Wind Measurement by Conventional Radar with a Dual Beam Pattern

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ABSTRACT

A method of measuring wind velocities by a dual beam modification to conventional (non-Doppler) radars is described. The two beams are disposed slightly to either side of the antenna bore-sight axis. This results in a double-peaked Doppler velocity (frequency) spectrum which is centered on the radial velocity component and whose peaks are spaced proportionally to the transverse velocity component. Such a dual beam Doppler radar is therefore able to measure the complete vector velocity. But the conventional radar can measure only the echo fluctuation spectrum. For any Doppler spectrum, there is a unique fluctuation spectrum, and in the case of the double peaked Doppler spectrum, the fluctuation spectrum has a secondary peak at a frequency corresponding to the peak spacing. Thus, the conventional radar can also measure the transverse velocity component. While spectrum contamination by turbulence and wind shear may cause the peaks to blend with one another, it is shown that the variance of the fluctuation spectrum for the dual beam mode minus that of the single beam is nevertheless uniquely related to transverse target velocity. Various applications to meteorology are described. The techniques are also directly applicable to the measurement of aircraft ground velocity using airborne radars.

1. Introduction

Several investigators have developed methods of measuring air motions in regions of detectable hydrometeors by Doppler radar (e.g., Lhermitte and Atlas, 1961; Gorelik *et al.*, 1962; and Caton, 1963). These methods involve the measurement of the Doppler component of the wind in the direction of the beam. Unfortunately, Doppler radars are not yet generally available to the meteorologist. Furthermore, the maximum radial velocity which can be measured unambiguously by Doppler radar is limited by the radar pulse repetition frequency (PRF). In this paper the feasibility of using a dual beam pattern on a conventional radar for measuring horizontal winds is investigated. The method involves measuring properties of the fluctuation spectrum (or the power spectrum of signal intensity fluctuations) when the axis between the beams is approximately perpendicular to the wind flow. Lhermitte (unpublished) has previously proposed a dual beam pattern with fairly wide beam spacing for determining wind speed. However, the use of a dual beam requires the assumption that the wind be uniform over the region between the beams, an assumption which becomes questionable as the beam spacing is increased.[1]

[1] Actually the general form of Lhermitte's theoretical development is similar in some respects to that derived here. Unfortunately, his unpublished note was unavailable to us and so the present treatment was done independently.

2. Principles

The basic principles are illustrated in Fig. 1. Consider two infinitely thin radar beams radiating from a single transmitter, feeding a single receiver, and separated by angle 2δ. The bisector of the two beams is directed normally to the wind vector W which is assumed constant across the small angle 2δ. It is of course understood that when the wind is not normal to the bisector, we are concerned only with the normal component. Echoes from targets moving across beam 1 with velocity W produce a small radial Doppler frequency shift $f_1 = +(2/\lambda)W\delta$ while those crossing beam 2 produce equal negative shifts $f_2 = -f_1$. The echoes from both arrive at the antenna simultaneously and so the intensity of the resultant echo fluctuates with frequency

$$F = f_1 - f_2 = 2f_1 = (2/\lambda)(2W\delta). \qquad (1)$$

In other words such a dual beam configuration would produce a well defined fluctuation frequency F which is uniquely related to the transverse component of the wind speed W. Note that while the conventional radar cannot measure either Doppler shift f_1 or f_2, it can readily measure the echo fluctuating frequency F. Note also that at small angles 2δ the fluctuation frequency corresponding to even the largest velocities is less than typical values of PRF/2, the maximum which can be measured unambiguously. (Example: $\lambda = 3$cm, $2\delta = 3$ deg. $W = 100$ m sec^{-1}, $F = 350$ cps.)

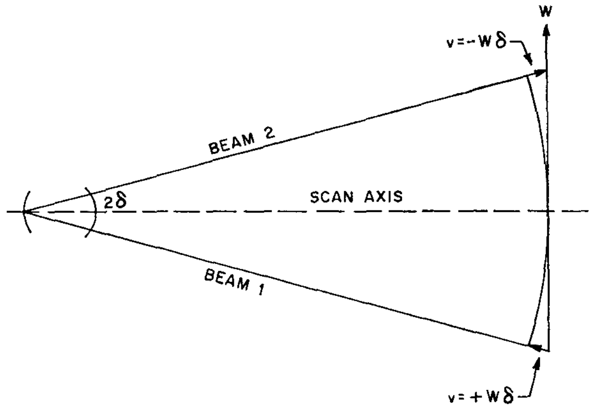

Fig. 1. Schematic representation of dual beam principle.

Of course, actual beams are not vanishingly thin, and so each beam will observe a spectrum of frequencies or velocities. Similarly, the incoherent radar will observe a spectrum of fluctuation frequencies instead of the well defined frequency noted above. In the absence of contaminating effects, the Doppler spectrum observed on each beam corresponding to a uniform wind will be an image of the two-way radiation pattern of the antenna. Since the pattern closely follows a Gaussian function the uncontaminated Doppler spectrum will also be Gaussian with breadth proportional to the cross-wind component. However, natural processes in the atmosphere broaden or contaminate the idealized spectrum; in general the net effect is equivalent to using a broader beam. If one now employs two beams, the uncontaminated Doppler spectrum will be an image of the two-lobed pattern of the antenna and the spacing between the peaks will be a unique function of the crosswind component. Natural contaminating processes will then broaden each lobe of the spectrum, but as long as two discrete peaks can be clearly identified their spacing will be unaffected. Similarly, the fluctuation spectrum will show a secondary peak which is uniquely related to the (uniform) crosswind speed, again provided that the wind field is uniform over the beam separation distance.

