Estimation of Surface Roughness Length and Displacement Height from Single-Level Sonic Anemometer Data

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ABSTRACT
The problem of finding joint values of both the roughness length $z_0$ and the displacement height $d$ is discussed in the context of the Monin-Obukhov similarity law for the wind speed profile. When focused on single-level datasets from one sonic anemometer (i.e., wind velocity, Reynolds stress, and sensible heat flux datasets at one height), it has been shown that this problem can be reduced to a simpler least squares procedure for one variable only. This procedure is carried out over a proper function of the data, representing the relative uncertainty of the roughness length, $\sigma_u/z_0$. This function is minimized with respect to $d$, giving a direct estimate of $d$, $z_0$, and their statistical uncertainty. The scheme is tested against two datasets in homogeneous and nonhomogeneous surface conditions.

1. Introduction
It has been established that the mixing-length approximation, in the framework of similarity considerations, describes the wind speed profile in the atmospheric surface layer in terms of the surface turbulent momentum and sensible heat fluxes. This is possible when local equilibrium conditions, generally related to a balance between turbulence production and dissipation, hold between wind speed and turbulent fluxes. This happens over homogeneous terrain when the ratio between horizontal fetch and measurement height is large enough to allow equilibrium to be reached (Panofsky and Dutton 1984; Wieringa 1993). In a more general way, this is also related to the problem of a “source area” for turbulent fluxes and averaged quantities measured at some height in the atmospheric surface layer—a subject of the current literature (Schuepp et al. 1990; Schmid 1994).

In the equilibrium case the shape of the wind profile appears to be governed by only one velocity and few height scales. In the Monin-Obukhov wind profile model scheme (Panofsky and Dutton 1984), the velocity scale (friction velocity $u_*$) is directly related to the surface momentum flux, while one length scale (Obukhov length $L$) is related to both heat and momentum fluxes. There must be another scale, however, related to the surface conditions that allow a boundary layer to exist that forces the wind speed to decrease to zero with height.

In flow over the ground, where the scale of the surface irregularities is much greater than the viscous scale $l = u_*/\nu$ ($\nu$ is the kinematic viscosity of air), the turbulent equilibrium conditions are not limited by the viscous sublayer. Instead, the presence of these “roughness elements” causes a local equilibrium breakdown by momentum transfer due to the local pressure gradients at a height comparable to their vertical dimension, thus determining a new boundary scale $z_0$ (roughness length). When this is the case, another operational problem can take place. An uncertainty arises about the “zero reference level” from which to measure the height $z$ above ground in the wind profile. If, anyway, $z$ is measured from the “true” surface it would need the introduction of a “displacement height” $d$. For example, in a flow over a forest, $d$ is related to the height of the trees, below which a wind profile in the previous sense does not hold. In this case, $z_0$ is related to the foliage shape and dimensions. As these heights do not, in principle, depend on atmospheric conditions (at least when the wind speed cannot affect the surface roughness characteristics: water and some crop surfaces are important exceptions) some attempts have been made to relate them to physical and geometric surface features (e.g., Thom 1971; Raupach 1994). Actually, from an operational point of view, and also in principle, the definition of those two heights is intrinsically related to the shape of the wind profile, so that their proper determination appears to be a fitting problem between flow data and theoretical similarity expressions. In the Monin-Obu-
khov similarity law, the windspeed profile \( U(z) \) is written as

\[
U(z) = \frac{u_0}{k} \ln[(z - d)/z_0] - \psi[(z - d)/L, z_0/L],
\]
(1.1)

where \( u_0 \) is the friction velocity scale, \( L \) the Obukhov length, \( k \) the von Kármán constant, and \( \psi[(z - d)/L, z_0/L] = \psi(z - d)/L \) is the integrated stability correction function (Panofsky and Dutton 1984). When applied to experimental data, this expression contains three kinds of parameters:

1) meteorological time averages, stochastically time dependent (\( \bar{U}, \bar{u}_a, \) and \( \bar{L} \));
2) surface variables, depending on the involved site and source area (\( z_0, \) and \( d \)); and
3) fixed parameters (the von Kármán constant and other nondimensional parameters in the explicit expressions of \( \psi \)), considered as universal constants in the theory.

