Experimental Determination of the Radar Cross Sections of Artificial Hailstones Containing Water

JÜRG JOSS
Osservatorio Ticinese della Centrale Meteorologica Svizzera, Locarno-Monti

AND ARMIN N. AUFDERMAUR
Swiss Federal Institute for Snow and Avalanche Research, Weissfluhjoch-Davos

(Manuscript received 28 May 1965, in revised form 15 July 1965)

ABSTRACT

The radar cross sections of particles grown in a hail tunnel (ice spheres with a spongy ice shell) and snow spheres dipped in water (spongy ice throughout) were measured at wavelengths of 3.21 cm, 4.67 cm, and 10 cm. The results were comparable and did not show obvious systematic differences either for the three wavelengths or for the two kinds of particles.

The average normalized cross section versus $\alpha(\alpha=\pi d/\lambda$, where d is the particle diameter and λ the wavelength) is given in the range of $0.4<\alpha<4$ for liquid water contents of 0 per cent (frozen particles), 5, 10, 20, and 30 per cent. For $\alpha>1$, a water content of more than 10 per cent was sufficient to produce a mean cross section equivalent to that of an all-water sphere.

1. Introduction

For the evaluation of radar data obtained from hailstorms, it is necessary to know the backscattering cross sections of the particles involved. Therefore, numerous theoretical computations and experiments have been carried out, confirming each other as long as it has been possible to find a mathematical model corresponding to the one used in the experiments, e.g., ice spheres, water spheres, and ice spheres with a water skin (Atlas *et al.*, 1960; Gerhardt *et al.*, 1961; Herman and Battan, 1961a, 1961b; Stephens, 1961). It has also been shown that, under certain conditions, hailstones may build up a spongy ice deposit, i.e., a framework of ice containing a considerable amount of water (List, 1960). Thus, an attempt has been made to compute the radar cross section of spongy ice (Battan and Herman, 1962). However, in this case, preliminary experiments by Joss and List (1963) did not agree with the theoretical results. Therefore, numerous experimental data over the whole range of interest were needed to determine the cross section of hailstones containing water. The purpose of this paper is to present these data.

The question of the experimental procedure adopted here and especially of the hailstone model could be subjects of serious disagreement. Since it is easier to discuss these points when the results of the experiments are known, the description, evaluation and results of the experiments will be given first.

2. Experimental procedure

Measurements were made on 71 individual ice spheres with spongy ice shells (EW: Eis-Wasser-Gemisch). The ice spheres at 0C were rotated and iced in the air stream of the hail tunnel on Weissfluhjoch (List, 1959) which had a wind speed of 12 m $\sec^{-1}$ and an air temperature of -25C. The particles reached a final diameter between 1.24 and 4.2 cm. The shell thicknesses (from 0.08 to 0.39 cm) and water contents (from 5.6 to 62 per cent) were varied by controlling the free liquid water content and icing time. Up to this point the procedure was very similar to the one described by Joss and List (1963).

In order to calculate the amount of ice in the ice-water mixture, density measurements were made in water at 0C before icing, after icing, and after freezing the stone in the microwave beams. (The calculations were made assuming that the ice of the ice-water mixture had the same density as the shell after freezing.) The amount of water in the mixture was found by subtracting the total amount of ice from the weight of the stone before the radar measurements. The amount of water divided by the total weight of the hailstone, called W_{tot}, varied from 5 to 40 per cent. We shall show that this characterizes most adequately the radar cross section.

Since a restriction of the backscattering results to a specific hailstone model was not desirable, 22 experiments were conducted with snowballs drenched with water (SK: Schneekugel). Spherical snowballs (1.4 to 3.0 cm in diameter) were weighed and hung in the microwave beam. They were then drenched for several seconds in water at 0C and subsequently allowed to freeze during the radar measurement. Afterward, the particles were weighed again, so that the amount of water soaked into them, and W_{tot}, which ranged from 7 to 55 per cent, could be calculated.