It is our purpose then to determine the Doppler and fluctuation spectra in the presence of contaminating factors corresponding to both a single beam and a dual beam configuration and to investigate possible methods of measuring wind velocity from properties of these spectra.

Table of symbols

2δ	beam spacing
λ	wavelength
W	wind speed
$2\theta_0$	beam width of radar
v	Doppler velocity
v_0	Doppler component of wind along axis of one beam $v_0 = W \sin\delta$
u	relative velocity
f	Doppler shift $f=2v/\lambda$
F	fluctuation frequency $F=2u/\lambda$
F_0	fluctuation frequency corresponding to Doppler component $2v_0 \cdot F_0 = 4v_0/\lambda$
$S(v)$	probability of velocity v
$S(u)$	probability of velocity u
σ_0^2	variance due to beam width broadening for a single beam
σ_1^2	variance due to turbulence, wind shear and variable fall speeds of the particles
σ^2	total variance $\sigma^2=\sigma_0^2+\sigma_1^2$
Σ^2	variance in terms of frequency $\Sigma=2\sigma/\lambda$
s^2	variance of the dual beam frequency spectrum in terms of relative velocity
k	$=2v_0/\sigma$
$\rho(\tau)$	autocorrelation function
τ	correlation time

3. Doppler and fluctuation spectra-one beam

Let us first consider the Doppler and fluctuation spectra corresponding to a single beam. The two way intensity pattern of an antenna can be expressed by the two-dimensional Gaussian function (Donaldson, 1964)

$$(G/G_0)^2 = \exp[-2.76\theta^2/2\theta_0^2]\exp[-2.76\phi^2/2\phi_0^2], \quad (2)$$

where G_0 is the antenna gain along its axis, and G is the gain at angles θ, ϕ in the horizontal and vertical directions, respectively, and $2\theta_0$, $2\phi_0$ are the conventional half power beam widths. The Doppler spectrum corresponding to the radial components of particles which are distributed uniformly throughout the beam and all moving with a single velocity perpendicular to the axis of the beam is readily shown to be (Lhermitte, 1963)

$$S(v)dv = \exp[-v^2/2\sigma^2]dv, \quad (3)$$

where σ^2 is the total variance. This is readily converted to a spectrum of Doppler frequencies (f) by use of the relations $f=2v/\lambda$ and $\sigma_f=2\sigma_v/\lambda$ where the subscripts and f and v signify frequency and velocity, respectively. Clearly, the spectrum is the image of the Gaussian shape of the beam because the particles on the axis of the beam, having zero radial velocity component are illuminated most intensely, while those at increasing distances from the beam axis, which produce increasingly large radial components, are illuminated less intensely.

To each Doppler spectrum there corresponds a fluctuation spectrum given by

$$S(u)du = \int_{-\infty}^{\infty} S(v)S(v+u)dv. \quad (4)$$

Here u represents the relative velocity between particles in the Doppler spectrum. While we have expressed the above as a spectrum of relative velocities, it is called a frequency spectrum because a relative velocity u causes a frequency fluctuation $F=2u/\lambda$. Note that when the Doppler spectrum is comprised of just two components $v_1=-v_2$, as in the case of the vanishingly thin beams discussed above, the fluctuation spectrum is comprised of a single line at $u=2v_1$, and a corresponding fluctuation frequency.

The convolution of a Gaussian function is also a Gaussian, but with twice the variance; thus the standard deviation of the fluctuation spectrum is $\sqrt{2}\sigma$. Henceforth we shall therefore speak of the variance or standard deviation of the Doppler spectrum although we may be concerned only with the measurement of the fluctuation spectrum.

If all particles within the pulse volume move with the crosswind W, then it may be shown from (2) and (3) (Hitschfeld and Dennis, 1956) that

$$\sigma_0=0.6\theta_0 W, \tag{5}$$

where σ_0^2 represents the variance of the uncontaminated Doppler spectrum given by (3), and θ_0 is half the beam width in radians. For example, with a one degree beam width and $W=10$ m sec^{-1}, σ_0 is 5.23 cm sec^{-1}, $\sqrt{2}\sigma_0=7.4$ cm sec^{-1}, and the corresponding standard deviation of the fluctuation spectrum of a 3 cm radar would be 4.9 cps. Of course, one could readily widen the beam to increase the spread of the fluctuation spectra. Clearly then, if all the particles in the pulse volume moved with a well defined crosswind W, it would be possible to measure the wind with a single beam simply by measuring the variance of the fluctuation spectrum. Unfortunately, however, other contaminating factors generally prevail so that the beam width broadening may be obscured.