Related to that, two kinds of experimental situations are usually found.

1) Neither surface nor meteorological variables are measured (besides \( U \)). This normally happens when fast response anemometers are not used. This case is usually limited in the literature to neutral stability conditions (\( \psi = 0 \)), in which \( u_0 \) is the only meteorological variable to be estimated (Robinson 1962). Even with this limitation, a several-level wind speed profile is required to get reasonable fitting statistics for the time-dependent \( u_0 \) at each measurement time. In addition, neutral-conditions must be somehow selected from the dataset (Kustas et al. 1989). As noted by Sozzi et al. (1998), this procedure can have the additional shortcoming of combining data from different heights, that is, different source areas for the meteorological variables. This could cause results over insufficiently uniform terrain to be unreliable. The problem of the reliability of the fitting procedure has been documented by Schaudt (1998), who used a least chi-square fit instead of a simple least square fit. A least chi-square fit can be properly used only if the variance of the wind speed \( U \) at any measurement level is known in each averaging time interval, which is normally difficult to obtain without the help of a fast-response anemometer.

2) When heat and momentum fluxes are also available, measurements at different times can be used together in the same statistics, because the problem is properly reduced to the determination of the geometric heights \( d \) and \( z_0 \), only, because \( u_0 \) and \( L \) can be directly estimated from the data. This can eliminate in principle the need to measure at different heights. Note, however, that if the fitting procedure is limited to a purely logarithmic law (neutral conditions, \( \psi = 0 \)), again at least two measurements heights are required. This is due to the mathematical form of (1.1) that links the two variables \( z_0 \) and \( d \) in the only multiplicative parameter \( a = \ln[(z - d)/z_0] \), so that they can be calculated separately only if at least two measurement heights are available. Jacobs and Van Boxel (1988) used the combination of a single-level flux measurement and a multilevel wind profile to get two independent relations between \( z_0 \) and \( d \). They were then solved graphically.

Some simple and interesting methods based on heat fluxes and velocity variance measurements in near-freeconvection conditions have been proposed. These can also be used in disturbed flows such as urban environments, because of the relative insensitivity of the source area for heat fluxes to small-scale temperature variations (Rotach 1994; de Bruin and Verhoef 1997). Because the similarity relations for the turbulent fluxes are independent of \( z_0 \), these methods are able to estimate \( d \) only.

In the last decade the use of sonic anemometers in surface-layer investigations has increased. When a sonic anemometer is used for single-level measurements, the vertical kinematic fluxes (\( \langle w\theta \rangle \) and \( \langle u\psi \rangle \)) can be available along with time-averaged wind speed \( U \) and temperature \( T \) at one height \( z \) (Cassardo et al. 1995), so that all the atmospheric variables in the Monin–Obukhov similarity law for wind speed are measured at the same time. For the kinematic turbulent fluxes, the brackets (\( \langle \cdot \rangle \)) indicate the usual time average and the small letters the turbulent fluctuations of horizontal (\( u \)) and vertical (\( w \)) velocity and potential temperature \( \theta \).] Sozzi et al. (1998) used an iterative multivariate fitting procedure to estimate \( z_0 \) for different direction sectors and the nondimensional parameters in the function \( \psi \) from single-level sonic anemometer data. They noted that the convergence of the fitting procedure is immediate (two steps) for the parameter \( z_0 \). As will become clear in the following paragraph, this is not accidental: actually \( z_0 \) and the parameters contained in \( \psi \) can be considered independently in the fitting procedure.

