NOTES AND CORRESPONDENCE

An Empirical Relation Describing Leaf-Area Density inside the Forest for Environmental Modeling

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ABSTRACT

The spatial distribution of the leaf-area density is a key parameter in describing the forest canopy characteristics, which has a strong impact on its radiation balance and the mass and energy exchange with the atmosphere as well. The objective of this short study is to define an empirical relation describing the vertical distribution of leaf-area density that can be used for an improved estimation of the turbulent transfer coefficient inside the forest as well as above it. To check the validity of the method proposed, the calculated values are compared with the observations using datasets from eight observational sites located in four different types of forest, covering a broad range of mean values of leaf-area indices between 2 and 18.

1. Introduction

Interaction between the land surface and atmosphere is strongly affected by the vegetation cover. Because more than 23% of the earth's surface is still covered by forest the study of meteorological processes inside and above the forest canopy is interesting, particularly for questions related to the problems of applied meteorology and climatology. For that reason, forest, as an underlying surface, is often met in environmental models of different scales. During the last decades, meteorological and other scientific communities have invested large efforts in order to better describe the interaction between forest trees and their environment. In those efforts special attention was devoted to considering wind profile and its attenuation inside the forest (Fons 1940; Daudet et al. 1999; Sakai et al. 2001), as well as the mass and energy transfer either inside the forest environment or inside the lower part of the atmosphere (Lianhong et al. 1999; Lee 1998; Law et al. 2001a,c). In the processes of interaction between the forest canopy and atmosphere, the amount of leaves and stems and their spatial distribution inside the forest play a crucial role. Consequently, many current vegetation–atmosphere, as well as environmental models require more specific information about the forest structure describing the leaf-area density variation with height in order to result in a better estimation of energy, mass, and momentum exchange (Mix et al. 1994; Zeng and Takahashi 2000). However, in practice it is extremely difficult to measure this quantity inside the forest canopy. Thus, some authors try either to provide alternative methods for measuring (Meir et al. 2000), or estimating (Law et al. 2001b) the leaf-area density L and leaf-area index (LAI) inside the different forest communities. For example, Levi and Jarvis (1999) suggested a simple empirical relation for LAI calculation, based on the inclusion of the forest optical characteristics, but met with difficulties in cases of a heterogeneous forest and higher values of LAI. In contrast to this and similarly established approaches, Gower et al. (1999) emphasized that the direct measurement of LAI distribution is the only reliable method for dense forest canopies having high values of LAI (LAI > 6).

In this note, we suggest an empirical relation describing the vertical distribution of leaf-area density $L(z)$, which is a good approximation even in the case of a heterogeneous forest, including high values of LAI. We checked the validity of assumptions introduced using eight datasets measured in four different types of forest. Results obtained are compared with observed leaf-area density distributions $L(z)$ used by Shaw and Schumann (1992), Dubov et al. (1978), and Kolic (1978).

2. Derivation of empirical relation and comments

The information about the vertical distribution of the leaf-area density $L(z)$ is very important in environmental

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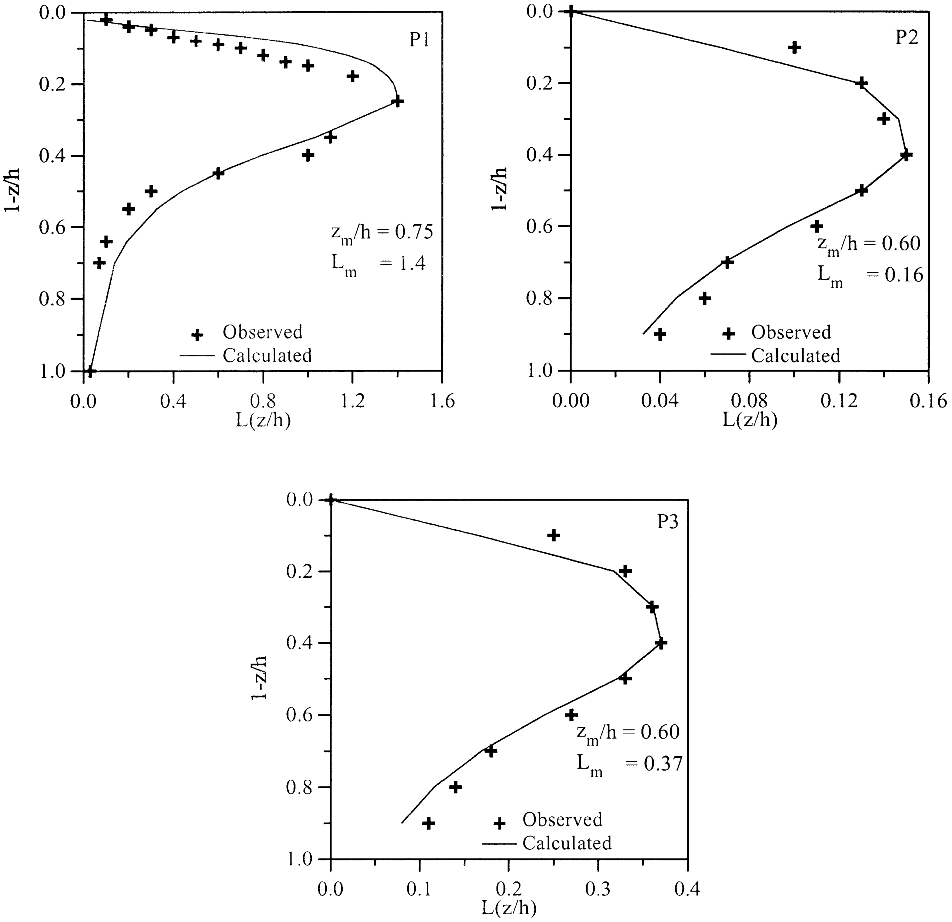