The contaminating factors which contribute to the broadening of the Doppler spectrum are turbulence, wind shear, and the spectrum of intrinsic fall speeds of the particles. Since these processes are independent, the total variance may be expressed as

$$\sigma^2=\sigma_0^2+\sigma_1^2, \tag{6}$$

where σ_0^2 is the variance due to beam-width broadening and σ_1^2 is the sum of all other contaminating factors (Atlas, 1964). Even with a 1-degree beam, measurements by Rogers (1956) show that $\sqrt{2}\sigma$ ranges from about 30 to more than 100 cm sec^{-1}, or about 4 to 14 times that due to the beam width broadening effect for a 10 m sec^{-1} crosswind. It is largely for this reason that we must resort to the dual beam configuration.

4. Doppler and fluctuation spectra—dual beams

Let us now consider two real beams separated by an angle 2δ with the axis between the two beams perpendicular to the horizontal wind. The Doppler shift producing component of the wind along the axis of one beam is v_0 and of the other $-v_0$ where

$$v_0=W\sin\delta\approx\delta W. \tag{7}$$

If each beam has a Gaussian radiation pattern, then by analogy to (3), the Doppler spectrum for the combined beams is given by

$$S(v)=\exp[-(v+v_0)^2/2\sigma^2]+\exp[-(v-v_0)^2/2\sigma^2], \tag{8}$$

where σ^2 is the total variance of the Doppler spectrum, as measured by either of the two lobes. The power spectrum of intensity fluctuations is according to Eq. (4)

$$\begin{aligned}S(u)=\int_{-\infty}^{\infty}&(\exp[-(v+v_0)^2/2\sigma^2]\\&+\exp[-(v-v_0)^2/2\sigma^2])\\&\times(\exp[-(v+v_0+u)^2/2\sigma^2]\\&\qquad+\exp[-(v-v_0+u)^2/2\sigma^2])dv.\end{aligned} \tag{9}$$

Multiplying out the terms we find for the exponents (omitting the multiplier $-1/2\sigma^2$):

$$2\left(v+v_0+\frac{u}{2}\right)^2+\frac{u^2}{2}$$

$$2\left(v-v_0+\frac{u}{2}\right)^2+\frac{u^2}{2}$$

$$2\left(v+\frac{u}{2}\right)^2+\tfrac{1}{2}(u+2v_0)^2$$

$$2\left(v+\frac{u}{2}\right)^2+\tfrac{1}{2}(u-2v_0)^2.$$

Taking the exponentials containing the terms without v outside the integral we find:

$$\begin{aligned}S(u)=C[2\exp(-u^2/4\sigma^2)&+\exp-(u-2v_0)^2/4\sigma^2\\&+\exp-(u+2v_0)^2/4\sigma^2].\end{aligned} \tag{10}$$

By normalizing so that $\int_{-\infty}^{\infty}S(u)du=1$ it is found that $C=1/8\sqrt{\pi}\sigma$. Clearly, the variance of each of the terms in (10) is twice that of the regular Gaussian Doppler spectrum in (3). However, the net variance is quite different and will be treated in Section 6.

Fig. 2 shows the normalized Doppler spectra (top) and the corresponding fluctuation spectra (bottom) for three values of $k=2v_0/\sigma$. The fluctuation spectrum is plotted only for positive values because the detection process makes no distinction between positive and negative frequencies. It contains the same information as the Doppler spectrum, provided that the latter is an even function. For the dual beam Doppler spectrum k represents the approximate spacing between the two peaks and in the fluctuation spectrum it represents the

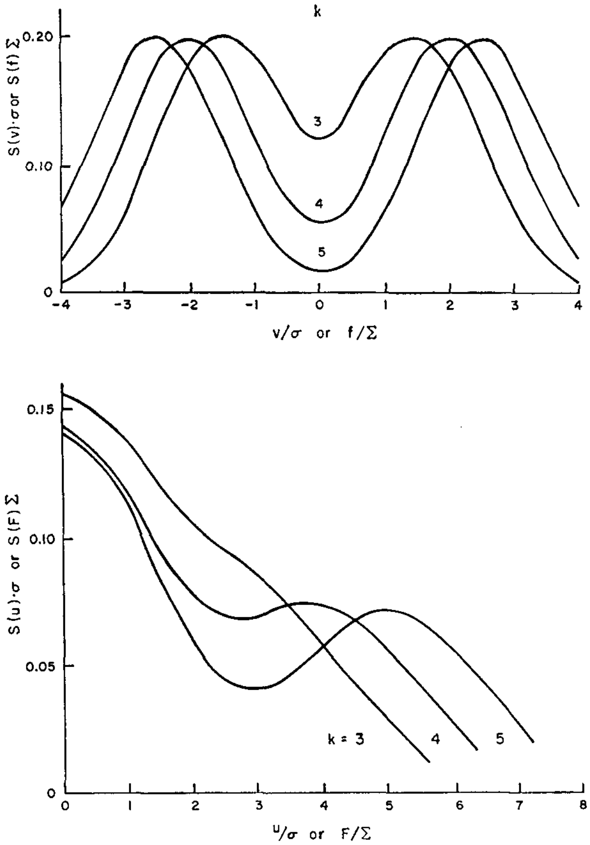

FIG. 2. Doppler spectra (upper) and fluctuation spectra (lower) of the dual beam radar. Peaks in the Doppler spectra are separated by approximately $k=2v_0/\sigma$. In the fluctuation spectrum the secondary maximum occurs at $u/\sigma=k$. In the abscissa u, v and σ are in terms of relative velocity, while f, F and Σ are in terms of frequency.