Focusing on the evaluation of \( z_0 \) and \( d \), in the scheme of the Monin–Obukhov theory, and choosing the least squares method as best-fit estimator, a mathematical formulation of the problem is

\[
\langle ku/u_0 - \ln[(z - d)/z_0] + \psi[(z - d)/L, z_0/L] \rangle_m = \min(z_0, d),
\]
(1.2)

where the operator \( \langle \cdot \rangle_m = (1/N) \sum_i \psi \) is intended to be the average over the dataset of \( N \) groups of time-averaged quantities \( U_i, T_i, u_{ai}, \) and \( L_i \) at the same height \( z \). Also, \( u_0 = -\langle u\psi \rangle \); \( \theta_0 = -\langle w\theta \rangle \); \( L = u_0 T/L \); \( k g \theta_0 \), where \( T \) is the absolute temperature, \( g \) is the gravitational acceleration, and \( k \) is the von Kármán constant; and \( \psi[(z - d)/L, z_0/L] = \psi[(z - d)/L, z_0/L] \) is the integrated stability correction function (Panofsky and Dutton 1984).

The expression \( \min(z_0, d) \) indicates the minimum with respect to both \( z_0 \) and \( d \), and this is in principle a bidimensional nonlinear least square problem. If there
were only one parameter to be found (say, \( z_0 \)) so the solution of (1.2) could be readily found by applying the minimum variance principle, which guarantees that the best estimate of \( \ln[(z - d)/z_o] \) in the sense of (1.2) is the average of the remaining part of the expression, that is, solving for \( z_o \):

\[
\begin{align*}
  z_o &= (z - d) \exp((-kU/u_b - \psi(z - d)/L, z_o/L)) \exp(z_o/L),
\end{align*}
\]

(1.3)

where \( z_o \) is the best estimate of \( z_0 \) in terms of least squares. Equation (1.3) could be solved iteratively, but, of course, this is not possible if \( d \) is not known.

An attempt to find \( d \) could be to apply the “minimum variance” principle again to the variance \( \sigma^2_{o,d} \) of \( z_o \), to determine its minimum with respect to \( d \). However, as is obvious from (1.3), the variance would depend on the square of \( z - d \) and would have a trivial minimum for \( d = z \). Instead, the fact that both average and standard deviation of \( z_o \) scale with \( z - d \) suggests that the relative uncertainty \( \sigma^2_{o,d}/\sigma^2_{o} \) could have a nontrivial minimum for some \( d \), thereby solving the problem in a direct and straightforward way. As shown in section 4, this is in fact the case, and in the next section it will be shown that this one-dimensional minimum problem is actually equivalent to the solution of (1.2).

2. Mathematical approach

Equation (1.2) can be written as

\[
\langle[S(z_o), d] - p(z_o, d)\rangle^2 = \min(z_o, d),
\]

(2.1)

where \( S = [kU/u_b + \psi(z - d)/L, \psi(z_o/L)] \) is a statistical quantity (function of the data) and \( p = \ln(z - d)/z_o \) is a parameter (function of \( z, z_o, \) and \( d \) only). The index \( m \) has been dropped from the data-average brackets to simplify the notation: the operator \( \langle \cdot \rangle \) will coincide with \( \langle \cdot \rangle \) in the remaining part of the paper.

Consider now the related problem,

\[
\langle[S(z_o), d] - p(z_o, d) - \langle S(z_o), d] - p(z_o, d)\rangle^2\rangle = \min(z_o, d).
\]

(2.2)

The fact that \( p \) is a parameter of the fitting procedure and not a measured quantity immediately implies that it is constant over the dataset. Then \( \langle p \rangle = p \), so that

\[
\langle[S(z_o), d] - p(z_o, d) - \langle S(z_o), d] - p(z_o, d)\rangle^2\rangle
\]

\[
= \langle[S(z_o), d] - \langle S(z_o), d]\rangle^2 = \sigma^2_{o,d}.
\]

(2.3)

Note now that (2.2) is equivalent to (1.2) with the condition

\[
\langle S(z_o), d] - p(z_o, d) = 0 \quad \text{or}
\]

\[
\ln[(z - d)/z_o] = [kU/u_b + \psi(z - d)/L, z_o/L],
\]

(2.4)

so that the problem of finding the minimum variance of \( S = kU/u_b + \psi(z - d)/L, z_o/L \) with respect to \( d \), with the constraint \( \langle S \rangle = \ln((z - d)/z_o) \), appears to be equivalent to (1.2), which defines \( z_o \) and \( d \), and which has been reduced to a one-variable conditioned minimum problem.