FIG. 1. Simulated (solid line) and observed (crosses) values of leaf-area density distribution $L(z/h)$ in the pine forest (P1, P2, P3).

modeling approaches of different spatial scales. However, this distribution is difficult to be either precisely measured (Meir et al. 2000) or estimated (Law et al. 2001b). Recently, the scientific community dealing with the environmental problems tends to derive empirical expressions for $L(z)$ based on available observational data archives. For example, to our knowledge, the empirically derived expression for $L(z)$ suggested by Levi and Jarvis (1999) is one of the useful expressions among their limited collection. In this note, we will attempt to extend this collection by introducing a simple empirical formula for $L(z)$ using three parameters, that is, tree height h, maximum value of leaf-area density L_m, and corresponding height z_m, which are the key parameters of the forest canopy structural characteristics according to Kolic (1978), Mix et al. (1994), and Law et al. (2001a). This formula could be applied in the broad range of forest canopies if we follow the classification that can be found in Kolic (1978). It is based on use of z_m and h parameters. According to Kolic, all forest canopies can be divided into three groups: 1) $z_m = 0.2h$ (oak and silver birch), 2) $0.2h < z_m < 0.4h$ (common maple), and 3) $z_m = 0.4h$ (pine), where in the bracket is a typical representative.

In order to define an empirical relation for $L(z)$, describing realistically as much as possible of the spatial variation of leaf-area density inside the forest, we have analyzed the eight datasets, including the leaf-area density distribution measured in different forest communities. In further text they will be denoted as follows: (a) pine (P1), silver birch (SB1), oak (01), and common maple (CM1), provided by Kolic (1978); (b) pine (P2) and pine (P3), used by Shaw and Schumann (1992); and (c) common maple (CM2) and common maple (CM3), used by Dubov et al. (1978). In establishing the formula for leaf-area density, the conditions for its design have been derived from the vertical distribution of this quantity keeping in mind its morphological nature. There are two conditions that the functional shape of $L(z)$ has to satisfy: (i) when $z \to 0$ and $z \to h$ then $L(z)$ must tend to zero, and (ii) $L(z)$ should have a maximum at $z = z_m$, that is, $L_m = L(z_m)$, where L_m is available either as a known parameter for certain type and oldness of tree or that can be calculated from LAI. Further, from the