approximate position of the secondary peak. It is approximate because the addition of energy from the the side of one beam to the peak of the second displaces the positions of the maxima from those which would occur with independent beams. Clearly $2v_0$ must be greater than σ by some factor in order to produce two distinct peaks in the Doppler spectrum or a secondary peak in the fluctuation spectrum. The ratio $k=2v_0/\sigma$ is then an important criterion for characterizing the efficiency of a dual beam system. Note that while the Doppler spectrum for $k=3$ shows two small peaks separated by about 3σ, the fluctuation spectrum shows no distinct secondary maximum although there is a slight bulge in the region near 3σ. For $k=4$ the peaks in the Doppler spectrum are further apart, and a secondary maximum occurs in the fluctuation spectrum at $u=3.90\sigma$; for $k=5$ it occurs at $u=4.90\sigma$. Hence as k increases the relative magnitude of the secondary maximum increases and its position lies closer to $u=k\sigma$.

Choosing conservative parameters such as beam width of 3° and a beam spacing of 6.25° for illustration, we may draw a family of spectra for various wind speeds and different values of σ_1, the contamination broadening. The results are shown in Figs. 3 and 4 for $\sigma_1=30$ and $\sigma_1=60$ cm sec^{-1}, respectively. To each wind speed there corresponds an intrinsic breadth σ_0 given by Eq. (5) and indicated on each curve. The fluctuation frequencies for various wavelengths between 0.86 and 10 cm are shown on the top scales. Note in both Figs. 3 and 4 that as the wind increases, the position of the secondary maximum changes proportionally, always occurring almost exactly at $2v_0=W\delta=k\sigma$. Comparison of Fig. 3 to Fig. 4 shows that for strong winds (30 to 40 m sec^{-1}) there is a distinct secondary maximum the location of which does not change with changes in the value of σ_1. For a wind of 20 m sec^{-1} there is a distinct secondary maximum for $\sigma_1=30$ cm sec^{-1} (Fig. 3) but only a bulge remains for $\sigma_1=60$ cm sec^{-1} (Fig. 4). At 10 m sec^{-1} a bulge is apparent for $\sigma_1=30$ cm sec^{-1} (Fig. 3) but no trace of its exists for $\sigma_1=60$ cm sec^{-1} (Fig. 4). Hence the position of the secondary maximum, when it exists, is a good indicator of the wind speed and the measurement of the crosswind becomes a problem of locating this maximum with precision. For winds of 10 to 20 m sec^{-1} a secondary maximum may be obtained by decreasing the beam width or increasing the beam spacing. It will be noted that we have selected a rather pessimistic situation in using a 3 degree radar beam. This choice is a realistic beam width to be expected with modest antenna size capable of being installed in most aircraft. With a one degree beam the presence of the secondary maximum would not be so readily obscured by contamination. However, we recognized that under conditions of strong wind shear the σ_1 may exceed 60 cm sec^{-1}.

The minimum beam separation for an adequate secondary maximum is at about $k=(2v_0/\sigma)=4$ or from (7) $\sin\delta=2\sigma/W$. In Fig. 5 this minimum beam separation is plotted as a function of wind speed for values of σ ranging from 30 to 100 cm sec^{-1}. This is independent of beam width provided that $\sigma_0=0.6\theta_0 W$ is less than the indicated value of σ. For a 10 m sec^{-1} wind the minimum beam separation ranges from about 7° to 23° whereas for 40 m sec^{-1} it ranges from less than 2° to 6°. Clearly a large beam spacing is desirable for measuring lighter winds, but this would limit the range since in that case both wind and variance may vary appreciably from one beam to the other at longer ranges, or the precipitation may not occupy both beams simultaneously.

Of course the above implies that we must measure the entire fluctuation spectrum and recognize a distinct secondary maximum. However, it will be shown later (Section 6) that it is not necessary either to measure the entire spectrum or to locate the secondary maximum in order to measure the cross wind.