The backscattering cross sections of the individual particles were measured in a manner similar to that described by Joss (1964), but this time alternately every two seconds at wavelengths of 3.21, 4.67 and 10 cm. This time-interval was found to be small enough so that information was not lost. The experiments were conducted outdoors in February and March at temperatures around −5C. A blower with a wind speed of about 15 m sec^{-1} was switched on during the experiments to simulate conditions of free fall and to produce regular freezing. For particles made in the hail tunnel, a period of 30 sec elapsed in still air between the determination of W_{tot} and the start of the backscattering measurements. However, this was negligible compared to the duration of the freezing process in the air stream of the blower (about 10 min). In the case of the snowballs, only a few seconds elapsed between the soaking of the snowball in water and the beginning of the cross section measurements.

3. Evaluation

It is assumed that, under the influence of the blower, the freezing of the particles and the resulting decrease of liquid water content proceeded linearly with time. This point will be discussed later. During freezing, the backscattering cross sections of a particle at the three wavelengths were changing more or less continuously. When they reached a constant value for all three wavelengths, it was assumed that all the water was frozen. Fig. 1 shows an example of the time history of the normalized cross sections, δ_n, as they were obtained for hailstone model EW 64.39, similar curves having been obtained for all the other models. Knowing the amount of water at the beginning, it was then possible to plot the normalized cross section versus liquid water content W_{tot} as it decreased during freezing. Fig. 2 shows the plot of five different spheres having approximately the same diameter, EW 64.39 having already been presented in Fig. 1.

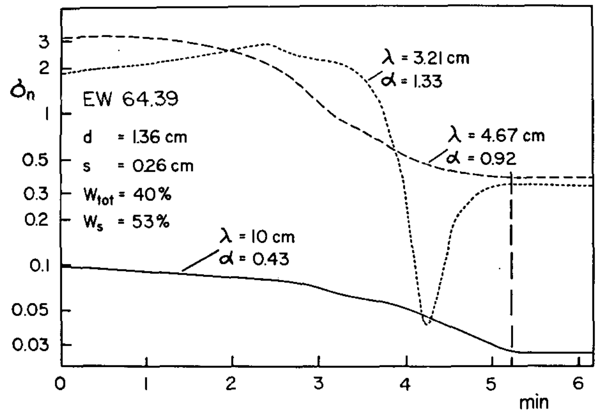

FIG. 1. Normalized cross section δ_n of EW 64.39 (diameter $d=1.36$ cm; shell thickness $s=0.26$ cm; water content of the shell $W_S=53$ per cent; water content of the total stone $W_{tot}=40$ per cent; wavelengths = 3.21 cm, 4.67 cm and 10 cm). After a little over 5 min (broken line), it is assumed that all the water in the stone was frozen.

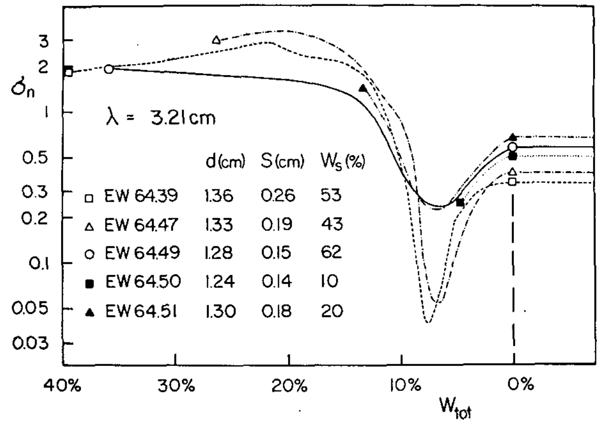

FIG. 2. Normalized cross section δ_n versus liquid water content W_{tot} of five different particles prepared in the hail tunnel and measured during freezing. EW 64.39 is also presented in Fig. 1.

