interval of 2.5°, and included the set of isobaric surfaces 1000, 850, 700, 500, 300, 200, and 100 hPa. For the present investigation, data over four midseason months (January, April, July, and October) were used. The mean annual values of the wind energy resource were estimated by averaging over these four months.

The availability of data at only one time per day, namely 1200 UTC, is not considered a serious drawback, since the diurnal changes in wind that are characteristic of the boundary layer are virtually absent in the upper atmosphere. Noon data are therefore sufficiently representative. The advantage of ECMWF data over aerological data is that the ECMWF data do not contain the measurement errors that characterize primary data and that could lead to further errors in the

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statistical estimation of the third-order moment of the wind speed distribution.

ECMWF data, recorded on magnetic tapes, were loaded on to an IBM/360 computer and then transfered to an IBM compatible 486 computer, where the computations were performed.

3. Method

The most widely used parameter to characterize the potential wind energy resource is the specific power density of the wind flow

$$P = \frac{1}{2}\rho v^3, \tag{1}$$

where ρ is air density and v the wind speed. Taking into account the fact that the annual fluctuations of air density about its mean value are small and of the order of 3%–5%, the mean wind energy can be estimated as follows:

$$\bar{P} = \frac{1}{2}\bar{\rho}\overline{v^3}. \tag{2}$$

Since it is not difficult to estimate mean air density, the calculation of $\bar{P}$ is reduced, in essence, to the estimation of the mean value of the cubed wind speed:

$$W = \overline{v^3}. \tag{3}$$

The estimation of W can be achieved in two ways. The first method is based on the use of a wind time series and the direct calculation of W over the sample:

$$\mathrm{WD} = \frac{1}{n}\sum_{i=1}^{n} v_i^3, \tag{4}$$

where n is the number of observations in the sample. The second method assumes the use of an analytical wind speed distribution law and the indirect calculation of the mean value of the cubed wind speed:

$$\mathrm{WI} = \int_0^\infty v^3 g(v)dv, \tag{5}$$

where $g(v)$ is the wind speed distribution density. The advantage of the latter method is that it reduces by up to two orders of magnitude the volume of data required. In this paper both methods of calculating the value W are used. Since the first method, based upon formula (4) is obvious, we expand upon the second one.

It was shown in earlier papers that the actual wind vector distribution in the free atmosphere is well described by a two-dimensional Gaussian distribution (Crutcher 1956; Bryukhan 1983). The possibility of using this distribution for the indirect calculation of the wind energy resource in the upper atmosphere and in the boundary layer was established in Bryukhan et al. (1986) and Bryukhan (1989). These factors allow us to apply the two-dimensional Gaussian distribution to the region investigated in the present work.

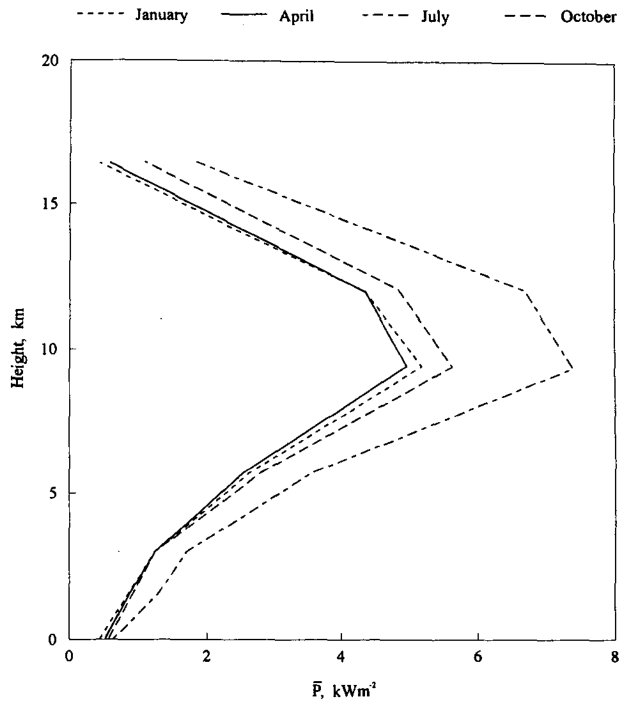

FIG. 1. Mean monthly vertical profiles of the specific wind power density P (kW m^{-2}) averaged over the region 0°–50°S, 0°–45°E for the period 1982–89.