5. The autocorrelation function

In some cases it may be more practical to measure the autocorrelation function $\rho(\tau)$ of the intensity $I(t)$

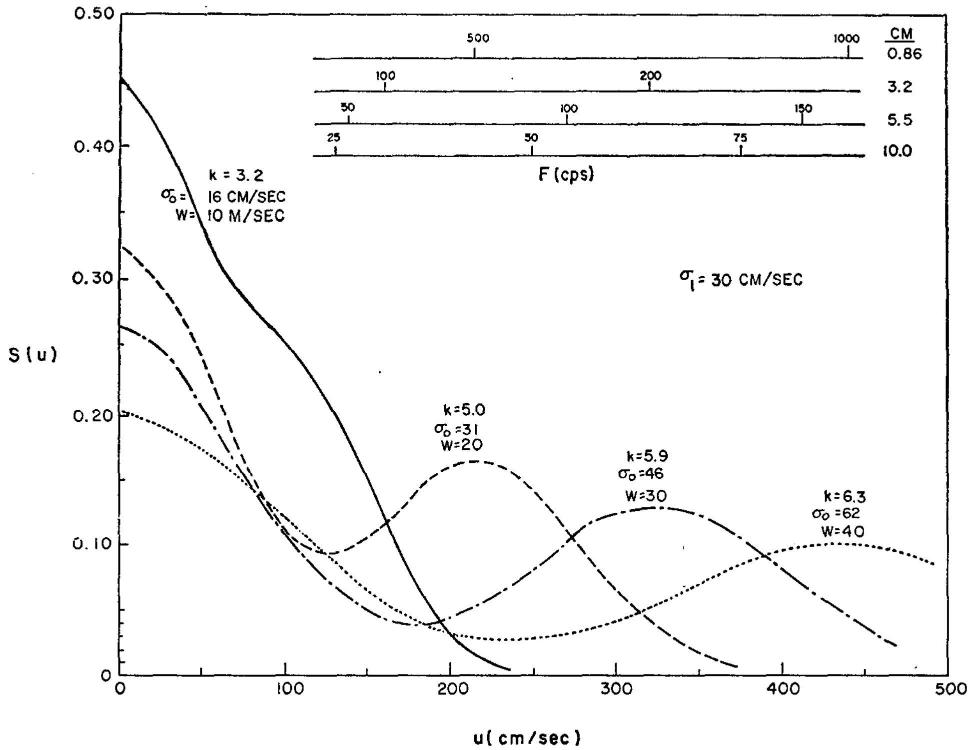

FIG. 3. Fluctuation spectra for different wind speeds for a beam width of 3°, beam spacing of 6.25° and $\sigma_1 = 30$ cm sec^{-1}. The standard deviation σ_0 due to beam width broadening is indicated. The frequency scales for different wavelengths are shown at the top.

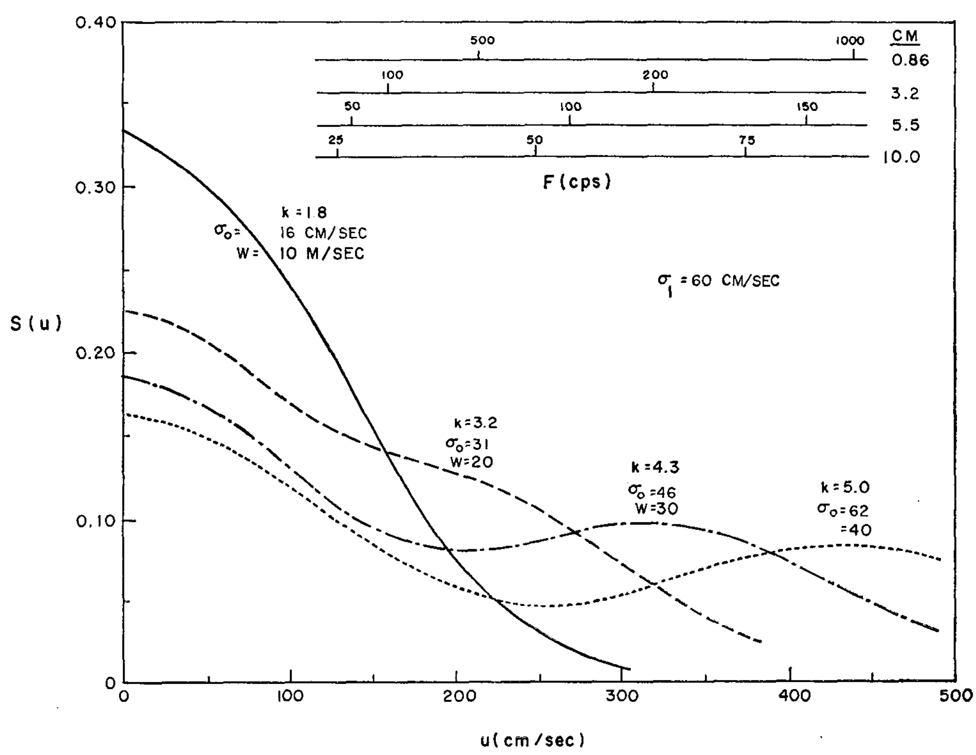

FIG. 4. Fluctuation spectra for different wind speeds for a beam width of 3°, beam spacing of 6.25° and $\sigma_1 = 60$ cm sec^{-1}. The standard deviation σ_0 due to beam width broadening is indicated. The frequency scales for different wavelengths are shown at the top.

