Modeling Turbulent Flow in an Urban Central Business District

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ABSTRACT

The Realistic Urban Spread and Transport of Intrusive Contaminants (RUSTIC) model has been developed as a simplified computational fluid dynamics model with a $k-\omega$ turbulence model to be used to provide moderately fast simulations of turbulent airflow in an urban environment. RUSTIC simulations were compared with wind tunnel measurements to refine and “calibrate” the parameters for the $k-\omega$ model. RUSTIC simulations were then run and compared with data from five different periods during the Joint Urban 2003 experiment. Predictions from RUSTIC were compared with data from 33 near-surface sonic anemometers as well as 8 sonic anemometers on a 90-m tower and a sodar wind profiler located in the Oklahoma City, Oklahoma, central business district. The data were subdivided into daytime and nighttime datasets and then the daytime data were further subdivided into exposed and sheltered sonic anemometers. While there was little difference between day and night for wind speed and direction comparisons, there was considerable difference for the turbulence kinetic energy (TKE) comparisons. In the nighttime cases, RUSTIC overpredicted the TKE but without any correlation between model and observations. On the other hand, for the daytime cases, RUSTIC underpredicted the TKE values and correlated well with the observations. RUSTIC predicted both winds and TKE much better for the exposed sonic anemometers than for the sheltered ones. For the 90-m tower location downwind of the central business district, RUSTIC predicted the vertical profile of wind speed and direction very closely but underestimated the TKE.

1. Introduction

The ability to understand and model the effects of buildings on atmospheric flow in urban areas has become increasingly important throughout the world as cities grow larger. To be able to quantitatively assess the impact of urban areas on weather and climate, it is necessary to be able to characterize and model the urban boundary layer. The important processes to be modeled depend on the scale being modeled (Rotach et al. 2002). At the smallest scales, the buildings and heat sources are modeled explicitly but require information on the atmospheric conditions, especially turbulence due to mesoscale circulations and boundary layer processes. For mesoscale models, estimates of the heat and momentum fluxes due to urban effects are needed, as is a characterization of the city in terms of surface roughness or drag coefficient. Urban-scale models could be used to provide better information to mesoscale models regarding the local influence of large building areas of a city.

Of particular interest at present is the ability to model the transport and dispersion processes not just at the larger scales but at the street level in cities where the effects of large buildings predominate. This ability applies not only to air pollution concerns, but also to the possibility of the release of toxic gases or aerosols in a city, whether intentional or accidental.

Computational fluid dynamics models have generally been used to model flow around a single object or building (Shah and Ferziger 1997; Calhoun et al. 2004) or through idealized street canyons (Liu and Barth 2002; Baik et al. 2003; Tsai and Chen 2004). While they can be applied to larger areas (Pullen et al. 2005), they require considerable computer resources.

For modeling larger urban areas quickly, rule-based mass-consistent flow models have been developed (Kaplan and Dinar 1996; Pardyjak and Brown 2001). While they are fast and include generalized building...
and street canyon effects, they give only an approximation of the flow in complex urban areas.

The Realistic Urban Spread and Transport of Intrusive Contaminants (RUSTIC) model is a moderately fast urban airflow and dispersion model used for evaluating the effects of accidental or intentional releases of toxic gases or aerosols, training of emergency responders, and aiding in safer building designs. It is being developed as a user-friendly urban flow model that can be coupled to a compatible random-walk particle-based atmospheric transport model (Diehl et al. 1982, 2007). It is capable of explicitly modeling building effects on flow and turbulence, yet can be run on a single-processor PC in a reasonable amount of time (0.5–24 h) depending on the area of coverage and resolution.

RUSTIC is being developed to provide a number of options that will allow the user to select the desired trade-off between fidelity and speed. The grid can be set up to provide finescale resolution in the center and much coarser resolution at the edges. A sequential nested-grid option allows RUSTIC to provide a quick solution at a coarse resolution and then to use that solution to initialize the model at a finer resolution.

For a given computer and atmospheric conditions, the required computation time has been found to be \( t_c = C N_t / \Delta x_{\text{min}} \) where \( t_c \) is the computation time in minutes, \( C \) is an empirical constant estimated from numerous RUSTIC simulations that depends on the computer and atmospheric conditions such as wind speed and stability, \( N_t \) is the total number of grid cells, \( t_s \) is the simulation time in seconds, and \( \Delta x_{\text{min}} \) is the minimum grid cell dimension in meters. For a desktop PC with a 2.66-GHz Pentium IV processor for daytime simulations with a mean wind speed of 6–8 m s\(^{-1} \), \( C \) is about 1.45 \times 10^{-5} \text{ min m}^{-1}. In this case for a grid with 60 \times 60 \times 30 = 108 000 total number of cells and a 5-m minimum grid cell dimension, run and then to use that solution to initialize the model at a finer resolution.

The eddy diffusion coefficient for momentum, \( K_m \), is obtained from the \( k-\omega \) turbulence model under the assumption of isotropy and is given by

\[
K_m = k / \omega. \tag{2}
\]