Note, incidentally, that this is a general observation and is particularly useful when, as in the present case, a function of the parameters only \( (p) \) is linear in the expression for the minimum, allowing an explicit solution for \( p \) of the equation for the average value (here in the form \( p = \langle S \rangle \)).

Furthermore, it has been established that \( \psi(z_o/L) = O(z_o/L) \) if \( z_o \ll |L| \), because the limit condition for neutral flow \( \psi(0) = 0 \) must hold. This means that in the usual surface layer conditions, when \( z_o \ll (z - d) < |L|, \psi((z - d)/L, z_o/L) = \psi((z - d)/L) - \psi(z_o/L) \approx \psi(z - d)/L \) (this is indeed the usual form of the stability correction in the surface layer; see, e.g., Garratt 1992; Panofsky and Dutton 1984). This means that \( S \) will be independent of \( z_o \), and the conditioned minimum problem for \( \sigma^2_{o}(d, z_o) \) becomes a simple one-dimensional minimum problem for \( \sigma^2_{o}(d, 0) \) in which the “constraint” \( \langle S \rangle = \ln[(z - d)/z_o] \) will only be used to find \( z_o \), after the value of \( d \) that minimizes \( \sigma^2_{o} \) has been found.

The quantity \( \sigma^2_{o} \) to be minimized with respect to \( d \) can attain a more intuitive meaning by noting that the constraint (2.4) is identical to (1.3), which defines the best value of \( z_o \).

If we estimate the standard deviation of \( z_o \) by taking \( z_o = z_o + \Delta z_o \) and using the “single-point” relationship \( z_o = (z - d)\exp(-S) \), and, expanding in Taylor series up to first order in \( S = \langle S \rangle + \Delta S \), taking into account (2.4) or (1.3), we find

\[
\sigma^2_{o} = (z - d)\exp(-S)\sigma_{s},
\]

so that

\[
\sigma_{s} = \sigma_{o,d}/\sigma_{o}.
\]

(2.5)

An estimate of the statistical uncertainty over \( d \) can also be determined by noting that it coincides with that of \( (z - d) \), which again can be estimated by means of a Taylor expansion, after having highlighted \( (z - d) \) in (1.3). After a straightforward calculation, the result is

\[
\sigma_{d} = (z - d)\sigma_{s},
\]

(2.6)

which shows that the uncertainty in \( d \) increases with the measurement height, as expected.

It can be noted that the minimization of \( \sigma d \) can be carried out with respect to any other parameter in place of \( d \) (e.g., a nondimensional constant of the function \( \psi \)) maintaining the relative independence from \( z_o \). This could explain the fast convergence of the iterative equation for \( z_o \) in the fitting procedure by Sozzi et al. (1998).

Eventually, an even more practical approximation can be used by noting that, if \( \sigma d \) is sufficiently small, the statistical averaging and exponential operators over \( S \) can be exchanged, as can be seen by using a second-order Taylor expansion around \( (-S) \) for the exponential function.
show the position of the minima of the corresponding curve and the $d$ (plus signs), as function of momentum flux, and temperature, taken at averages of wind speed, sensible and latent heat flux, momentum, sensible heat, wind speed, and temperature measurements, coupled with a Campbell KH20 krypton hygrometer for latent heat flux determination (which, in the current study, is only used to correct the temperature flux data; see the end of this section).

The mast was placed in a small clearing (less than 100 m of radius) on the university campus. The area surrounding the clearing is covered by trees with a height between 5 and 10 m (mainly pine, eucalyptus, cypress, and some olive trees), at a minimum distance of about 50 m from the mast (short olive trees and cypresses), up to more then 1 km in all directions. Some isolated two-story buildings are also present in the area around the clearing.

The surface cover is not completely uniform on the small scale (different trees with patches of shorter vegetation and some buildings are present), but no strong dependence in the roughness parameters on the direction can be detected by eye inspection at least within 1 km from the mast.