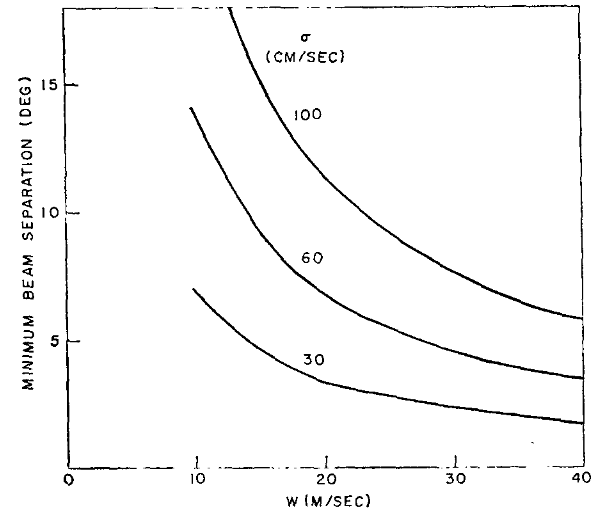

FIG. 5. Minimum beam separation required for an adequate secondary maximum ($k=4$) in the fluctuation spectrum. This is independent of beam width.

rather than the fluctuation spectrum. Let us therefore consider the autocorrelation function corresponding to the fluctuation spectrum. In the case of the vanishingly thin beams, there is a discrete fluctuation frequency $F_0=(4/\lambda)W\delta$ and there will be a discrete line in the autocorrelation function at $\tau=1/F_0$. For real beams the autocorrelation of the signal intensity is given by:

$$\rho(\tau)=\int_{-\infty}^{\infty} S(F)\cos 2\pi F\tau dF, \tag{11}$$

where the fluctuation spectrum is now given in terms of the fluctuation frequency $F=2u/\lambda$:

$$S(F)=\frac{1}{8\sqrt{\pi}\Sigma}\left[2e^{-\frac{F^2}{4\Sigma^2}}+e^{-\frac{(F-F_0)^2}{4\Sigma^2}}+e^{-\frac{F+F_0}{4\Sigma^2}}\right], \tag{12}$$

where $F_0=4v_0/\lambda$ and $\Sigma=2\sigma/\lambda$, Σ^2 is the variance in terms of fluctuation frequency, and σ^2 is a variance in the Doppler velocity spectrum for one beam. Inserting (12) in (11) we may integrate each term separately with the result:

$$\rho(\tau)=e^{-(2\pi\Sigma\tau)^2}(\tfrac{1}{2}+\tfrac{1}{2}\cos 2\pi\tau F_0). \tag{13}$$

The autocorrelation for the single beam is the Gaussian term (Lhermitte, 1963). For the dual beam the autocorrelation is modified by the term in the parentheses which in effect splits the function into two parts with one half unmodified and the other half modified by a cosine function which depends on the beam spacing. The autocorrelation first becomes zero at $\tau=1/2F_0$ and then reaches a secondary maximum before $\tau=1/F_0$. Clearly both the first minimum and the secondary peak are well defined in terms of $F_0=(4/\lambda)W\delta$ and thus may provide a unique measure of wind speed.

Fig. 6 shows the autocorrelation for the single beam and for dual beams with $k=3$ and 4. For $k=4$ the autocorrelation drops off to zero at $\Sigma\tau=0.125$ and then rises to a secondary maximum at about $\Sigma\tau=0.2$. The locations of both the minimum and the maximum provides a means of measuring the wind. For $k=3$ (i.e., at lower wind speeds), $\rho=0$ at $\Sigma\tau=0.167$ and then rises to a very small secondary maximum at about $\Sigma\tau=0.25$; in this case, it may not be possible to determine the location of either the maximum or the minimum with sufficient accuracy.

6. Variance of the fluctuation spectrum

For many practical purposes it is neither desirable nor necessary to measure the entire fluctuation spectrum or the autocorrelation function. Instead it is much simpler to measure only the variance of the fluctuation spectrum using the R-meter (Rutkowski and Fleisher, 1955; Atlas, 1964).

The variance may be determined analytically by substituting (10) in the equation:

$$s^2=\int_{-\infty}^{\infty} u^2 S(u)du-(\bar{u})^2.$$

Recalling that $\bar{u}=0$ since $S(u)$ is an even function we find that

$$s^2=2\sigma^2+2\delta^2W^2. \tag{14}$$

The same result may be derived from (8), for which the variance is readily found to be equal to $\sigma^2+v_0^2$. This is doubled in the convolution process so that the variance of intensity fluctuations corresponding to the Doppler spectrum (8) is $2(\sigma^2+v_0^2)$. In order to demonstrate how the second term is interpreted physically, consider the case of the infinitely narrow dual beams in which there are two values of Doppler velocity at $v=\pm\delta W$, each weighted $\frac{1}{2}$, so that the Doppler variance is clearly δ^2W^2. For the fluctuation spectrum due

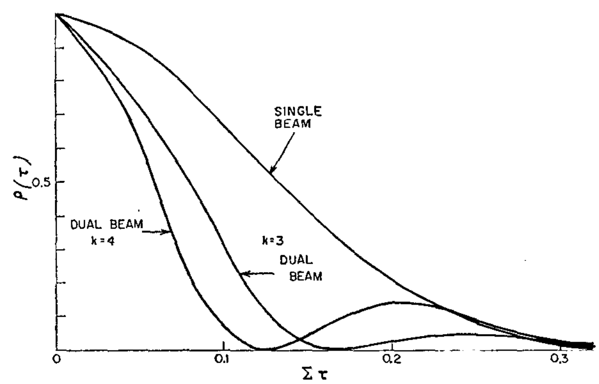

FIG. 6. Autocorrelation functions for the single beam and for the dual beams with $k=3$ and 4. The wind speed may be determined by the position of the minimum or the secondary maximum.

to the convolution process there are two values at $u = \pm 2\delta W$ each weighted $\frac{1}{4}$ and a value at $u=0$ weighted $\frac{1}{2}$. The variance for this configuration is $2\delta^2 W^2$ or twice the Doppler variance.