It is also assumed that

\[
K_H = K_m, \tag{3}
\]

where \( k \) is the turbulence kinetic energy (TKE), \( \omega \) is the specific dissipation, and \( K_H \) is the eddy diffusion coefficient for heat. The values of \( k \) and \( \omega \) are obtained from a \( k-\omega \) turbulence model (Wilcox 1998). This model has been modified to include the generation of TKE by thermally driven processes:

\[
\frac{\partial k}{\partial t} = -U_i \frac{\partial k}{\partial x_j} + \tau_{ij} \frac{\partial U_j}{\partial x_j} - \beta^2 k \omega
\]

\[
+ \frac{\partial}{\partial x_j} \left[ \left( \nu + \sigma k / \omega \right) \frac{\partial k}{\partial x_j} \right] - \frac{k g \partial \theta}{\omega \partial \theta} \frac{\partial \theta}{\partial x_j} \tag{4}
\]

\[
\frac{\partial \omega}{\partial t} = -U_i \frac{\partial \omega}{\partial x_j} + \sigma \frac{\partial U_j}{\partial x_j} - \beta \omega^2
\]

\[
+ \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{k}{\omega} \right) \frac{\partial \omega}{\partial x_j} \right] \tag{5}
\]

where \( \nu \) is the kinematic viscosity of air and the Reynolds stress is given by

\[
\tau_{ij} = \frac{1}{2 \nu} \left( \frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) - \frac{2}{3} \delta_{ij} k \delta_{ij}, \quad \delta_{ij} = \begin{cases} 1, & i = j \smallskip \text{ or } \nu \smallskip 0, & i \neq j \end{cases}. \tag{6}
\]
The closure constants and functions as given by Wilcox are
\[
\alpha = 0.52, \beta = \beta_0 f_\beta, \beta^* = \beta_0^* f_\beta^*, \sigma = 0.5, \sigma^* = 0.5,
\]
\[
\beta_0 = 9/111, f_\beta = 7/8, \beta_0^* = 0.09, \quad (7)
\]
\[
f_{\beta^*} = \begin{cases} 
1, \chi_k \leq 0 \\
1 + 680 \chi_k^2, \chi_k > 0
\end{cases} \quad \text{and} \quad \chi_k = \frac{1}{\alpha} \frac{\partial k}{\partial \omega} \frac{\partial \omega}{\partial x_i} \frac{\partial x_i}{\partial \omega}, \quad (8)
\]

These constants were adjusted as follows from the values given by Wilcox, to provide the best fit to comparisons with data from a wind-tunnel experiment that is described in the next section:
\[
\alpha = 0.35, \beta = \beta_0 f_\beta, \beta^* = \beta_0^* f_\beta^*, \sigma = 0.8, \sigma^* = 0.8,
\]
\[
\beta_0 = 9/111, f_\beta = 7/8, \quad \text{and} \quad \beta_0^* = 0.09. \quad (9)
\]

In its standard form, the k-ω turbulence model overestimates k in the vicinity of a stagnation point in front of a bluff body. This was investigated by Durbin (1996) who described it as the stagnation point anomaly. His solution was later refined by Medic and Durbin (2002). A turbulent time scale was defined such that \( k_m = C_m u^2 T_r \), where \( u \) is a scale velocity and \( T_r \) is the turbulent time scale. For normal conditions it is assumed that \( k = u^2 \) and \( \omega = 1/C_m T_r \). A limit is then placed on the turbulent time scale to ensure that the turbulent stress tensor \( S \) always has nonnegative eigenvalues so that the normal stress components are always positive:
\[
T_r = \min \left( \frac{1}{C_m \omega}, \frac{\alpha}{\sqrt{6 C_m |S|}} \right), \quad \alpha \leq 1. \quad (10)
\]

The recommended range of values for \( \alpha \) is 0.6–1.0. For RUSTIC a value of 1.0 produced the best results. Young and Ooi (2004) found that larger values of \( \alpha \) were required to correctly predict the drag coefficient for Reynolds numbers \( > 10^6 \), as would be the case for atmospheric flows.

This is implemented in RUSTIC by setting
\[
\omega_{\text{limited}} = \max \left( \omega, \frac{\sqrt{6|S|}}{\alpha} \right), \quad (11)
\]
which results in the specific dissipation being increased in front of building walls, thereby reducing the turbulent kinetic energy.

One of the biggest limitations of the two equation k-ω model is the assumption of isotropy for the turbulence. Certainly, in an urban setting near large building walls and in street canyons one would expect there to be many places where the turbulence would be decidedly nonisotropic.

RUSTIC uses a computational grid consisting of six-sided cells with rectangular cross sections. Velocities are computed as normal to the cell wall centers, while scalar values are computed at the cell centers. Five different types of cell walls are recognized, each with their own boundary conditions. There are open boundaries with no restriction to flow. Inflow boundaries and their adjacent cells have fixed values and are specified at initialization. Outflow boundaries and adjacent cells are also specified at initialization, but the outflow varies in response to pressure changes and thus ensures there is no net accumulation or loss of mass during the simulation. The fourth cell boundary type is the solid boundary at the surface, building walls, or rooftops. These boundaries permit no flow across them and restrictions are placed on the turbulence and shear values within the cells adjacent to solid boundaries in accordance with the surface characteristics. The fifth type of boundary is usually used for the upper grid boundary. At these boundaries, the pressure and turbulence values are held constant. For numerical stability purposes, some flow is allowed across the boundary, but it is restricted.

Surface and rooftop characteristics are contained in a two-dimensional array of structures. Each surface structure contains the elevation, surface roughness, friction velocity, heat flux, and surface classification. Building walls are treated differently and a single surface roughness value is applied to all.

When RUSTIC is initialized with a single vertical wind profile, a value of \( u_\theta \) is computed for that profile. The \( u_\theta \) values are assigned to solid boundary surfaces and are used to adjust the TKE, k, and specific dissipation \( \omega \) values in boundary adjacent cells toward a value of \( K_m \) that is consistent with the \( u_\theta \) value. In turn, as the wind and turbulence values change as the model converges, the \( u_\theta \) values are also periodically updated to better reflect the actual conditions in that cell. The relation
\[
\frac{k}{\omega} = K_m = u_\theta \kappa (z_c + z_0) \quad (12)
\]
is used as the “nudging goal” for the values of k, \( \omega \), and \( u_\theta \). The von Kármán constant \( \kappa \) is 0.4, \( z_c \) is the distance of the cell center from the solid boundary, and \( z_0 \) is the roughness length associated with that surface.

b. Grid generation procedure

A grid generation procedure was developed that allows a grid to be quickly produced even for very com-
plex urban regions. A two-step procedure is required. The first step merges the building description data with the digital elevation data to form a regional terrain file. This file is a two-dimensional array of the elevations of the surface or building rooftops. The terrain cells must be smaller than the final grid cells. The second step uses a file that describes the grid structure to generate the model grid from the terrain and building data.