The second dataset was obtained in a similar way for the same 30-min averaging time from the top of a 9-m mast near the beach of Frigole (about 10 km northeast of Lecce). The measurement site is located a few hundred meters from the coastline in a flat area covered with bushes of Mediterranean vegetation (about 1-m average height) and a few patches of low trees (short pines closer to the coast and olive trees farther inland). As in the former case, quite uniform cover conditions appeared to hold, by eye inspection, for more then 1 km from the mast in the inland direction, although some nonhomogeneities were present on smaller scales (patches of trees). A sand beach and a small water basin were present between the mast site and the coastline.

Flux data were collected continuously for about two weeks at each site. They were obtained directly by eddy correlation in real time (every 30 min) through homemade software that also provided a rotation in the “streamline” reference frame (McMillen 1988) to eliminate the effect of vertical misalignment and to give the proper value of the Reynolds stress in the wind speed–oriented reference frame. Sonic temperature corrections (Cassardo et al. 1995) and Webb corrections (Webb et al. 1980) were then applied to get the best estimates of sensible heat flux.

4. Results

a. Global estimates

Three practical procedures were compared to estimate the best values of $d$ and $z_o$, following suggestions in section 2.

1) The exact procedure of minimizing $\sigma_x(d, z_o)$ with respect to $d$ under the constraint that $z_o(d) = (z - d) \exp(-S)$. This implies a calculation of $\sigma_x$ changing $d$ step by step from slightly negative values up to the maximum allowed $d = z$, while using in $\sigma_x(d, z_o)$ the value of $z_o$ calculated iteratively at each step by Eq. (2.4). This is equivalent to solving the exact least square problem of Eq. (1.2) with respect
to both $z_0$ and $d$, as discussed, and it has also been proven numerically.

2) Minimizing $\sigma_s$ step by step with respect to $d$ as above, but now with the simpler procedure of using the approximation 
$$
\psi((z-d)/L, z_0/L) \approx \psi((z-d)/L, 0) = \psi(z-d)/L,
$$
so that $\sigma_s^2 = \sigma_s^2(d, 0)$ is independent on $z_0$ (unconditioned minimum, one does not need to calculate $z_0$ at each step). In these two cases, the final value of $z_{aw}$ is given by (2.4), with $d$ corresponding to the minimum of $\sigma_s$.

3) The approximate procedure of finding the minimum of $\sigma_s^2/d z_{aw}$ with respect to $d$, using the same approximation as in point 2, which gives directly the values of $z_{aw}$, $\sigma_s$, and $d$.

In all cases the Businger–Dyer forms for the $\psi$ functions have been used (see, e.g., Panofsky and Dutton 1984) and wind speed data less than 1.5 m s$^{-1}$ have been dropped from the datasets. A 0.5-m step was always used in the minimum procedure with respect to $d$, that is, well below its statistical uncertainty (see below). The results are compared in Figs. 1 and 2, for the two different datasets.

Figure 1 presents the plots of $\sigma_s(d, z_0)$, $\sigma_s(d, 0)$, and $\sigma_s/d z_{aw}$ as functions of $d$ for the first dataset as obtained by cases 1–3, respectively. The results for case 1 are $z_0 = 0.51$ and $d = 7.5$ m, for case 2 are $z_0 = 0.42$ and $d = 8.5$ m, and for case 3 are $z_0 = 0.65$ and $d = 6.5$ m.

Figure 2 is analogous to Fig. 1 for the second dataset, where no global minimum has been found, so that it has been divided into two subsets separating offshore from onshore winds. For offshore winds (Fig. 2), a minimum is found at $z_0 = 0.52$ and $d = 1$ m in case 1, $z_0 = 0.50$ and $d = 1.5$ m in case 2, and $z_0 = 0.68$ m in case 3, but $d$ is slightly negative, because of the approximations involved and the statistical uncertainty (see below). No apparent minimum exists for onshore winds. This last result could be due to the closeness of the mast to the coastline, because this can cause a lack of equilibrium conditions between surface fluxes and wind speeds. An estimate of the required minimum fetch $F$ for the wind velocity $U(z)$ to be determined by the new roughness $z_0$ (over land), after a roughness change on the coastline, can be given by (Wieringa 1993)

$$
F = 2z_0((10z_0/[\ln(10z_0)] - 1) + 1). \quad (4.1)
$$

This gives $F \approx 700$ m for $z_0 = 0.6$ m and $z = 9$ m, increasing for decreasing $z_0$, so that the measurement point at $z = 9$ m probably lies in a transition layer in which neither sea nor ground equilibrium profiles hold for onshore flow conditions.

