The Significance of Mountain Lee Waves as Seen from Satellite Pictures

SIGMUND FRITZ

U. S. Weather Bureau, Washington, D. C.

(Manuscript received 30 June 1964, in revised form 30 September 1964)

ABSTRACT

TIROS pictures of mountain lee waves produced by airflow over four different mountains are discussed. The wavelengths of the observed cloud patterns are compared to the mean wind in the troposphere as suggested by Corby (1957). In three of the four cases examined, Corby's relationship worked well. In the fourth case, it was found that smaller thermal stability was associated with an average wind speed lower than Corby's relation suggests. This is in agreement with theory.

Other criteria, such as proper wind direction, needed to produce mountain waves are also found in these cases. Thus pictures of mountain waves may often be useful in estimating certain atmospheric parameters in those mountainous areas where conventional data may be unavailable.

1. Introduction

It is well known (Alaka, 1960) that waves will form in the atmosphere when a wind blows across an extended mountain range, if conditions of thermal stability and wind flow are favorable.

Furthermore, when sufficient moisture is present, clouds will appear in a wave pattern which can be photographed from satellites. TIROS has televised such lee-wave cloud patterns on several occasions (e.g., Döös, 1962; Conover, 1964; Fritz and Lindsay, 1964), and it is interesting to compare pictures with existing theories and empirical results. If they agree, pictures can be applied to deduce information about the atmosphere in areas where conventional data may be lacking. When the pictures disagree with earlier results perhaps physical factors can be found to account for the discrepancies or, alternatively, theory may have to be modified to account for the satellite observations.

The main purpose of this paper is to use satellite pictures of mountain lee waves to estimate the average wind speed in the troposphere. To do this we already have Corby's (1957) radiosonde study which relates wavelength of the lee waves to average wind speed in the troposphere. As we shall show, such application of Corby's results fits the satellite and wind data in three of the four cases examined below. One of the cases did not fit his results well enough; however, further examination shows that consideration of the thermal stability could reduce the discrepancy.

2. Earlier results

For mountain lee waves, Corby (1957) found empirically from radiosonde ascents that the wavelength is linearly related to the mean wind speed, averaged through a deep layer of the troposphere. Sawyer (1960) found a similar relationship between the wavelength and the mean wind, averaged over the height interval from 0 to 10 km. Sawyer's study is based on theory and more than one wavelength is permissible from his calculation for a given atmospheric situation; the relationship which most nearly corresponded to Corby's involved the shortest computed wavelength.\footnote{Sawyer's (1960) Fig. 11 shows a line attributed to Corby (1957). This line is different from Corby's (1957), and according to private correspondence with Sawyer and Corby, Corby's (1957) is correct.}

WMO Technical Note No. 34 (Alaka, 1960) justifies Corby's linear relationship in the following manner. Theory suggests that the wavelengths of any lee-wave will have a magnitude somewhere between the minimum and maximum values of the quantity $2\pi/\sqrt{F(z)}$ or $2\pi u/\sqrt{g \bar{S}}$ found in the troposphere. The parameter $F(z)$ is Scorer's parameter. Here

- $u$ is the undisturbed horizontal wind speed
- $S = (1/\theta) (\partial \theta/\partial z)$ is the static stability
- $\theta$ is potential temperature
- $z$ is the vertical coordinate
- $g$ is the acceleration of gravity.

The Technical Note suggests that, although the static stability in shallow layers varies widely, the mean stability through the whole troposphere does not. The implication from this is that the wavelength, $\lambda$, is given by

$$\lambda = (2\pi/g)^{1/2} (\bar{u}/\bar{S})^{1/4},$$

From this the Technical Note apparently assumes

$$\lambda = (2\pi/g)^{1/2} (\bar{S})^{-1/4} \bar{u} = k \bar{u},$$

where $k$ is a constant if $\bar{S}$ is assumed constant. The averages in Eqs. (1) and (2) are taken in the vertical
Fig. 1. Mountain lee waves photographed in the lee of the Andes Mts. The long bright area located just at the left edge of the wave pattern, is composed mainly of clouds located along the mountain. A black dot, showing the position of Puerto Montt, Chile, has been added about half way between the center cross-mark and the top of the picture, near the left edge of the bright “mountain” cloud. The wavelength of the lee waves to the east of Puerto Montt is about 8 st. mi. The temperature and wind soundings are also shown. The numbers along the wind sounding represent the wind direction in tens of degrees.