The input data are from digital elevation model data for the terrain and/or building data. The buildings are described in terms of footprints and heights. The algorithm reads the digital elevation model file, if present, and interpolates it into a 2D array. Complex buildings are created using overlapping footprints. To prevent lower building sections from overwriting the taller building sections, the footprints are sorted by rooftop height from lowest to highest. The building data are merged with the elevation array by adding the building height to the surface elevation for the cells that are inside of the footprint.

An “empty” three-dimensional grid is generated by specifying the boundaries in the x, y, and z dimensions. It is then modified by determining whether or not a cell is outside, inside, or partially occupied by a building footprint or the ground surface. If a cell is outside of a building and above ground, then nothing needs to be done. If it is wholly inside a building or below ground, the entire cell is marked as inactive.

The procedure is more complex if the cell is partially occupied by a building. The volume of a cell is reduced by the amount occupied by the building and the areas of the cell walls are also reduced by the amount blocked. If a wall is entirely inside a building, it is marked as a solid boundary. In addition, minimum cell volumes and dimensions can be specified so that if the volume of the cell is reduced below the minimum percentage or the smallest cell dimension is decreased below the minimum dimension, the entire cell will be marked as inactive.

c. Model initialization

Currently, RUSTIC has three options for initialization. The simplest option uses only wind direction and wind speed at a reference height together with a specified surface roughness and displacement height. A one-dimensional boundary layer model is then used to calculate the vertical profiles of wind speed, turbulent kinetic energy, and specific dissipation, taking into account the vertical heat flux in the upwind environment, which may be specified in the input file as simply stable, unstable, or neutral. The resultant profile is then applied uniformly to all grid columns.

The second option is similar but allows a vertical profile of wind direction and speed from a profiler or rawinsonde to be used together with separate values of the surface heat flux upwind (used to compute inflow turbulence) and in the model domain (used for computing buoyancy effects on turbulence and momentum). A still more sophisticated option utilizes an interface routine that allows RUSTIC to be initialized from a fifth-generation Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model (MM5; Grell et al. 1994) output file.

RUSTIC can be run with either steady-state or time-varying boundary conditions. At present only two limited options exist for running RUSTIC in the time-varying mode. One option allows the user to specify a frequency, amplitude, and wavelength for a sine wave separately for each of the velocity components. The second option uses a file of varying wind speeds and directions. A start time and frequency for updating the winds is specified. Each time a new wind direction and speed are read, the one-dimensional boundary layer model is run to produce a new vertical wind and turbulence profile. The new profile is then applied to all of the inflow and outflow boundaries.

3. Comparison with wind-tunnel data

For initial testing and “calibration,” RUSTIC was compared with wind-tunnel data for flow around a cube in a channel from a study by Martinuzzi and Tropea (1993). The cube was confined in a channel with a length of 176H, a width of 24H, and a height of 2H, where H is the dimension of the cube. All three components of the velocity were measured at numerous locations in the channel and in the vicinity of the cube.

To reduce the computation time, the simulation used a grid of 24H in length, 8H in width, and 2H in height. The lateral boundaries were open but top and bottom boundaries were specified as solid. The grid cells were cubes 0.1H on a side. Martinuzzi and Tropea (1993) stated that their tests were conducted for Reynolds numbers ranging from 80 000 to 115 000 based on the channel height of 2H and the mean inflow velocity or bulk velocity, U\textsubscript{b}. The RUSTIC wind velocity was then scaled to provide a Reynolds number of 100 000 based on the channel height. The model was run for 60 s of simulation time, sufficient time for air entering the grid at the start with velocity, U\textsubscript{b}, to pass all the way to the end of the grid. Flow velocity profiles in the channel simulated by RUSTIC were compared with the wind-tunnel measurements.

The initial simulation was made without the correction for the stagnation point anomaly and with the standard values for the closure constants. The plot of TKE along the x axis in the center of the channel in Fig. 1
**Fig. 1.** Cross section through the center of a cube in a channel along the axis of flow. Contours are of TKE (J kg\(^{-1}\)). The simulation was performed using the original version of the Wilcox \(k-\omega\) model. The direction of the airflow is from left to right.

**Fig. 2.** Along-channel (left) nondimensional TKE and (right) velocity (nondimensional) are shown for comparison between RUSTIC and wind tunnel measurements of a cube in a channel using standard constants for the \(k-\omega\) model. The solid line shows the wind tunnel results and the dashed line is the RUSTIC simulation. (a), (b) Vertical profiles in the center of the channel located a distance 0.25\(H\) in front of the cube, where \(H\) is the height of the cube. (c), (d) Vertical profiles in the center of the channel located a distance 0.08\(H\) behind the cube. (e), (f) Vertical profiles in the center of the channel located a distance 3\(H\) behind the cube.
shows the large values of TKE that form around the upwind edge of the cube due to the stagnation point anomaly. The results were then compared with the wind tunnel vertical profile data. For comparison purposes the velocities were normalized by dividing by the bulk velocity, $U_b$, and the TKE values were scaled by dividing by $U_b^2$. Figures 2a and 2b are profiles in the center of the channel $0.25H$ upwind of the front edge of the cube. As can be seen, the peak TKE value predicted by RUSTIC exceeds the measured peak value by an order of magnitude. The along-channel velocities, while in better agreement, nonetheless, indicate a greater slowing of the flow in front of the cube in the RUSTIC simulation relative to the measurements.

The next pair (Figs. 2c and 2d) are vertical profiles in the center of the channel at $0.08H$ behind the downwind edge of the cube. While the model TKE values are closer to the observed values than upwind, they are still too large especially in the wake immediately behind the cube. The model also fails to produce as much acceleration of the flow over the top of the cube and the return flow in the wake is weaker than observed as well.