According to Bryukhan (1983) the joint distribution density of the magnitude v and direction ψ of the random wind vector can be written as follows:

$$f(v, \psi) = \frac{1}{\pi L \sigma^2}\exp(-a^2 + 2bv - c^2v^2), \tag{6}$$

where

$$a^2 = \frac{V_r^2}{2(1-r^2)}\left(\frac{\sin^2\theta}{\sigma_x^2} - \frac{r\sin 2\theta}{\sigma_x\sigma_y} + \frac{\cos^2\theta}{\sigma_y^2}\right),$$

$$b = \frac{V_r}{2(1-r^2)}\left[\frac{\sin\psi\sin\theta}{\sigma_x^2} - \frac{r\sin(\psi+\theta)}{\sigma_x\sigma_y} + \frac{\cos\psi\cos\theta}{\sigma_y^2}\right],$$

$$c^2 = \frac{1}{2(1-r^2)}\left(\frac{\sin^2\psi}{\sigma_x^2} - \frac{r\sin 2\psi}{\sigma_x\sigma_y} + \frac{\cos^2\psi}{\sigma_y^2}\right),$$

$$\sigma = (\sigma_x^2 + \sigma_y^2)^{1/2},$$

$$L = \frac{2\sigma_x\sigma_y(1-r^2)^{1/2}}{\sigma_x^2 + \sigma_y^2}. \tag{7}$$

Here V_r and θ are the magnitude and direction of the monthly mean resultant wind vector, σ_x and σ_y are the standard deviations of the zonal and meridional wind components, and r is the coefficient of mutual correlation between the zonal and meridional components.

These five characteristics compose one of a variety of sets of independent parameters of the distribution (6). In the case of $\sigma_x = \sigma_y$ and $r = 0$, the distribution (6) is circular, whereas in all other cases it is elliptical.

By integration of the joint probability density (6) over all wind directions, it is possible to derive an expression for the wind speed distribution density:

$$g(v) = \frac{1}{\pi L \sigma^2} \exp(-a^2) \int_0^{2\pi} v \exp(2bv - c^2 v^2) d\psi. \quad (8)$$

In the case of the circular distribution of the wind vector expression (8) is transformed according to Weil (1954) as follows:

$$g(v) = \frac{2v}{\sigma^2} \exp\left[-\frac{(v^2 + V_r^2)}{\sigma^2}\right] I_0\left(\frac{2vV_r}{\sigma^2}\right). \quad (9)$$

The use of formula (5) allows the derivation of expressions for the calculation of WI in the case of elliptical

$$\mathrm{WI} = \frac{1}{4\pi L \sigma^2} \exp(-a^2) \int_0^{2\pi} \frac{1}{c^5} \{5y + 2y^3 + \frac{\pi^{1/2}}{2} (3 + 12y^2 + 4y^4) \exp(y^2)[1 + \mathrm{erf}(y)]\} d\psi \quad (10)$$

and circular

$$\mathrm{WI} = \frac{\pi^{1/2}}{4} \sigma^3 \exp\left(-\frac{\alpha^2}{2}\right)\left[(3 + 6\alpha^2 + 2\alpha^4) I_0\left(\frac{\alpha^2}{2}\right) + 2\alpha^2(2 + \alpha^2) I_1\left(\frac{\alpha^2}{2}\right)\right] \quad (11)$$

wind distribution laws (see Bryukhan 1989). Here I_0 and I_1 are the standard Bessel functions with imaginary variance of orders 0 and 1. In expression (10), $y = b/c$, and in expression (11), $\alpha = V_r/\sigma$. The circular wind distribution has practical implications because it demands only two distribution parameters, namely V_r