An estimate of $\sigma_s/d z_{aw}$, the relative uncertainty in $z_{aw}$, is just given by the ordinate value at the minimum of the curve, as discussed in section 2, Eq. (2.5). Equation (2.6) gives a standard deviation for $d$ of about 2 m in both cases examined. This statistical uncertainty comes mostly from the use of all wind data in all stability conditions, which is inherent in the present method and, probably to a lesser extent, from the small-scale non-homogeneity of the surface and the uncertainty of the canopy height.

As expected, all the figures show that approximating $z_0$ with 0 in the integrated stability function has little effect in establishing the minimum of the curve and the corresponding values of $d$ and $z_0$. The discrepancy between the exact and approximate values is less than the statistical uncertainty. Because of the small-scale non-homogeneity in the vegetation at the two sites, it is not easy to compare these results with values found in the literature.

Some results collected by Stull (1988, p. 380) found a $z_0$ value between 0.2 and 0.5 m for “many trees, edges, and few buildings.” Garratt (1992, p. 290) collected a series of $d$ and $z_0$ estimates from the literature. The labels “woodland” and “pine forest” that approximate the conditions of the first dataset show $z_0$ between 0.3 and 0.9 m, while $d$ should be between 0.6 and 0.75 times the canopy height (between 6 and 7.5 m in this case, taking the maximum canopy height $h = 10$ m). For areas with vegetation of 1–2 m in height (second dataset), the same table reports $z_0 = 0.2$.

A good review and discussion of the literature devoted to the estimation of roughness parameters can be found in Wieringa (1993), in which the quality and reliability of the datasets is also discussed. A detailed table on pp. 346–347 of Wieringa’s (1993) paper provides data for surface conditions closest to those of the present datasets.
1) First dataset “low buildings and trees, about 8 m in height” \( z_0 = 0.7 \) m (no \( d \) reported), “pine forest, 10 to 12-m height” \( z_0 = 1 \) and \( d = 9.6 \) m, and “pine forest, 10-m height” \( z_0 = 1 \) and \( d = 7 \) m. These values (\( d/h \approx 0.7-0.8 \), if \( h \) is the maximum canopy height) compare reasonably well with the present results: \( d = 7.5 \), \( z_0 = 0.51 \) m (\( h = 10 \) m), taking also into account the standard deviations \( s_d \approx 2 \) m and \( s_{z_0} \approx 0.2 \) m.

2) Second dataset “bushland, \( h = 3 \)-m height” \( z_0 = 0.36 \) and \( d = 2.4 \) m and “bushland, \( h = 2.3 \)-m height” \( z_0 = 0.43 \) and \( d = 1.8 \) m.

The trend in \( z_0 \) between the reported values (\( \Delta z_0/\Delta h \approx -0.10 \)) would suggest \( z_0 = 0.56 \) for the present dataset (\( h = 1 \) m), and the scaling of \( d \) with \( h \) (\( d/h \approx 0.8 \)) suggests also \( d = 0.8 \) for \( h = 1 \). They compare reasonably well with the present results: \( d \approx 1 \), \( z_0 = 0.52 \) m, but it should be stressed that it is very difficult to assess a proper quantitative comparison from a mainly qualitative description of the sites involved, and that the expected \( d \) is less than its statistical uncertainty in this case (\( \sigma_d \approx 2 \) m and \( \sigma_{z_0} \approx 0.14 \) m).

b. Sector estimates and wind speeds comparison

Because of the uncertainty in comparing the present estimates of the roughness parameters with the literature data, a better quantitative validation of the procedure has been attempted. The above procedure has been applied to a single direction sector in each dataset, to reduce the scatter, and then the measured wind speeds have been compared with those calculated by the Monin–Obukhov similarity, using the measured \( u_* \) and \( L \) and the calculated \( z_0 \) and \( d \).