The last pair (Figs. 2e and 2f) is the vertical profiles at $3H$ downwind of the rear edge of the cube. Even here, the maximum TKE value is about twice that of the observed maximum value. The vertical profile of the along-channel component of the wind is similar in shape to that observed but the velocities are less.
The first step to improving the model performance was to correct the $k-\omega$ model for the stagnation point anomaly using Eq. (12), as discussed earlier. While this eliminated most of the large accumulation of TKE at the front of the cube, the downwind distribution was still unrealistic. By trial and error, it was found that adjusting the values of the $k-\omega$ model constants from the values in Eq. (7) to those in Eq. (9) produced a much better fit of the model data to the observations. The resulting distribution of TKE is shown in Fig. 3.

The difference between the initial solution and the adjusted solution is shown in Fig. 4. The result (Fig. 4a) is a decrease in TKE all around the cube, but primarily in front of the upwind face and over the top. The changes in the TKE distribution also resulted in changes in the velocity distribution as shown by the net difference in the $v$ component of the velocity in Fig. 4b. The biggest change was an enhancement of the recirculation zone and shear over the top of the cube. It also enhanced the circulation at the upwind base of the cube.

Figure 5 shows the comparison of the new model configuration with the wind-tunnel observations. In front of the cube (Figs. 5a and 5b) not only does the turbulence profile now closely match the wind-tunnel data, but so does the velocity profile. Immediately behind the cube (Figs. 5c and 5d) the situation is also greatly improved although some differences still re-
The peak of the turbulence at the rear top of the cube is overestimated, as is the amount of turbulence in the wake area. The acceleration of the air coming over the top of the cube in the simulation matches the measurements very well, but the return flow in the wake of the cube is still less than the measurements.

Farther downwind (Figs. 5e and 5f), the amount of turbulence in the wake is just about right, but the peak value is near the top of the cube in the model and near the bottom of the channel in the wind-tunnel measurements. The biggest difference at this location is in the velocity profiles. The wake effects on the flow appear to persist too far downwind in the model as compared with the wind tunnel.

4. Simulations with Joint Urban 2003 data

a. Setup and initialization

In July 2003, an intensive study of airflow and dispersion was conducted in Oklahoma City, Oklahoma. Descriptions of the locations, instrumentation, and objectives are given in Allwine et al. (2004) and Brown et al. (2004). Data from 3D sonic anemometers, sodars, and boundary layer towers were used for comparison with RUSTIC simulations.

A building database was obtained from the University of Oklahoma’s Geography Department. This consisted of building footprints and heights accurate to about 1 m. The database covered a 1.6 km × 1.6 km area centered on the Oklahoma City central business district (CBD).

Simulations of the CBD were run with RUSTIC for five different release periods on three separate days. Three of the times were during the day and two were at night. The daytime periods were 1600–1700, 1800–1900, and 2000–2100 UTC 9 July 2003. The nighttime periods were 0400–0500 UTC 19 July 2003 and 0800–0900 UTC 28 July 2003. For each of the time periods, two simulations were run: one using a single grid covering most of the CBD, and the other using three nested grids ranging in coverage from areas well outside the CBD down to a localized area centered in the CBD.

The grid for the simulations covered a region of 1200 m × 1200 m × 200 m in the CBD (Fig. 6) with grid cell numbers of 190 × 190 × 33. Grid cells were 5-m cubes near the surface in the center of the grid expanding to 10-m cubes at the corners of the grid. The simulations were initialized using the vertical wind profiles from the Argonne National Laboratory (ANL) mini-sodar, which was located near the southern edge of the grid (see Fig. 6 for location).

For the nested-grid simulation, three grids with horizontally uniform grid cells of 45, 15, and 5 m were used (Fig. 7). The area of coverage for the outermost grid was 3600 m × 3600 m × 300 m with the number of grid cells being 80 × 80 × 32. Grid dimensions in the vertical varied from 5 m near the surface to 15 m at the top. The middle grid was 2100 m × 2100 m × 300 m in size with 140 × 140 × 32 grid cells. Grid dimensions in the vertical varied from 5 m near the surface to 15 m at the top. The innermost grid was 800 m × 800 m × 200 m with 160 × 160 × 33 grid cells. Grid dimensions in the vertical varied from 5 m near the surface to 10 m at the top. The nested-grid simulations were initialized using the wind profile from the Pacific Northwest National Laboratory (PNNL) sodar in Wheeler Park (see Fig. 6 for the location).

For each simulation, the sodar data were averaged over a 1-h period to obtain the initialization profile. In addition to the vertical wind profile, RUSTIC is able to utilize surface heat flux data to estimate the initial vertical profile of TKE and the specific dissipation as well.
as to compute the buoyancy production of TKE in the simulation.

Upwind heat flux values, used for initialization, were obtained from the Indiana University sonic anemometers located on an 80-m tower in a suburban area, 6 km upwind of the CBD (Grimmond et al. 2004). For the CBD, heat flux values were computed from 33 sonic anemometers within the CBD. All values were averaged over a 1-h period. The inflow wind and turbulence profiles were obtained by using a 1D boundary layer model in conjunction with the upwind suburban heat flux values. The CBD heat fluxes for the most part represented street canyon values that were probably much lower than the rooftop values. However, only the mean CBD heat flux was used in the grid, as differential heating effects are not incorporated into RUSTIC at present. The heat flux values used for each simulation are listed in Table 1.

The main task of the RUSTIC model is to supply winds and turbulence information to a dispersion model. In this paper only the airflow capabilities of RUSTIC are examined. A companion paper (Hendricks et al. 2007) evaluates the use of RUSTIC for dispersion modeling. To determine how well RUSTIC is able to model winds and TKE near the surface, the model results were compared with sonic anemometer data from the CBD.