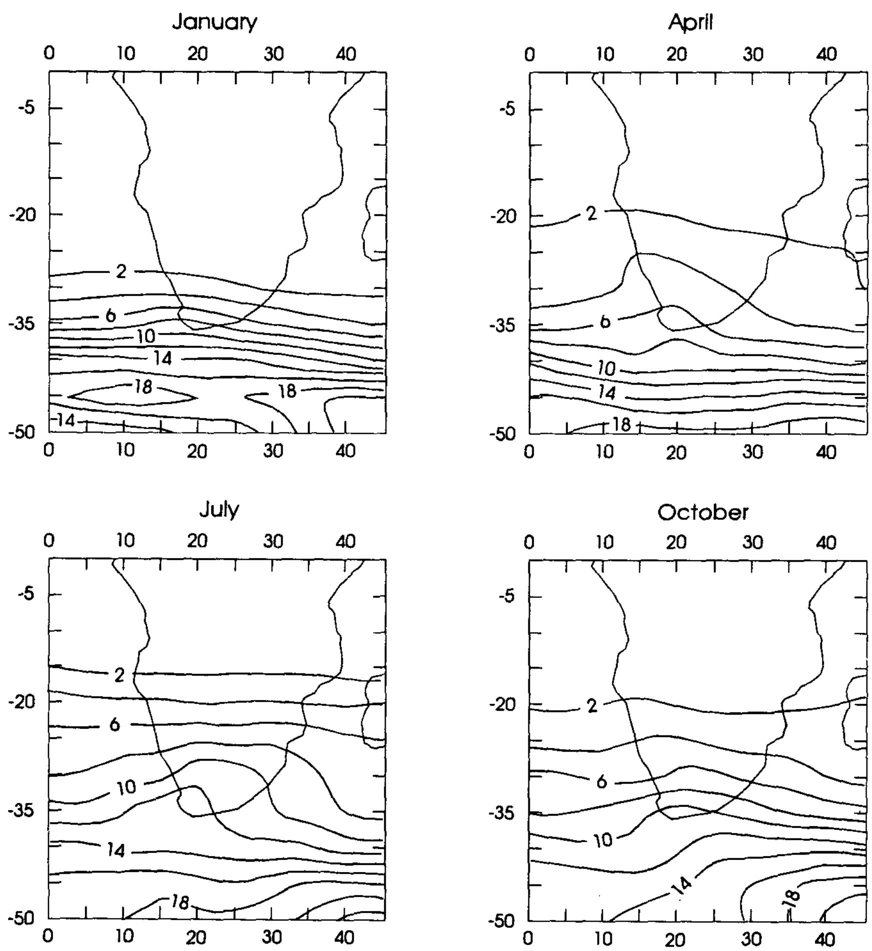

FIG. 2. Mean monthly specific wind power density P (kW m^{-2}) on the 300-hPa surface for the region 0°–50°S, 0°–45°E for the period 1982–89.

and σ and furthermore is valid in the upper atmosphere (Crutcher 1956; Bryukhan 1983).

The indirect calculation has a further advantage relating to the simplicity of the estimation of usable wind energy, which is proportional to a value

$$\mathrm{WU} = \int_0^\infty g(v)F(v)dv, \tag{12}$$

where $F(v)$ is the performance function of the wind turbine.

4. Results and discussion

All the estimates of the wind energy resource have been based on the general formula (10). To determine the isobaric surface generally experiencing the highest wind energy potential, mean seasonal vertical profiles of the value P were calculated:

$$\mathrm{PG} = \frac{1}{399}\sum_{j=1}^{21}\sum_{k=1}^{19}\bar{P}_{jk}. \tag{13}$$

The averaging in formula (13) is undertaken over the latitude–longitudinal network of 21 × 19 grid intersections. The calculated profiles for the midseason months are presented in Fig. 1, where it is evident that the intensity of the wind is generally greater by about 25%–30% in winter than in other seasons. In all seasons