It is apparent from (14) that the wind may be determined by measuring the variances corresponding to the single beam and the dual beam configuration and subtracting one from the other. Aside from the advantage of simpler implementation than either the measurement of the fluctuation spectrum or the autocorrelation function, it is clear that its use does not depend upon recognizing a well defined peak or minimum in either of these functions. The only requirement is that the difference (dual beam variance minus single beam variance) be significantly above noise in the variance measurements. Furthermore the method has the great advantage of being completely *independent of either beam width or spectral broadening by contamination* provided that these factors do not affect the variance noise. The only important implicit assumption is that the variance be equal on both beams.

The most practical implementation of this method would involve the use of a third axial beam feeding an independent but identical receiver. The axial beam would then be used to measure the single beam variance $2\sigma^2$, while the two off-axis beams simultaneously measure dual beam variance s^2, the difference being recorded automatically as a direct measure of wind.

In Fig. 7 we have plotted the net standard deviation $(s^2-2\sigma^2)^{\frac{1}{2}}$ versus the crosswind, W, for various beam spacings. The choice of beam spacing will then determine the accuracy with which W can be measured in the presence of errors or noise in the net standard deviation. A complete error analysis depends on the spatial variability of σ and v_0 in storms, on which the

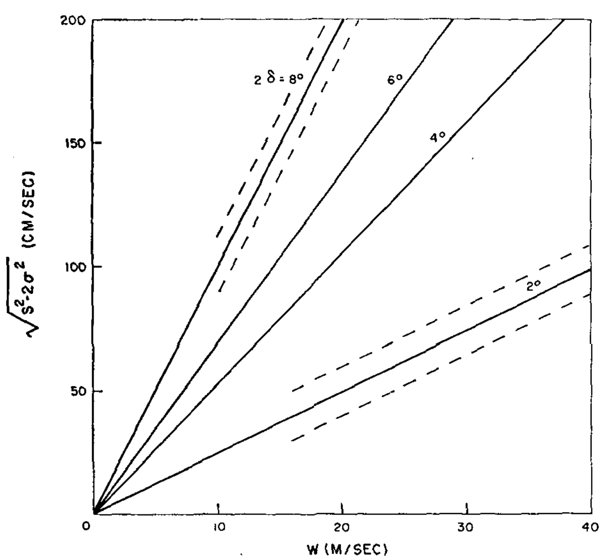

FIG. 7. Wind speed for different beam separations (2δ) as determined from the difference in variances: s for the dual beam spectrum and σ for the Doppler single beam spectrum.

assumption of uniformity in the wind field also depends. If the wind were not uniform within the beam the value of the variance would be increased. Variance measurements by Rogers (1956) at different elevations in various storms indicate an uncertainty in the measurements of about ± 10 cm sec^{-1}. The partition of this uncertainty between electronic and meteorological noise is unknown, but is irrelevant. The corresponding limits of error in wind speed measurement are indicated for the two extreme curves in Fig. 7, assuming that the "noise" is independent of the magnitude of the standard deviation. Thus, a measurement with a 2° beam separation gives a wind speed subject to an error of ± 4 m sec^{-1} while a similar measurement with an 8° separation is subject to an error of ± 1 m sec^{-1}. Clearly one would like to employ the curve with the greatest slope, and thus the smallest errors. However, this implies a large beam spacing which may cause real differences in both wind and variance from one beam to the other.

7. Applications

There are, of course, a number of interesting applications of the methods described above. Foremost of these is its use in the remote measurement of hurricane winds. In this case, one would direct the antenna axis perpendicular to a spiral band; i.e., in the region where the wind may be expected to be nearly normal to the antenna bore-sight axis. However, since the orientation of the spiral band is only a rough indicator of wind direction, measurements would have to be made at other points on either side of the perpendicular direction in order to sense the maximum crosswind.

In use from an aircraft, the preferred mode of operation would be to fly parallel to the spiral band under observation but opposite to the wind with the antenna directed normal to the aircraft velocity vector. In this way, the velocity measurement would be of the sum of aircraft and wind speeds, and can be accomplished with greater accuracy than for the lower speeds, provided that the fluctuation frequency corresponding to this high velocity does not exceed half the pulse repetition frequency. Of course the aircraft ground speed must be subtracted; but this is readily done. Indeed the dual beam system appears to be ideally suited for the measurement of true ground velocity simply by tilting the beams downward to observe ground or sea return and varying the antenna azimuth to sense a maximum in the dual beam variance.