Wind speed, wind direction, and TKE values were computed from 33 different sonic anemometers in the CBD operated by Dugway Proving Grounds (DPG), ITT Corporation (ITT), the University of Utah (UU), and Arizona State University (ASU). The locations of the anemometers used for the comparisons are shown in Fig. 8. Data were averaged for 1-h periods corresponding to the simulation times. The wind direction from the 250-m level of the PNNL sodar was taken as indicative the prevailing wind direction. For the five cases, the wind direction ranged from 184° to 216°. The PNNL sodar 250-m wind direction and speed for each of the simulation periods is listed in Table 2. The vertical profiles of wind speed and wind direction measured by both the ANL minisodar and the PNNL sodar are plotted in Fig. 9.

b. Results

Since RUSTIC was run with steady-state boundary conditions and the turbulence field changed little after 5 min of simulation time, the effects of the large-scale turbulence are not included in the model results. The \( k-\omega \) turbulence model generates turbulence associated with the buildings and shear at the surface. An investigation was done to find for what averaging time scale the measured TKE correlated the best with the RUSTIC predictions, as well as with the least error. TKE values for each period were computed using

\[
\tau = 0.5\left( \langle u'^2 \rangle + \langle v'^2 \rangle + \langle w'^2 \rangle \right),
\]

Table 1. Heat flux values used to initialize RUSTIC for each of the five simulation cases. Upwind heat flux values were used to characterize the model inflow and the grid values were used as lower boundary values in the model grid.

<table>
<thead>
<tr>
<th>Simulation date</th>
<th>Time (UTC)</th>
<th>Upwind HF (W m(^{-2}))</th>
<th>Grid HF (W m(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Jul 2003</td>
<td>1600</td>
<td>153</td>
<td>68</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>1800</td>
<td>240</td>
<td>75</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>2000</td>
<td>162</td>
<td>79</td>
</tr>
<tr>
<td>19 Jul 2003</td>
<td>0400</td>
<td>–23</td>
<td>–5</td>
</tr>
<tr>
<td>28 Jul 2003</td>
<td>0800</td>
<td>–18</td>
<td>11</td>
</tr>
</tbody>
</table>
where $e$ is the TKE and $u'$, $v'$, and $w'$ are the turbulent components of the $u$, $v$, and $w$ velocity components. The angle brackets represent turbulence time-scale averaging and the overbar is for averaging over the observational time period of 60 min. Turbulence values were computed using time-scale averaging periods of 9.4, 18.8, 37.5, 60, 75, 150, 300, 600, 1200, and 2400 s. Only the daytime simulations for 9 July 2003 were used in this investigation, since, as will be shown later, RUSTIC did not estimate the TKE values well for the nighttime simulations. For each sonic anemometer location, the RUSTIC-estimated TKE was compared with the measured TKE. For each averaging period, the correlation coefficient, $r$, and the relative mean error, RME, were computed, where

$$RME = \frac{|O - P|}{O}. \quad (14)$$

In Eq. (14), $O$ is the observed value and $P$ is the predicted value. The comparison is shown in Fig. 10. For the single-grid simulations, the maximum value of $r = 0.66$ for an averaging time of 150 s while the minimum error was RME = 0.44 for 60-s averaging times. The results for the nested-grid simulations showed that the maximum value of $r = 0.58$ for averaging times from 37.5 through 150 s and the minimum error RME = 0.38 for an averaging time of 37.5 s. Overall, the correlations were near the maximum for averaging times from 60 through 300 s. The error was near the minimum for times from 37.5 through 150 s. For the purposes of this study it was decided to use an averaging time of 75 s for computing the TKE from the sonic anemometer measurements. For this case, $r = 0.65$ for the single-grid simulations and $r = 0.58$ for the nested-grid simulations. Similarly for the single-grid simulations, RME = 0.44 and for the nested-grid simulations RME = 0.40.

The next part of the investigation was to compare the winds and TKE for all simulations with the sonic anemometer measurements. The metrics used for the comparing wind speed and TKE were the correlation

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (UTC)</th>
<th>Wind direction (°)</th>
<th>Wind speed (m s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Jul 2003</td>
<td>1600</td>
<td>216°</td>
<td>8.9</td>
</tr>
<tr>
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<td>10.5</td>
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<td>10.6</td>
</tr>
<tr>
<td>19 Jul 2003</td>
<td>0400</td>
<td>205°</td>
<td>12.4</td>
</tr>
<tr>
<td>28 Jul 2003</td>
<td>0800</td>
<td>196°</td>
<td>16.5</td>
</tr>
</tbody>
</table>
Fig. 9. Observed sodar wind profiles for the simulations. The solid line is the ANL sodar and the dashed line is the PNNL sodar at (a) 1600 UTC 9 Jul 2003, (b) 1800 UTC 9 Jul 2003, (c) 2000 UTC 9 Jul 2003, (d) 0400 UTC 19 Jul 2003, and (e) 0800 UTC 23 Jul 2003.
coefficient \( r \), the normalized mean square error NMSE, the fractional bias FB, factor of 2 FAC2, and factor of 5 FAC5 (Hanna 1989; Chang and Hanna 2004). For the wind direction the scaled averaged angle (SAA) was used (Calhoun et al. 2004). These metrics were computed separately for the single-grid and the nested-grid simulations using all five of the intensive observing period (IOP) release periods. Table 3 lists the comparison for the wind and Table 4 lists the comparison for the TKE. Overall, the single-grid simulations compared slightly better with the sonic anemometers than did the nested-grid simulations. This was illustrated best by the NMSE. For the single-grid simulations the wind speed NMSE = 0.59 and for the nested-grid simulations NMSE = 0.70. For the TKE this was even more pronounced with the single-grid NMSE = 0.45 and the nested-grid NMSE = 1.04. For wind direction, the single-grid SAA = 46° and the nested-grid SAA = 52°. The correlation in wind speed between RUSTIC and the sonic anemometers was very low. In the single-grid case \( r = 0.30 \) and in the nested-grid case \( r = 0.29 \). For TKE there was more difference with \( r = 0.44 \) for the single-grid cases and \( r = 0.10 \) for the nested-grid cases.