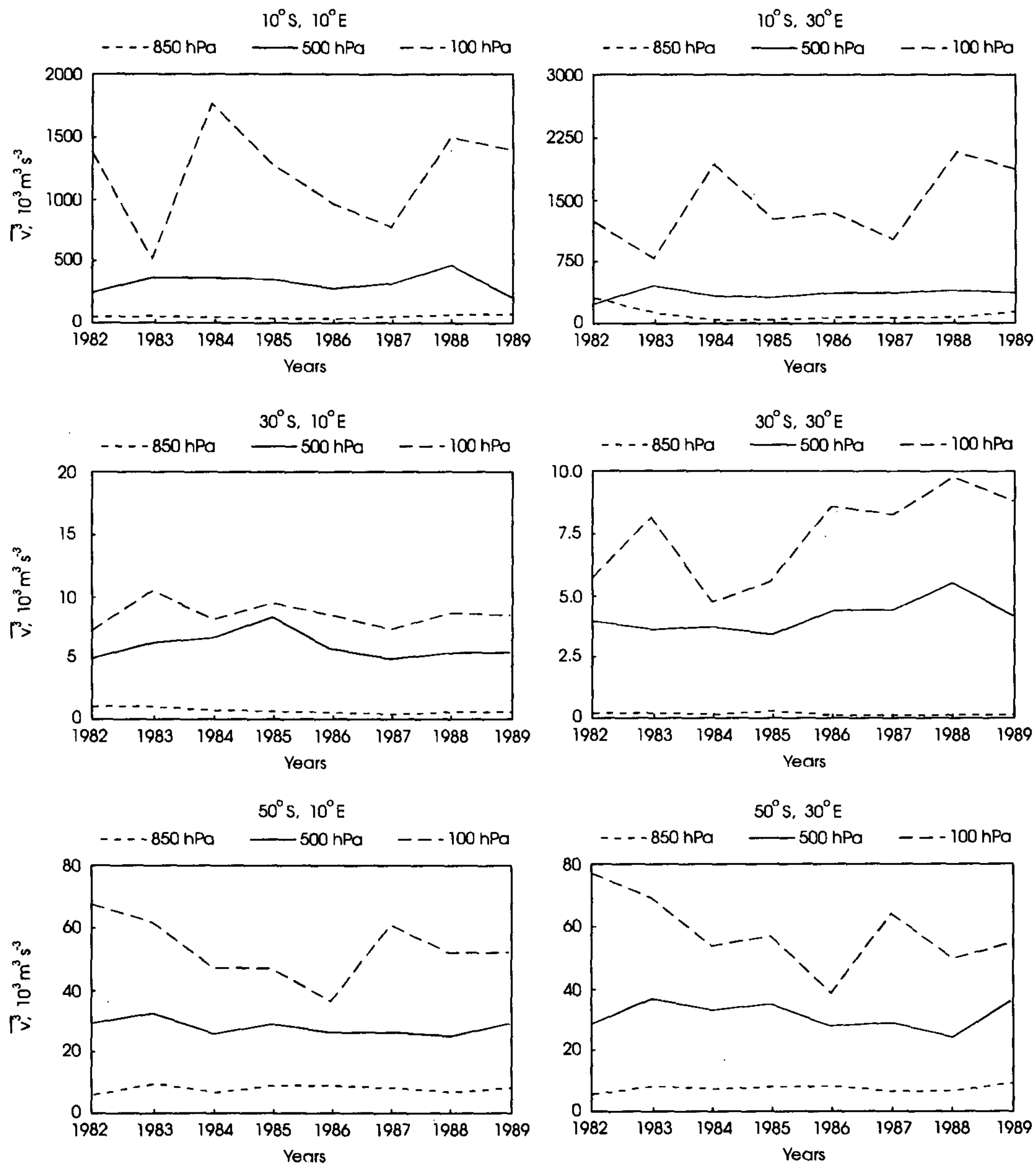

FIG. 3. Interannual changes in v^3 over the period 1982–89 on three isobaric surfaces for selected grid intersections.

the maximum values of wind energy occur near the 300-hPa surface.

Maps of the mean wind energy resource for the 300-hPa surface are presented in Fig. 2. Minimum values of $\bar{P}$ are found above the equatorial zone, equatorward of 20°S. From summer to winter, the zones of moderate and high values of $\bar{P}$ shift northward as a consequence of the large-scale shift of pressure belts associated with the pattern of solar heating. Highest continental wind energy resources are found over South Africa, with values about 8–9 kW m^{-2} in summer and 12–13 kW m^{-2} in winter. Wind energy resources increase to the south, and the subtropical jet stream is clearly shown near 45°S in summer. Comparison of data presented in Fig. 2 with the results of investigations of wind energy resources near ground level over South Africa (see Diab 1988; Jury and Diab 1989) allow us to conclude that the mean wind energy resource in the upper atmosphere is at least an order of magnitude greater than that near the surface.

Graphs of the interannual change in the wind energy resource (Fig. 3) demonstrate the greater interannual variability evident at the 100-hPa surface in contrast to the lower layers and highlight the importance of averaging the wind energy resource over a substantial period. While there is clearly some conformity in the interannual pattern in different longitudinal zones, there are marked differences between latitudes. This suggests that there is an interannual latitudinal shift in the zone of maximum winds in the stratosphere.