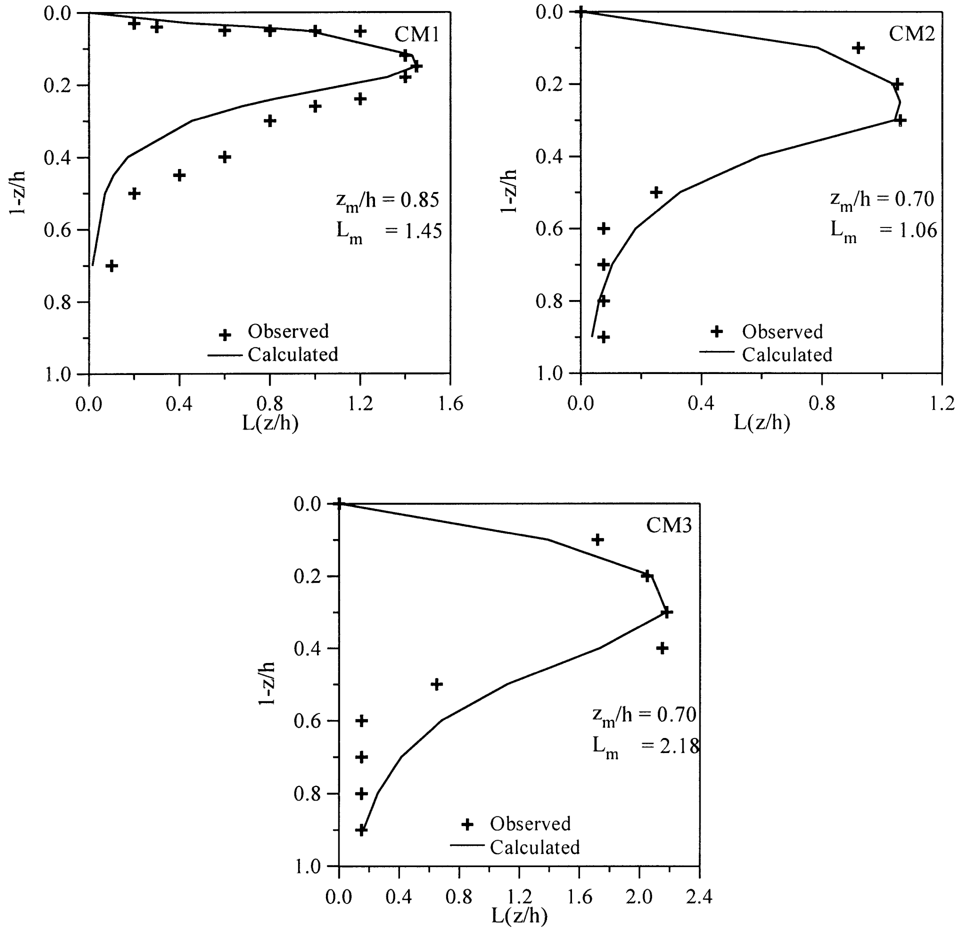

FIG. 2. Simulated (solid line) and observed (crosses) values of leaf-area density distribution $L(z/h)$ in the common maple forest (CM1, CM2, CM3).

reason of simplicity we would rather use the argument $1 - z/h$ instead of the vertical coordinate z, performing the analysis on the interval $1 - z/h \in [1, 0]$ instead of $z \in [0, h]$. In reaching the empirical relation for $L(z)$ we employed a mathematical procedure established by Planck in discovering the formula for pure-temperature radiation, which is described in detail in Whittaker (1961). According to this procedure, the analysis is

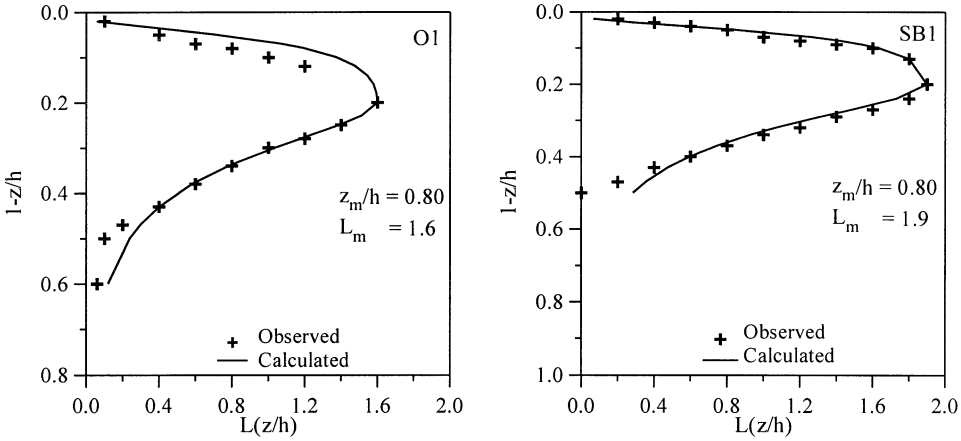

FIG. 3. Simulated (solid line) and observed (crosses) values of leaf-area density distribution $L(z/h)$ in the silver birch (SB1) and oak (O1) forests.

TABLE 1. Error analysis of the simulated leaf-area density for the eight datasets and the dataset obtained by their unification.

Statistical parameter according to Eqs. (3)–(6)	Leaf-area density ($m^2\ m^{-3}$)			
	ν	ν_{BR}	η	$\hat{\eta}$
Dataset				
P1	0.21	0.17	0.46	0.42
P2	0.01	0.01	0.04	0.04
P3	0.03	0.03	0.10	0.09
CM1	0.26	0.25	0.46	0.45
CM2	0.17	0.16	0.36	0.44
CM3	0.31	0.30	0.73	0.90
O1	0.21	0.17	0.51	0.50
SB1	0.12	0.12	0.55	0.54
All datasets	0.16	0.15	0.40	0.42