Next, the dataset was split into day simulations and night simulations to see what effect this had on the comparisons. For the nighttime conditions, one might expect the isotropic turbulence assumption of the \( k-\omega \) model to break down as vertical mixing becomes suppressed. As shown in Table 1, the upwind sensible heat flux for the nighttime cases were indicative of stable conditions upwind of the CBD and more nearly neutral conditions within the CBD. During the daytime periods, conditions were more unstable especially upwind of the CBD. The results for the comparisons between day and night are listed in Tables 5 and 6. For the wind, there is little difference but for the TKE there is considerable difference. For the single-grid simulations in the daytime cases \( r = 0.65 \) and NMSE = 0.31 while for the nighttime cases \( r = 0.13 \) and NMSE = 0.86. Considerable difference was found for the fractional bias as well. In the daytime cases the TKE is underestimated (FB = -0.28) and in the nighttime cases the TKE is overestimated (FB = 0.34). Similar differences were found for the nested-grid simulations.

Another aspect that was investigated was the effect of the sonic anemometer location relative to the proximity of large buildings. In this case, the daytime simu-

### Table 3. Wind speed and direction comparison metrics for all five simulation periods.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single grid</td>
<td>0.587</td>
<td>0.300</td>
<td>-0.022</td>
<td>0.524</td>
<td>0.866</td>
<td>45.7</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.697</td>
<td>0.286</td>
<td>0.004</td>
<td>0.506</td>
<td>0.829</td>
<td>51.9</td>
</tr>
</tbody>
</table>

### Table 4. TKE comparison metrics for all five simulation periods.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single grid</td>
<td>0.449</td>
<td>0.435</td>
<td>-0.065</td>
<td>0.610</td>
<td>0.963</td>
</tr>
<tr>
<td>Nested grid</td>
<td>1.037</td>
<td>0.104</td>
<td>0.145</td>
<td>0.591</td>
<td>0.909</td>
</tr>
</tbody>
</table>

Fig. 10. Variation of (left) the RME and (right) the correlation coefficient \( r \), with the TKE averaging time for three daytime RUSTIC simulations for 9 Jul 2003. The solid line is from the RUSTIC single-grid simulations and the dotted line is from the RUSTIC nested-grid simulations.
lations were divided into two groups referred to as exposed and sheltered. The exposed anemometers are indicated in Fig. 8 as gray symbols and the sheltered anemometers are the black symbols. The exposed anemometers were selected for their placement either upwind or downwind of the CBD in locations where they were not completely surrounded by high buildings. Two of them were located on the roof of a parking garage 15.5 m above street level and the rest were either 2.5 m above street level or on lamp posts about 8 m above street level. The sheltered anemometers were selected because of their location within the CBD where the tall buildings provided shelter from the south-to-southwest winds that occurred during the periods of this study. The mean horizon angle varied from 7° to 29° with a mean of 17° for the exposed anemometers and 11° to 56° with a mean of 37° for the sheltered anemometers. The overlap in horizon angles results from decisions made concerning the location of the anemometer relative to the nearby buildings in conjunction with the wind direction. In some cases, an anemometer was considered exposed even though it was located close to a building, if the building wall was nearly parallel to the wind and there were no blockages in the upwind direction. Conversely, in some cases anemometers with generally low horizon angles were specified as blocked because they were in locations where the local flow was greatly affected by buildings upwind of the location.

This time the difference between the two cases was significant not just for the TKE comparisons, but also for the wind speed and direction comparisons (Tables 7 and 8). For the single-grid simulations and the exposed anemometers for the wind speed, \( r = 0.73 \) and NMSE = 0.29, while for the sheltered anemometers, \( r = 0.05 \) and NMSE = 0.94. For the nested-grid simulations, the results were similar: for the exposed anemometers, \( r = 0.59 \) and NMSE = 0.36 and for the sheltered anemometers, \( r = 0.07 \) and NMSE = 0.97.

Wind direction is also simulated much better in the exposed locations: SAA = 35° and 40°, respectively, for the single-grid and nested-grid simulations. While for the sheltered locations, SAA = 52° and 55° for the single-grid and nested-grid simulations, respectively.

The TKE values also differed dramatically between these two sets of anemometer locations. For the exposed locations, \( r = 0.77 \) and NMSE = 0.20 for the single-grid simulations, while for the sheltered locations the results were \( r = 0.21 \) and NMSE = 0.50.

A minisodar operated by UU was located on top of a parking garage in the CBD (see Fig. 6) about 200 m north (downwind in these simulations) of a 150-m-high building. The vertical profile from the UU sodar was compared with four RUSTIC simulations. Sodar data for the last simulation time on 28 July 2003 were only available for the lowest 20 m, so that case was not used for the comparison. The vertical wind profile comparisons are depicted in Fig. 11. The best results are for the 1800 and 2000 UTC 9 July 2003 simulations and the worst is for 0400 UTC 18 July 2003, a nighttime case.