Considering the potential usefulness of the indirect calculation of the wind energy resource, a comparative analysis of these results with the results of the direct calculation was made. Graphs showing the comparison between WI and WD calculated by formulas (4) and (10), respectively, are presented in Fig. 4. The data points in each graph are representative of calculations over 399 grid locations and 4 midseason months. Surprisingly close accordance between the results of the different methods of calculating the mean cubed wind speed is evident. Figure 5, where the maps of the mean

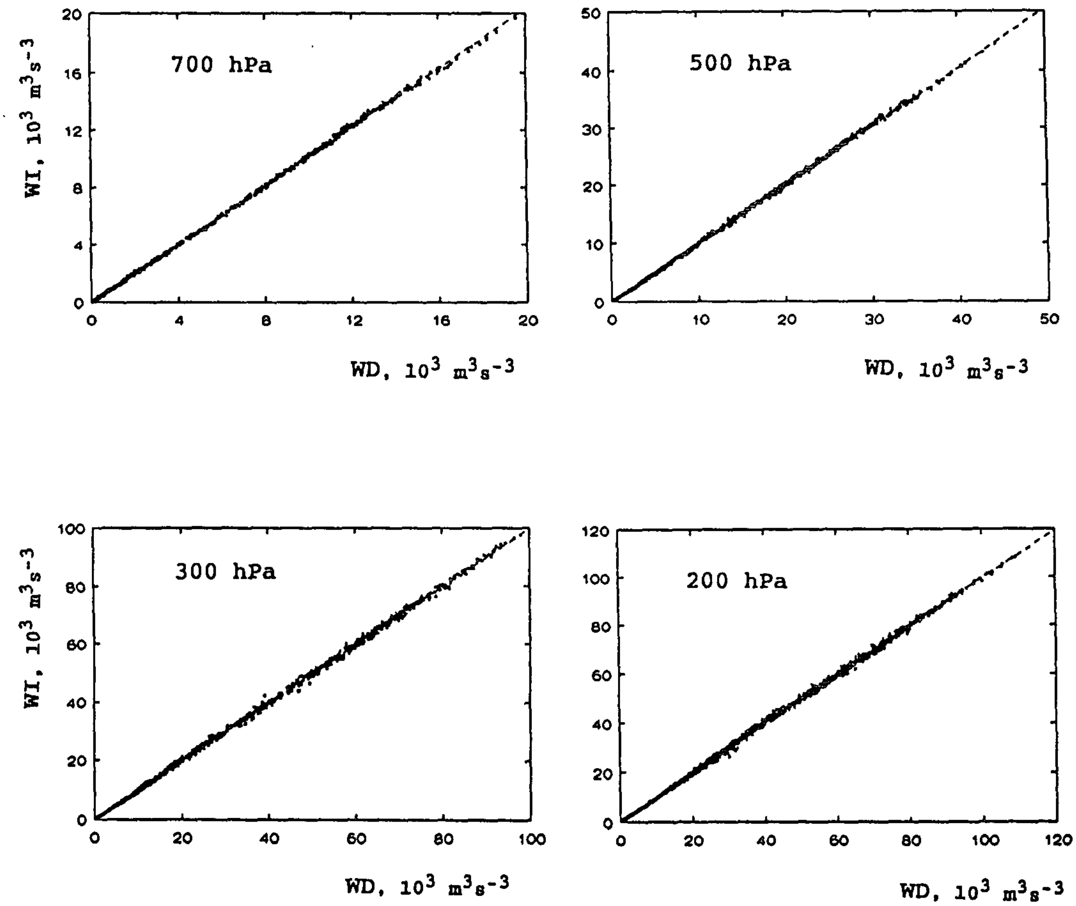

FIG. 4. Comparison between direct (WD) and indirect (WI) calculated v on different isobaric surfaces. The results include data for the months January, April, July, and October for the period 1982–89 and over the region 0°–50°S, 0°–45°E.

annual cubed speed on the 300-hPa surface are illustrated, further confirms the agreement.

The results presented in Figs. 4 and 5 provide qualitative evidence of good agreement between the empirical wind vector distribution and the two-dimensional Gaussian distribution. To estimate this accordance more objectively, the following statistical characteristics of the deviations averaged over the latitude–longitudinal network have been calculated:

$$\delta = \left[\frac{1}{399}\sum_{j=1}^{21}\sum_{k=1}^{19}\left(\frac{\mathrm{WI}_{jk}-\mathrm{WD}_{jk}}{\mathrm{SW}_{jk}}\right)^2\right]^{1/2}, \quad (14)$$

where

$$\mathrm{SW} = \frac{\sigma_w}{n^{1/2}}. \quad (15)$$

The standard error of the statistical estimation of value W is defined in formula (15).

The value δ shows the relationship of the error of the mean cubed speed calculation method to the standard error of the mean cubed speed estimation in general over the domain. It is obvious that the two-dimensional Gaussian distribution can be considered as an acceptable approximation of the actual wind vector distribution if δ is of the order of 1. The criterion δ is a rather strong criterion of feasibility of the two-dimensional Gaussian distribution, since the standard error estimation is applied to data series that are uncorrelated in time. In practice, due to the autocorrelation of the wind, the standard error SW is larger than the error calculated by formula (15). This means that the indirect calculation of the mean cubed speed can be considered reliable where δ also slightly exceeds 1.