Fig. 2. Mountain lee waves photographed in the lee of the Sierra Madre Occidental Mountains in Mexico. Some geographical outlines showing Mexico and the southern United States have been superimposed. A black dot showing the position of El Paso has also been added. The wavelength near the center cross-mark is about 13 st. mi. The temperature and wind soundings for El Paso are shown. The numbers near the wind sounding are the wind direction in tens of degrees.
Fig. 3. Mountain lee waves in the lee of the Cascade Mountain Range. The boundaries of Oregon and of some other geographical entities have been superimposed. The position of Salem, Oreg., is shown by a white dot. The wavelength of the waves, near the lower part of the picture, is about 12 st. mi. The temperature and wind soundings for Salem are shown; the numbers along the wind sounding are wind directions in tens of degrees.

Fig. 4. Mountain lee waves generated in the lee of the Appalachian Mountains. Lake Erie and Lake Ontario can be seen near the top of the picture; the coasts of New Jersey and Maryland are visible just below the center crossmark. White dots show the position of Washington (W) and Pittsburgh (P) and a black dot shows the position of Huntington (H). The wavelength of the waves was about 13 st. mi. The temperature and wind soundings for Washington, D. C., are shown near the time of the picture; the numbers near the wind sounding are the wind direction in tens of degrees. The wind sounding for Pittsburgh, Pa., has been included to show that strong winds existed north of the wave pattern.
through the depth of the troposphere. Eq. (2) is an approximation to Corby's (1957) empirical relationship, although his line does not go through the point, \( \lambda = 0, \bar{u} = 0 \). Since we may wish to derive the mean wind from the wavelength observed in satellite pictures it might be desirable to write Eq. (2) in the form

\[
\bar{u} = G(\bar{S}) \lambda,
\]

where

\[
G = g^1/(2\pi).
\]

3. TIROS pictures of lee-waves

With these considerations as a basis, the satellite pictures of mountain lee waves were examined for possible relationships between \( \lambda \) and \( \bar{u} \). Figs. 1–4 show examples of mountain waves photographed by TIROS satellites in several parts of the western hemisphere. The wind and temperature soundings associated with them are also shown.

The soundings are not always taken at the same time as the pictures. Nor is it evident that they should be, for comparison with theory. Sometimes the wind and temperature distributions were changing rapidly with time, and if one uses the wind and temperature data taken west of the mountains (when available), then it may be that the lee wave generated and observed by the satellite east of the mountain should be associated with soundings taken at some earlier time. However, the data shown in this article seemed to be the most pertinent of those available.

In each of the Figs. 1 through 4 the wavelength can be measured. The distance across a group of well-defined waves in the photographs was measured by Fujita's method (1963). The wavelength was taken to be the measured distance divided by the number of waves in the interval. It is estimated that this average wavelength can be determined to within about one mile. The wavelengths were plotted against \( \bar{u} \), the average wind speed in the troposphere, on Corby's (1957) graph, as shown in Fig. 5. The unlabelled dots are the results found by Corby, and the straight line is the one he

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**Fig. 5.** Relation between the wavelength of mountain lee waves and the mean wind speed averaged through the troposphere. The plain dots represent Corby's (1957) data; the other symbols represent results from the satellite pictures in Figs. 1–4, except that point \( H_1 \) refers to a satellite picture (not shown) taken by TIROS VI over the Appalachians about two hours earlier than the picture in Fig. 4. (See Fritz and Lindsay, 1964, for that picture.) The wind \( H_2 \) at 1800Z was available only up to 10,000 ft, the remainder of the wind sounding up to the tropopause was interpolated from the 1200Z and 0000Z soundings.
drew to fit his data. The other points, labelled with letters, are the results obtained from measurements in Figs. 1–4 and from corresponding wind soundings at the designated locations. With the exception of the case in the lee of the Appalachian Mountains, the results fit Corby’s data quite well. However, it will be noted that even in these other cases, the points from the satellite pictures fall to the left of Corby’s line, and Corby’s points themselves fall to the left when the wavelength is large. Perhaps the slope of the line should be somewhat steeper; or perhaps a curved line might ultimately fit the data better.

To explain the discrepancy for the Appalachian mountain waves of Fig. 4, the wind and temperature data were examined further. As mentioned earlier in connection with Eq. (2), the expectation that \( \lambda \) would be related to \( \bar{a} \) depends on the fact that \( \bar{S} \), the mean stability in the troposphere, is relatively constant with time and does not vary from place to place. If \( \bar{S} \) should vary, then this might alter the relationship of Fig. 5.