Both the single-grid and the nested-grid simulations compared well for the afternoon cases, 1800 and 2000 UTC (see Tables 9 and 10), with NMSE values in the 0.01–0.02 range, \( r \) values in the 0.96–0.99 range, and SAA values from 3.8° to 10.4°. For the morning case

### Table 5. Wind speed and direction comparison metrics for both daytime and nighttime cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-grid day</td>
<td>0.640</td>
<td>0.266</td>
<td>0.073</td>
<td>0.490</td>
<td>0.888</td>
<td>45.2</td>
</tr>
<tr>
<td>Single-grid night</td>
<td>0.623</td>
<td>0.237</td>
<td>0.064</td>
<td>0.606</td>
<td>0.894</td>
<td>50.2</td>
</tr>
<tr>
<td>Nested-grid day</td>
<td>0.730</td>
<td>0.229</td>
<td>0.178</td>
<td>0.520</td>
<td>0.837</td>
<td>48.5</td>
</tr>
<tr>
<td>Nested-grid night</td>
<td>0.878</td>
<td>0.231</td>
<td>-0.107</td>
<td>0.500</td>
<td>0.848</td>
<td>64.0</td>
</tr>
</tbody>
</table>

### Table 6. TKE comparison metrics for both daytime and nighttime cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-grid day</td>
<td>0.308</td>
<td>0.652</td>
<td>-0.281</td>
<td>0.602</td>
<td>0.949</td>
</tr>
<tr>
<td>Single-grid night</td>
<td>0.864</td>
<td>0.132</td>
<td>0.343</td>
<td>0.621</td>
<td>0.985</td>
</tr>
<tr>
<td>Nested-grid day</td>
<td>0.335</td>
<td>0.581</td>
<td>-0.353</td>
<td>0.684</td>
<td>0.980</td>
</tr>
<tr>
<td>Nested-grid night</td>
<td>2.057</td>
<td>0.230</td>
<td>0.804</td>
<td>0.455</td>
<td>0.803</td>
</tr>
</tbody>
</table>

### Table 7. Wind speed and direction comparison metrics for both sheltered and exposed cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposed cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single grid</td>
<td>0.294</td>
<td>0.731</td>
<td>-0.322</td>
<td>0.694</td>
<td>1.000</td>
<td>35.0</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.357</td>
<td>0.591</td>
<td>0.295</td>
<td>0.750</td>
<td>0.944</td>
<td>39.6</td>
</tr>
<tr>
<td>Sheltered cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single grid</td>
<td>0.937</td>
<td>0.050</td>
<td>-0.086</td>
<td>0.371</td>
<td>0.823</td>
<td>51.5</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.974</td>
<td>0.065</td>
<td>0.122</td>
<td>0.371</td>
<td>0.758</td>
<td>54.6</td>
</tr>
</tbody>
</table>

### Table 8. TKE comparison metrics for both sheltered and exposed cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>NMSE</th>
<th>( r )</th>
<th>FB</th>
<th>FAC2</th>
<th>FAC5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposed cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single grid</td>
<td>0.196</td>
<td>0.768</td>
<td>-0.019</td>
<td>0.694</td>
<td>0.889</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.266</td>
<td>0.714</td>
<td>-0.262</td>
<td>0.694</td>
<td>1.000</td>
</tr>
<tr>
<td>Sheltered cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single grid</td>
<td>0.498</td>
<td>0.207</td>
<td>-0.495</td>
<td>0.548</td>
<td>0.984</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.414</td>
<td>0.177</td>
<td>-0.417</td>
<td>0.677</td>
<td>0.968</td>
</tr>
</tbody>
</table>
(1600 UTC) and especially for the nighttime case (0400 UTC), the results were not as good. For these cases the nested-grid simulations were better than the single-grid simulations, NMSE = 0.11 and 0.35, FB = 0.11 and 0.01, \( r = 0.97 \) and 0.98, and SAA = 8.5° and 19.2° for the nested grid and single grids, NMSE = 0.69 and 0.02, FB = −0.03 and 0.07, \( r = −0.16 \) and 0.97, SAA = 18.39° and 19.21°.

The final comparison was with the 90-m pseudo-crane tower operated by LLNL and equipped with eight sonic anemometers at heights from 7.5 to 83.2 m above the ground (Lundquist et al. 2004). The location of the tower is indicated by a star in Fig. 7. As the location of the tower was north of the grid domain used for the previous simulations, two special simulations were performed to make a comparison possible. Only the 1800 UTC 9 July 2003 simulation, which had produced the best results in the previous cases, was run.

The single grid for the simulations covered a region of 806 m × 1667 m × 301 m in the CBD with grid cell numbers of 120 × 250 × 38 (Fig. 12). Grid cells were 6-m cubes near the surface in the center of the grid expanding to 15-m cubes at the corners of the grid. The simulation was, as before, initialized using the vertical wind profile from the ANL minisodar (Fig. 9b). In the nested-grid case, the 45- and 15-m resolution grid simulations run previously were utilized and for this case only another 5-m grid simulation was required. It had the same domain size as before but was relocated to include the pseudo-crane tower (Fig. 13). The results are shown in Fig. 14 with the numerical comparisons summarized in Table 11. The simulated winds matched the measurements quite closely. For wind speed, NMSE = 0.02 and 0.01, FB = −0.06 and −0.10, and \( r = 0.976 \) and 0.996 for the single-grid and nested-grid simulations, respectively. The wind directions were quite close as well: SAA = 6.0° and 3.4° for single grid and nested grid, respectively. However, the model did

<table>
<thead>
<tr>
<th>Simulation date</th>
<th>Time (UTC)</th>
<th>NMSE</th>
<th>FB</th>
<th>( r )</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Jul 2003</td>
<td>1600</td>
<td>0.35</td>
<td>0.14</td>
<td>0.85</td>
<td>19.21</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>1800</td>
<td>0.01</td>
<td>−0.07</td>
<td>0.98</td>
<td>3.84</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>2000</td>
<td>0.01</td>
<td>−0.07</td>
<td>0.97</td>
<td>8.40</td>
</tr>
<tr>
<td>18 Jul 2003</td>
<td>0400</td>
<td>0.69</td>
<td>0.03</td>
<td>−0.16</td>
<td>18.39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulation date</th>
<th>Time (UTC)</th>
<th>NMSE</th>
<th>FB</th>
<th>( r )</th>
<th>SAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 Jul 2003</td>
<td>1600</td>
<td>0.11</td>
<td>0.11</td>
<td>0.97</td>
<td>8.52</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>1800</td>
<td>0.02</td>
<td>−0.12</td>
<td>0.99</td>
<td>7.24</td>
</tr>
<tr>
<td>9 Jul 2003</td>
<td>2000</td>
<td>0.02</td>
<td>0.10</td>
<td>0.96</td>
<td>10.35</td>
</tr>
<tr>
<td>18 Jul 2003</td>
<td>0400</td>
<td>0.17</td>
<td>0.20</td>
<td>0.80</td>
<td>9.16</td>
</tr>
</tbody>
</table>
not generate nearly as much TKE downwind of the CBD as was observed as shown by the FB = −0.60 and −1.10 for the single-grid and nested-grid simulations, respectively. In addition, the correlation was poorer for the TKE than for the wind: $r = 0.53$ and 0.76 for the single-grid and nested-grid simulations, respectively.