made in a coordinate system having logarithmic axes. We assumed that our curve can be represented by a linear function for $z < z_m$, while $z \geq z_m$ is used because its shape has a hyperbolic behavior, depending on z_m, L_m, and parameter n. This parameter was found from analysis of minimum root-mean-square error (rmse) for different values of parameter n, including all eight datasets. Results of analysis pointed out that the best choice is $n = 0.5$ for range $z \geq z_m$ and $n = 6$ for $z < z_m$. Therefore, we have found that the leaf-area density distribution $L(z)$ can be properly described by the function having the following form:

$$L(z) = L_m\left(\frac{h - z_m}{h - z}\right)^n \exp\left[n\left(1 - \frac{h - z_m}{h - z}\right)\right],$$

where

$$n = \begin{cases} 6 & 0 \leq z < z_m, \\ \dfrac{1}{2} & z_m \leq z \leq h. \end{cases} \tag{1}$$

As we mentioned above, the value of L_m that we need in Eq. (1) can be obtained, for example, from the forest phenology calendar. However, this parameter can be also obtained from Eq. (2), if the LAI index is available. Thus,

$$\begin{aligned} \mathrm{LAI} &= \int_0^h L(z)\,dz \\ &= \int_0^h L_m\left(\frac{h - z_m}{h - z}\right)^n \exp\left[n\left(1 - \frac{h - z_m}{h - z}\right)\right] dz. \end{aligned} \tag{2}$$

After we calculated this integral we can easily reach the numerical value of L_m. The results of comparison between the calculated and observed values of the leaf-area density distribution are presented in Fig. 1 for pine, Fig. 2 for common maple, and Fig. 3 for silver birch and oak forest. Very simple inspection of these figures indicates that the suggested empirical expression for the $L(z)$ profile quite realistically describes the leaf-area density changes with the height inside the forest canopy. Let us note that such agreement between the calculated and measured values is achieved for different types of forest communities.

In order to quantify the modeled values of leaf-area density we have performed an error analysis of simulated values, based on a method discussed in Pielke (1984) and later used by Mahfouf (1990) and Mihailovic et al. (2000). Following them, we computed several statistical quantities as follows:

$$\nu = \left[\frac{1}{N}\sum_{i=1}^{N}(\Gamma_i - \hat{\Gamma}_i)^2\right]^{1/2}, \tag{3}$$

$$\nu_{BR} = \left\{\frac{1}{N}\sum_{i=1}^{N}[(\Gamma_i - \overline{\Gamma}) - (\hat{\Gamma}_i - \overline{\hat{\Gamma}})]^2\right\}^{1/2}, \tag{4}$$

$$\eta = \left[\frac{1}{N}\sum_{i=1}^{N}(\Gamma_i - \overline{\Gamma})^2\right]^{1/2}, \quad \text{and} \tag{5}$$

$$\hat{\eta} = \left[\frac{1}{N}\sum_{i=1}^{N}(\hat{\Gamma}_i - \overline{\hat{\Gamma}})^2\right]^{1/2}. \tag{6}$$

Here, Γ is the variable of interest (leaf-area density in this study) while N is the total number of hourly data values. An overbar indicates the arithmetic average, while a caret refers to an observation. The absence of a caret indicates a simulated value. The rmse is ν, and ν_{BR} is rmse after a bias is removed. Root-mean-square errors give a good overview of a dataset, with large errors weighted more than many small errors (Mahfouf 1990). The standard deviations in the simulations and in the observations are given by η and $\hat{\eta}$. An rmse that is less than the standard deviation of the observed value indicates skill in the simulation. Moreover, the values of η and $\hat{\eta}$ should be close if the simulation is to be considered realistic. The statistics for the values of the leaf-area density are listed in Table 1. It indicates that the unbiased rmse for leaf-area density is $0.16\ m^2\ m^{-3}$. This value was obtained when the statistical analysis was applied on a single dataset obtained by a combination of all eight available datasets. A comparison of η and $\hat{\eta}$ shows that the difference between them is very small, that is, $0.02\ m^2\ m^{-3}$. This analysis shows that the suggested empirical function simulates the leaf-area density well.

The benefits of the proposed empirical expression for $L(z)$ are evident for a broad range of practical and scientific activities in environmental and closely related sciences, such as the biophysical parameterization of vegetation in atmospheric, ecological, and agricultural models of all scales (Lalic and Mihailovic 1998, 2002a,b; Mihailovic 2002); the designing of biometeorological systems for giving the messages regarding the occurrence of plant diseases (Mihailovic et al. 2001); the prediction of the spread of forest fires (Curry and Fons 1938); and the parameterization of spore, pollen, and particle movement within and just above the canopy

(Pingtong and Hidenori 2000; Pinard and Wilson 2001) for the purpose of biological and ecological models.

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