The normalized cross sections, δ_n, at water contents W_{tot} of 0, 5, 10, 20 and 30 per cent were read out of each individual curve, always assuming a linear freezing rate. In Fig. 3 the values for 20 per cent are shown. Because no systematic difference could be seen between the different wavelengths or between the different models (see discussion), the linear average of all the points for a given water content was calculated. This average for $W_{tot}=20$ per cent is shown as the broken line in Fig. 3.

4. Results

The average values of the normalized cross section δ_n versus the size parameter α are presented in Fig. 4 for water contents of 0 per cent (completely frozen), 5, 10, 20 and 30 per cent. The theoretical curve for an all-water sphere, as calculated by Stephens (1961)

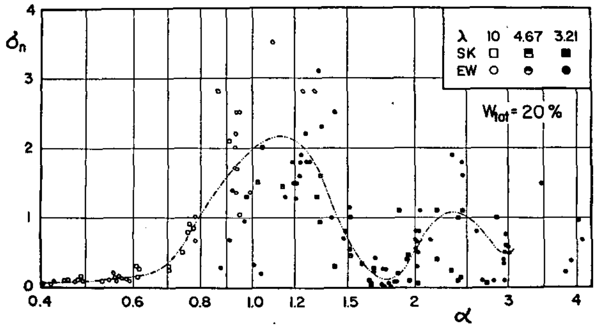

FIG. 3. Normalized cross section δ_n versus size parameter $\alpha(=\pi d/\lambda)$ from cross-section measurements during freezing at $W_{tot}=20$ per cent. The broken line shows the average result, shown again in Fig. 4. The wide scattering of the single measurements demonstrates that the average curve should not be used to determine the cross section of a single stone. The curve is intended to estimate the mean influence of spongy ice on the total cross section as measured by weather radar. SK stands for snow spheres drenched with water (wet core) and EW for particles made in the hail tunnel (solid core).

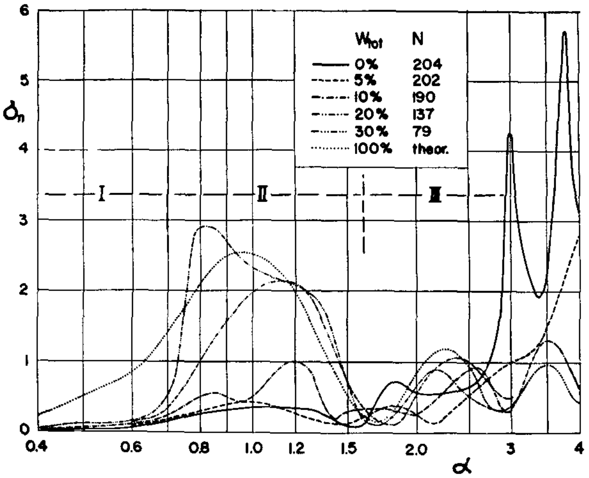

Fig. 4. Averaged normalized cross section δ_n versus size parameter for liquid water contents W_{tot} of 0 per cent (frozen particles), 5, 10, 20 and 30 per cent. For comparison the theoretical result for the water sphere (Stephens, 1961) is added. N represents the number of measurements available for averaging at a given water content.

for $\lambda = 3.21$ cm, is added for comparison. There seem to be three different regions:

$$\text{I: } \alpha < 0.7 \quad \text{II: } 0.7 < \alpha < 1.6 \quad \text{III: } \alpha > 1.6$$

In Region I cross sections were measured having on the average about two, three and 3.5 times the value of the frozen particle for $W_{tot} = 10$, 20 and 30 per cent, respectively. The cross sections measured for the completely frozen particles were on the average 0.35 db lower than the theoretically derived cross sections (Stephens, 1961).

In Region II the presence of water in the particles increases their cross section, except for very low water contents, where a decrease of a factor of ten can be observed (see Figs. 1 and 2). But this decrease will have only little effect on radar investigations in clouds because only a few hailstones of the whole population will have at a given time the right size and the exact water content to produce this minimum. Particles containing a W_{tot} of over 10 per cent frequently showed cross sections which were equal to or even larger than those of the all-water sphere.