TABLE 1. Mean values of δ, averaged over the latitude–longitude network for the period 1982–89.

Isobaric surfaces (hPa)	Month: January	April	July	October
1000	10.60	7.60	16.22	11.11
850	2.69	2.22	9.58	2.53
700	1.12	1.18	0.72	0.94
500	0.78	0.95	0.58	0.74
300	0.75	0.84	0.68	0.60
200	1.08	1.03	1.08	1.09
100	1.96	1.41	5.08	4.19

It follows from Table 1, where the results of calculation of δ for 4 months and 7 isobaric surfaces are presented, that the two-dimensional Gaussian approximation of the empirical wind vector distribution is acceptable at least for the atmospheric layer 700–200 hPa.

5. Conclusions

On the basis of daily ECMWF data over the period 1982–89 the results of the calculation of the wind energy resource in the upper atmosphere over the southern Africa region bounded by latitudes 0° and 50°S and longitudes 0° and 45°E are presented.

In general, the height of maximum wind energy is located near the 300-hPa isobaric surface. Wind speeds increase from summer to winter at all levels but are best defined at the 300-hPa level. There is a northward shift of the zone of maximum wind speeds in winter, and highest continental wind speeds are experienced over the southern tip of the subcontinent. Values on

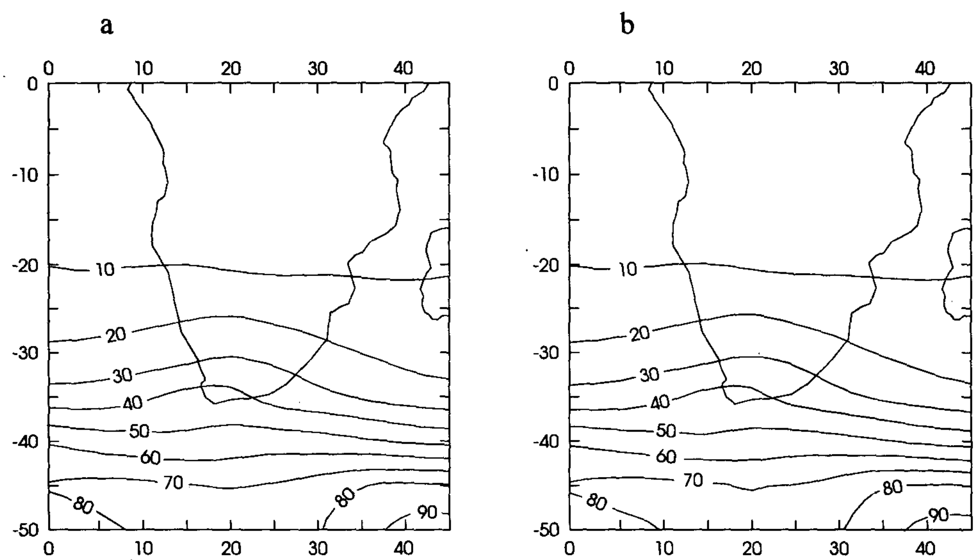

FIG. 5. Mean annual v^3 (10^3 m^3 s^{-3}) on the 300-hPa surface estimated according to the (a) direct method and (b) indirect method.

the 300-hPa surface exceed the wind energy resource near the surface by at least an order of magnitude.

Marked interannual changes in the wind energy resource are evident on the 100-hPa surface but not at the other levels. This is indicative of a latitudinal shift in the zone of maximum winds in the stratosphere.

The comparison between the results of the direct calculation of the wind energy resource and the indirect calculation based on the Gaussian distribution of the wind vector has shown very good correspondence for the atmospheric layer 700–200 hPa. It is therefore possible to recommend the indirect method of calculating the wind energy resources in the upper atmosphere for use in other areas.

Acknowledgments. The European Centre for Medium-Range Weather Forecasts is acknowledged for the provision of ECMWF data, which were supplied as part of a special program on Southern African Climatic Change: Analysis, Interpretation and Modelling (SACCAIM) funded by the Foundation for Research Development, Pretoria. Acknowledgements are also due to the Department of Mineral and Energy Affairs for continued support of wind energy research in South Africa. Mr. Frank Sokolic is thanked for his assistance with data manipulation.

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