c. Discussion

As might be expected, RUSTIC wind predictions agree best with the measurements in exposed locations near the ground or above the rooftops and fare poorest for locations within the street canyon areas. Comparing model winds and observed winds at street level in areas surrounded by tall buildings can be difficult. The observed winds for locations in street intersections may be bimodal so that the mean wind may actually result in a direction and a speed that are not representative at all of the “normal” wind at that location. For this study the anemometer winds were vector averaged, giving result-

![Fig. 12. The 6-m-resolution grid used for the single-grid comparisons with the LLNL crane tower sonic anemometers. The grid domain size is 803 m × 1798 m.](image)

![Fig. 13. The 5-m-resolution grid used for the nested-grid comparisons with the LLNL crane tower sonic anemometers. The grid domain size is 800 m × 800 m.](image)

...
ly resolve the flow as the canyons were generally only five cells wide.

Predictions of TKE not only were poor for sheltered locations in the street canyons but also for the nighttime simulations. While problems in the street canyons would be expected for all of the reasons above, the difficulty with the nighttime simulations is not as obvious. In addition, RUSTIC tends to underpredict the TKE during the daytime and overpredict the TKE at night.

For the exposed locations in the daytime the TKE was underpredicted by $FB = -0.02$ for the single-grid simulations and $FB = -0.26$ for the nested-grid simulations compared to $FB = -0.50$ and $FB = -0.42$, respectively, in the sheltered locations. The greater underprediction for the sheltered locations likely goes along with the general problem of predicting local winds in the sheltered areas. Although the underprediction for the exposed locations in the CBD is relatively small, for the most exposed location of all, the LLNL crane tower location, the TKE was underpredicted by $FB = -0.60$ and $-1.10$. It seems likely that the $k-\omega$ model simply fails to produce enough turbulence in the CBD due to shear around the buildings. In addition, much of the building-induced turbulence in the model may have dissipated by the time it reached the LLNL crane tower location. However, the tower is located slightly west of the main tall building region of the CBD and the 250-m wind direction from the PNNL sodar for that simulation was $184^\circ$. It seems likely that the TKE at the LLNL crane tower was not being strongly influenced by the CBD at that time and the larger TKE values that were measured there resulted from boundary layer processes not modeled well by RUSTIC, such as buoyancy and differential surface effects.

The poor correlation between the model and observations for the TKE values combined with the overprediction of the magnitude of the TKE is somewhat puzzling. One possibility is that more stable conditions at night suppress the vertical turbulence component so that the $k-\omega$ model, with its assumption of turbulence isotropy, is not able to produce a realistic simulation for those conditions. However, examining the vertical wind speed profiles for the nighttime cases (Figs. 11e and 11f) reveals that they are distinctly different from the daytime profiles (Figs. 11a–c). Whereas the daytime profiles are reasonably close to a logarithmic profile, the nighttime profiles are nearly linear. The linear profile likely is the result of the formation of the low-level jet (LLJ) at night in the Oklahoma City area as studied by Lundquist and Mirocha (2006). The peak velocity of

<table>
<thead>
<tr>
<th>Wind Speed</th>
<th>Wind direction</th>
<th>TKE</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMSE</td>
<td>FB</td>
<td>r</td>
</tr>
<tr>
<td>Single grid</td>
<td>0.02</td>
<td>-0.061</td>
</tr>
<tr>
<td>Nested grid</td>
<td>0.01</td>
<td>-0.096</td>
</tr>
</tbody>
</table>
this jet was generally between 300 and 600 m above the ground. The initialization routine for RUSTIC provides a least squares logarithmic curve fit to the observed wind speed profile. While this produced good results for the daytime simulations, it resulted in poor fits at night as in Fig. 15, which compares the ANL minisodar wind speed profile with the RUSTIC profile at the same location. Compared to a more linear profile, the logarithmic curve will result in more shear near the surface and less shear higher up. This is a possible explanation for why RUSTIC produces too much TKE at the surface in the nighttime simulations.

5. Conclusions

In general, RUSTIC produced good simulations for the daytime cases. It was able to closely reproduce the vertical profile of the wind speed both downwind of the city as well as downwind of a large building. Near ground level, it was able to provide good predictions for locations not surrounded by tall buildings.

Some problems did occur in the street canyons and street intersection areas near the ground within the central tall building area. While this is the most difficult area to model, it is also the most important to be able to model. Further investigation needs to be done to determine whether this is a problem with the $k-\omega$ turbulence model or a grid resolution problem. This will be tested by using nested-grid simulations to produce more detailed simulations of localized street canyons to see if any improvement results.

The TKE predictions were poor for the nighttime simulations. This also needs to be investigated further to determine if it is a deficiency in the $k-\omega$ turbulence model, a result of the way that the RUSTIC input winds are initialized, or the result of not accounting for larger-scale boundary layer processes. The initialization procedure definitely needs to be improved to handle cases in which the vertical profile of the wind speed is linear rather than logarithmic.

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