For the first dataset a 30° width sector has been chosen along the northern direction, which appears to have a more uniform tree cover and no buildings, and also has the advantage that incoming wind is mainly from the north in the dataset (typical of high pressure conditions in the Otranto channel). For the second dataset again a 30° width sector has been chosen almost perpendicular to the coast (southwestern direction).

The results for the computation of \( z_0 \) and \( d \) are shown in Fig. 3 for the first dataset and Fig. 4 for the second dataset. Procedure 1 gives \( z_0 = 0.37 \) and \( d = 8 \) m for the former and \( z_0 = 0.78 \) and \( d = 0 \) m for the latter. As expected, the statistical uncertainty \( \sigma_z \) appears to be reduced in both cases, as compared with the global results.

Figures 5 and 6 show the Monin–Obukhov similarity modeled wind speeds versus the measured ones. The straight lines are minimum chi-square (\( \chi^2 \)) best-fit lines that enable the reliability of the used model parameters (\( z_0 \) and \( d \)) to be tested, where

\[
\chi^2 = \frac{1}{2} \left[ \frac{1}{N - P} \right] \sum_{i=1}^{N} \frac{(\hat{U}_{Mi} - U_{Mi})^2}{\sigma_i^2}. \tag{4.2}
\]

Here, \( U_{Mi} \) are the measured wind speeds, \( \hat{U}_{Mi} = (u_*/k)(S - \psi_i) \) are the wind speeds estimated from (1.1), \( N \) is the number of data, and \( P = 2 \) is the number of
parameters estimated from the dataset by the model equations \((z_0, d)\). To test the consistency of the estimates of \(z_0\) and \(d\), the measured data are considered as reference values, and the statistical uncertainty \(\sigma\) of each estimated wind speed \(U_E\) is considered to be dependent on the uncertainty of the roughness parameters only and is calculated as follows:

\[
\sigma^2 = (u_*/k)^2[\sigma_0^2 - 2 \text{cov}(S, \psi_i) + \sigma_0^2]
\]

where \(\sigma_0^2\) is the variance (function of \(\sigma\)) of the function \(\psi_i\) and \(\sigma_0\) indicates the covariance between \(S = \ln[(z - d)/z_0]\) and \(\psi_i = \psi_i[(z - d)/L_i]\). The last two equalities hold for \((z - d) \ll |L|\) (a condition verified in the used datasets), in which case it can be inferred from the results of section 2 that

\[
\text{cov}(S, \psi_i)/\sigma_0^2 = O[(z - d)/L_i]\approx 0\quad \text{and}\quad \sigma_0^2/\sigma_0^2 = O[(z - d)/L_i].
\]

The regression coefficients are \(r = 0.94\) in Fig. 5 (\(N = 100\) data) and \(r = 0.87\) in Fig. 6 (\(N = 9\) data). For the minimum chi-square of (4.2), whose expected value is 1, it was obtained: \(\chi^2 = 1.27\) in Fig. 5 and \(\chi^2 = 1.65\) in Fig. 6, which correspond to confidence coefficients of 0.95 and 0.90 in the \(\chi^2\) distribution for \(N - P = 98\) and \(N - P = 7\) degrees of freedom, respectively.

In synthesis, with the estimated parameters \(z_0\) and \(d\) and their calculated uncertainties, the Monin–Obukhov wind speed law fits the two datasets within a confidence of 90% and 95%, respectively.

5. Concluding remarks

Surface roughness and displacement height can be determined from the similarity wind profile law from single-level measurements of wind speed and fluxes by solving a simple and straightforward one-dimensional minimum problem. The quantity to be minimized has a direct physical meaning and allows the estimation of the statistical uncertainties of the two parameters. The procedure requires no specialized software, very little computational effort, and can also be used to estimate different parameters in the stability correction functions. Further testing of this procedure will assist in determining its general applicability.

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