Corresponding to Fig. 3 and 4, respectively, the temperature soundings for Salem, Ore., and for Huntington, W. Va., are shown in Fig. 6. The Huntington sounding is clearly less stable than the Salem sounding; the Washington, D. C., sounding was similar to the one at Huntington.

To examine this further, the stability, \( S \), was computed for the soundings related to the waves in Figs. 1–4. This was done for every 50-mb layer from the surface to the tropopause. The stability data of every three adjacent 50-mb layers were then averaged to form overlapping means. The results of the smoothed stability computations for Huntington, W. Va., and Salem, Ore., are shown in Fig. 7. The figure also contains \( \bar{S} \), the value of \( S \), averaged from the surface to the tropopause, for the four stations shown in Figs. 1–4 (except that Huntington is shown in Fig. 7 instead of Washington, D. C.).

Fig. 7 shows that at almost all levels the stability was less at Huntington than at Salem. And it is also interesting to note that \( \bar{S} \) was almost the same for Salem, El Paso, and Puerto Montt, Chile. But at Huntington \( \bar{S} \) was substantially smaller both at 12Z, 18 April, and 00Z, 19 April. Since the wave pattern occurred in the middle of the day (about 1800Z), surface heating might have tended to make the more unstable sounding representative of the conditions which produce the waves in the picture.

If we assume that the relation in Eq. (3) approximates Corby’s (1957) results, then we could account for the low average wind speeds at Huntington as follows. The value \( \bar{a} \) for El Paso, Salem and Puerto Montt agree fairly well with Corby’s results in Fig. 5. Therefore, it may be valid to assume that \( \bar{S} \) for these stations represents the value to be expected most of the time; or \( \bar{S} = 4.5 \times 10^{-6} \text{ ft}^{-1} \). Then since \( \lambda \) for Huntington is about the same as \( \lambda \) for El Paso, Eq. (3) would suggest that

\[
\bar{a}_H = \bar{a}_E (\bar{S}_H/\bar{S}_E)^\gamma,
\]

where the subscripts \( H \) and \( E \) refer to Huntington and El Paso. If we take,

\[
\bar{a}_E = 38 \text{ m sec}^{-1}, \quad \bar{S}_E = 4.5 \times 10^{-6} \text{ ft}^{-1},
\]

\[
\bar{S}_H = 3 \times 10^{-6} \text{ ft}^{-1} \text{ then } \bar{a}_H = 31 \text{ m sec}^{-1},
\]
Among the criteria which are considered favorable for wave formation, we may cite the following (Alaka, 1960):

1) The wind direction should be nearly perpendicular to the mountain throughout a deep layer of the atmosphere. In the cases discussed (Figs. 1–4) the wind direction was nearly perpendicular to the mountain.

2) The atmosphere should be statically stable. The presence of an inversion at levels near the top of the mountain crest, is favorable for wave formation, a condition frequently found near frontal zones. Although fronts are not mandatory for wave formations, frontal zones are often found near areas where lee waves are observed. In Figs. 1–4, an inversion was present in each case at heights between 10,000 and 20,000 ft.

3) Scorer's parameter, $F(z)$, should decrease with height. This will be satisfied as $u$ increases with height. Thus a marked vertical wind shear is a frequent characteristic of lee-wave situations. This criterion for the decrease of $F(z)$ will also be met if the stability, $S$, decreases rapidly with height. Hence this criterion may also be found in lee wave situations where a pronounced inversion is present in the lower atmosphere, for $S$ is then large in the inversion and smaller in the region above.

It will be noted from Figs. 1–4 that the wind generally increased with height, sometimes quite rapidly and that $S$ would decrease above the inversion which was present in each figure.

The wind and thermal fields, variable in both time and space, have already been mentioned as a complicating factor. The theory usually assumes a mountain with simple contours. But in nature the terrain features are quite complex and may introduce more than one wave pattern.

Such a situation may be illustrated in Fig. 8, which is a picture taken by TIROS VI, about three hours before the picture in Fig. 3 was taken by TIROS V. Fig. 8 shows that wave patterns extended from Oregon across all of northern Nevada; the next frame, not shown here, indicated that some waves existed even in southwestern Idaho and across northern Utah.

Over this large region, extending over 400 miles in a east-west direction, many mountain barriers exist which could generate lee waves. Thus it is doubtless necessary to consider the wind and temperature regime in the vicinity of each major mountain complex, and its interaction with the terrain features, to understand fully the details in the satellite pictures.
REFERENCES


Fritz, S., and C. V. Lindsay, 1964: Lee-wave clouds photographed over the Appalachians by TIROS V and VI. *Soaring*, 28, 14–17.