In Region III the cross section for the all-water sphere is a good approximation for spongy ice spheres containing 10 per cent or more water. Even 5 per cent of water reduces the cross section considerably.

5. Discussion

Many particles contribute simultaneously to the power received by a weather radar. This tends to average the form factor (directional variation of the backscattering cross section due to a nonspherical form), but not the influence of the dielectric constant, which depends on the liquid water content of the individual particles. Therefore, when measuring single targets of spongy ice, it is reasonable to keep the form factor as small as possible by using spherical particles, and to average over the remaining scattering (uneven water distribution must be considered, too) by measuring many particles. The particles examined had excentricities smaller than 2 mm and were allowed to move in the radar beam, so that different positions were measured and averaged for the evaluation.

The measurements on hailstones with various arrangements of the spongy layers showed that the composition of the inner portion of the sphere (up to $\frac{1}{2}$ of the stone's diameter) had little effect on the cross section. Such a characteristic is probably observed because the inner sphere, with a diameter of $d/2$, occupies only $\frac{1}{8}$ of the volume of the sphere. Therefore, W_{tot} proved to be a representative parameter of the hailstone regardless of the distribution of water within the sphere. When investigating layers of spongy ice, no obvious systematic deviations between different hailstone models were obtained. In fact, four kinds of particles were tested as the measurements were going on during the freezing process, i.e., EW with a solid core and a spongy layer, SK wet throughout, and both subsequently with a solid shell formed during freezing from the outside inwards.

Fig. 2 shows a typically good agreement between five stones with different initial liquid water contents and different shells during freezing. A considerable scattering for the extreme values of the cross section as well as for the minima in Fig. 2 is to be expected since measurements at and near extreme values are most sensitive to the arrangement of the ice-water mixture. The large scatter of the individual points is shown in Fig. 3.

If the detailed cross sections of ice-water mixtures are desired, one should go back to theoretical calculations similar to those carried out by Battan and Herman (1962). But this time the dielectric constant should be determined experimentally, because the discrepancy between their previous calculations and our results is probably only due to their use of the Debye equation for the dielectric characterization of ice-water mixtures. The doubts already raised by Battan and Herman (1962) themselves are confirmed by the results of Cumming (1952), by our own preliminary measurements of the complex dielectric constant, and by the findings of Howorka (1964) at a lower frequency.

The deviation of the dielectric constant of water at different frequencies suggests a wavelength dependence of the backscattering cross sections of ice-water mixtures too. When looking at the cross sections during freezing of different stones for given α-values but different wavelengths, an agreement is found which excludes a large dependence on the wavelength. On this basis it is concluded that the mean effect of dispersion of the dielectric constant is relatively minor and of the same order of magnitude as that observed by Stephens

(1961) for water spheres. However, the experimental procedures and the scattering of the results make it impossible to verify such a small effect.

The most critical point of our measurements seems to be the determination of W_{tot} during freezing. Due to a temperature gradient in the stone, the freezing rate diminishes toward the end. In such a case our linear extrapolation attributes a higher W_{tot} to the measured cross section than was actually present in the stone. This error is largest in the case of big, wet snow spheres. But particles with a solid core and a thin shell of spongy ice can also show a considerable deviation from a constant freezing rate due to the fact that the water tends to accumulate at the bottom of the stone. Theoretical calculations for the measured particles show that an error of the order of 5 per cent W_{tot} must be expected. Although Fig. 2 shows that the cross sections of stones with small water contents correspond well to the curves of initially very wet stones, a deviation could be observed in other cases. Therefore, the water contents given in Fig. 4 tend to be too high. This may be one reason the preliminary study of Joss and List (1963), where the cross sections during freezing were not considered, showed that a W_{tot} of only 5 per cent was sufficient to account for an all-water cross section in the range of $\alpha=1.3$. At the same time, they obtained a rather low W_{tot} due to its determination with a centrifuge.