The method may also be applied to vertically pointing "cloud" detection radars such as the AN/TPQ-11. In this case, the plane of the dual beams would be rotated around a vertical axis to sense a maximum variance, whose direction is that of the wind speed. Of course, the receiver is gated so that signals may be measured at a single altitude. However, the gate may be stepped sequentially in range to measure winds at all altitudes occupied by detectable scatterers.

For purposes of measuring the vertical velocities in severe convective storms, the two beams may be arrayed one above the other and the storm viewed from the side. Clearly, in this case, large beam spacings would not be tolerable. However, with the narrow beams commonly employed on vertically scanning radars such as the AN/TPS-10, the AN/MPS-4 or the AN/FPS-6, small beam spacings could be employed. The accuracy of measurement may then be limited, especially for weak vertical velocities (Fig. 7), but the lack of any other suitable means for measuring vertical drafts recommends the method nevertheless.

Finally, it is apparent that the techniques may be used with a Doppler radar to obtain both the radial and transverse velocity components simultaneously, thereby providing a complete measurement of the wind velocity vector. In this case, two basic alternatives are possible. The entire Doppler spectrum corresponding to the two spaced beams may be measured to provide curves such as those in Fig. 2 (top), the only difference being that the spectra will be centered at a velocity corresponding to the radial component instead of zero. The spacing of the two peaks remains a measure of the transverse component. The other alternative is to use a central beam to measure both the radial component in the normal Doppler mode and the single beam variance of the spectrum, while the two off-axis beams are fed to a conventional receiver (without coherent phase detection) for the measurement of dual beam variance as described earlier. In this arrangement, all the required information can be obtained by straightforward instrumentation without measuring the entire spectrum.

Clearly, none of the above applications have yet been attempted, and so we are somewhat hesitant to assure their successful performance. Nevertheless, the possible limitations to successful implementation have been discussed and appear to provide no insurmountable obstacles.

8. Summary

A method has been devised which permits the measurement of the crossbeam velocities of distributed targets (e.g., clouds, precipitation, ground clutter) by a simple modification of the antenna radiation pattern of a conventional (non-Doppler) radar. The technique employs two beams squinted slightly to opposite sides of the antenna bore-sight axis. In its simplest form, the echoes from one beam produce a small positive Doppler frequency shift, while the echoes from the other produce an equal negative shift. The combined echoes at the output of a conventional receiver then fluctuate with twice the Doppler shift and provide a measure of the crossbeam target velocity.

Realistic beams viewing distributed targets do not produce a single Doppler frequency but a spectrum of frequencies. Furthermore in the case of precipitation targets, turbulence and wind shear broaden the spectrum. A detailed analysis is performed to evaluate the nature of the spectrum for realistic beams with and without such contamination. The dual beam pattern is shown to produce a symmetrical double-peaked Doppler spectrum which is centered at the radial velocity of the targets and whose peaks are spaced by an amount directly proportional to the transverse velocity. Thus a dual beam Doppler radar is shown to be capable of measuring the complete horizontal velocity vector of the targets. The contaminating influences of turbulence and wind shear have the effect of broadening the individual beams or their corresponding spectra, and so tend to wipe out the minimum between the individual peaks. However, until that minimum disappears entirely, the spacing between the peaks remains an accurate measure of the transverse target velocity.

In the case of a conventional non-coherent radar which is incapable of measuring the Doppler spectrum, we consider the fluctuation spectrum and the autocorrelation function. For a symmetrical Doppler spectrum there is a unique corresponding fluctuation spectrum and autocorrelation function. The fluctuation spectrum corresponding to the double-peaked dual beam Doppler spectrum is peaked at zero frequency and has a secondary maximum at a frequency equal to the peak spacing, and so also provides a unique measure of the transverse target velocities. A corresponding secondary maximum also occurs in the autocorrelation function. Until these secondary maxima are wiped out by contamination broadening, their positions remain accurate measures of the transverse velocity.

Because of the effects of spectrum contamination, and since the measurement of the entire spectrum (Doppler or fluctuation) or autocorrelation function is complicated, we resort simply to a measurement of the variance of the fluctuation spectrum.

For a single beam the variance is a function of the beam width and contamination broadening and can be lumped together as the intrinsic variance $2\sigma^2$, where σ^2 is the variance of the corresponding Doppler spectrum. For the dual beam pattern, the variance is increased by $2\delta^2W^2$ (in velocity) where δ is the beam spacing and W the transverse target velocity. The difference, which is readily measured by simple instrumentation, is therefore a unique measure of that velocity and is completely unaffected by either beam width or contamination broadening of the intrinsic spectrum, provided that both the wind and the variance are constant over the beam spacing.

A brief discussion is included of various applications of the technique in meteorology. These include the measurement of hurricane winds from both ground-based and airborne radars, the measurement of horizontal wind with a vertically pointing radar, the estimation of vertical velocities in convective storms, and the measurement of the complete wind vector with

Doppler radar. The method is also applicable to the measurement of aircraft ground velocities by simple modification of conventional airborne radars.

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