6. Discussion and conclusions

The determination of the backscattering cross sections of single spongy hailstones is very complex. The resolution and accuracy of the reported measurements are not appropriate to clarify the theoretical aspects. But the results should be of practical use for radar meteorology, since they give an idea of the backscattering behavior of hailstones in a wet growth regime. For example, when evaluating radar data it is important to know that spongy hailstones having diameters between $\lambda/4$ and $\lambda/2$, and containing more than 10 per cent of water, contribute a strong radar echo, which can be an order of magnitude above the all-ice reflectivity. Therefore, a relatively large radar echo must be expected from a hail spectrum containing spongy hail in the above size range. The problems of explaining exceedingly large radar reflectivities by the presence of these particles are discussed in detail by Atlas *et al.* (1960).

The arrangement and evaluation of the experiments are suited to measure the average cross sections shown in Fig. 4. This seems reasonable because one can not expect a homogenous population of ideal and identical hailstones in a thunderstorm. The occurrence of hailstones containing water will not be discussed here. We take it for granted that wet growth happens in nature, although the importance and frequency of this process are not yet clarified.

Acknowledgments. The research reported in this document has been sponsored in part by the Air Force Cambridge Research Laboratories under Contract AF61 (052)-559 through the European Office of Aerospace Research (OAR), United States Air Force, and by the Swiss Commission on Hail Research, the Swiss National Fund, Project No. 2071.

We are indebted to Mr. W. C. Mayes for carrying out the density measurements and to Dr. K. A. Browning of the Air Force Cambridge Research Laboratories for his cooperation in the experiments.

The work was carried out in collaboration between the Osservatorio Ticinese and the Federal Institute for Snow and Avalanche Research, and would not have been possible without the interest and support of the directors of these institutes, Mr. J. C. Thams and Dr. M. R. de Quervain.

REFERENCES

Atlas, D., W. G. Harper, F. H. Ludlam and W. C. Macklin, 1960: Radar scatter by large hail. *Quart. J. R. Meteor. Soc.*, **86**, 468–482.

Atlas, D., K. R. Hardy and J. Joss: 1964: Radar reflectivity of thunderstorms containing spongy hail. *J. Geophys. Res.*, **69**, 1955–1961.

Battan, L. J., and B. M. Herman, 1962: The radar cross sections of "spongy" ice spheres. *J. Geophys. Res.*, **67**, 5139–5145.

Cumming, W. A., 1952: The dielectric properties of ice and snow at 3.2 centimeters. *J. Appl. Phys.*, **23**, 768–773.

Gerhardt, J. R., C. W. Tolbert, S. A. Brunstein and W. W. Bahn, 1961: Experimental determinations of the back-scattering cross-sections of water drops and of wet and dry ice spheres at 3.2 centimeters. *J. Meteor.*, **18**, 340–347.

Herman, B. M., and L. J. Battan, 1961a: Calculations of Mie backscattering of microwaves from ice spheres. *Quart. J. R. Meteor. Soc.*, **87**, 223–230.

——, and ——, 1961b: Calculations of Mie backscattering from melting ice spheres. *J. Meteor.*, **18**, 468–478.

Howorka, F., 1964: Dielektrische Messung des freien Wassergehaltes der Schneedecke. Dissertation, Leopold-Franzen University, Innsbruck, Austria.

Joss, J., 1964: Die Bestimmung der Rückstreuquerschnitte von Eis-Wasser-Gemischen bei einer Wellenlänge von 5.05 cm. *Z. angew. Math. Phys.*, **15**, 509–539.

——, and R. List, 1963: Zur Radarrückstrahlung von Eis-Wasser-Gemischen. *Z. angew. Math. Phys.*, **14**, 376–380.

List, R., 1959: Der Hagelversuchskanal. *Z. angew. Math. Phys.*, **10**, 381–415.

——, 1960: Zur Thermodynamik teilweise wässriger Hagelkörner. *Z. angew. Math. Phys.*, **11**, 273–306.

Stephens, J. J., 1961: Radar cross-sections for water and ice spheres. *J. Meteor.*, **18**, 348–359.