Response of Superpressure Balloons to Vertical Air Motions[1]

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1. Introduction

For years, meteorologists have used superpressure, constant-volume balloons as tracers of air motions (Booker and Cooper, 1965; Angell, 1963) utilizing the assumption that these balloons provide a first approximation to the three-dimensional air flow.

The accuracy of superpressure balloons in representing air trajectories is an important consideration. Since the balloon will seek to fly at a constant density surface, it resists vertical air motions to an extent governed by the restoring and drag forces on the balloon. It is desirable to have as large a vertical drag coefficient as possible so the balloon will follow the vertical air currents closely.

By using certain measurable balloon constants a balloon's trajectory through a sinusoidal wave can be simulated considering the drag and restoring forces which act on the balloon.

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2. Properties of superpressure balloons

A superpressure balloon is designed to float near a constant altitude and is made of an inelastic material so that its volume is essentially constant with excess internal pressure. The balloon will seek a density level where the weight of the air displaced by it is equal to the weight of the balloon, inflating gas and all the attachments. If the balloon is displaced above that density level, it becomes negatively buoyant and seeks to return to its original level. The only way for the balloon to be displaced from the equilibrium level is by vertical air currents, a change of mass of the assembly, or a change of volume of the balloon itself. If the balloon is designed such that its mass cannot change and it is free of leaks, it can be expected to remain at its equilibrium level indefinitely except for temporary displacements due to vertical air movements.

3. Balloon simulation logic

A FORTRAN program has been written to simulate the action of a superpressure balloon within a sinusoidal wave. The program assumes a constant balance between the drag force D, that would pull the balloon away from its equilibrium level H_0, and the restoring force R, that would return the balloon to H_0. Inertia effects are negligible here due to the very small accelerations within the considered waves. This assumption would probably not be justified in simulating balloon reaction to turbulence.

The drag force acting on the balloon is given by

$$D = R = \tfrac{1}{2}\rho_a C_d A W_d^2, \quad (1)$$

where ρ_a is air density, C_d, the drag coefficient, A, the cross sectional area, and W_d, the air flow relative to the balloon.

The restoring force is given by

$$R = V(\rho_a - \rho_g) - W_b, \quad (2)$$

where V is the volume, ρ_a, the density of free air, ρ_g, the density of the inflating gas, and W_b, the weight of the balloon and attachments. For a given balloon and equilibrium altitude H_0, the restoring force may be expressed in terms of the displacement of H_0.

With a given sinusoidal wave and a maximum displacement H_x, the instantaneous displacement of an air parcel H_a may be expressed as

$$H_a = H_x \sin\frac{2\pi vt}{\lambda}\zeta, \quad (3)$$

where v is the horizontal wind velocity, t is time, and λ, the wavelength. The vertical air velocity, W_a, may be obtained by differentiating (3) to obtain

$$W_a = H_x \cos\frac{2\pi vt}{\lambda} dt. \quad (4)$$

The balloon vertical velocity, W_b, may be defined as $W_a - W_d$, and is found from Eqs. (1) and (4), to give

$$W_b = H_x \cos\frac{2\pi vt}{\lambda} dt - \left(\frac{2R}{\rho_a C_d A}\right)^{\frac{1}{2}}. \quad (5)$$

The program assumes that an air parcel and the balloon start from the same point on the H_0 level and move downstream through a sinusoidal wave at the same horizontal rate. The trajectory of the air parcel is specified. The corresponding balloon trajectory is found by solving Eq. (5) at many points and integrating with time.

The effects of adiabatic changes of ρ_a and the small known changes of V with superpressure were considered in our program.

4. Determining the balloon constants

Solutions have been obtained for the 0.17 m^3 pillow balloons developed at the Pennsylvania State University and described by Booker and Cooper (1965).

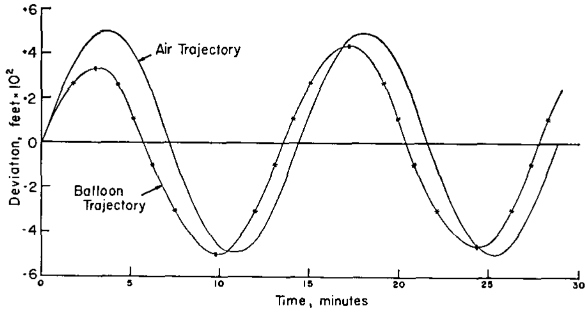

FIG. 1. Balloon and air parcel trajectories.

Solutions for superpressure balloons of other shapes, sizes, and materials would require a determination of the balloon constants in Eq. (1) before the above solutions could be used.

The drag coefficient for balloons may be determined for non-turbulent air by accurately tracking balloons approaching their equilibrium level in still air. The tracking data yield measurements of W_b which are assumed to be equal to W_d in still air. The area perpendicular to the flow A may be measured from the shadow cast by the inflated balloon held in sunlight. The drag force D was again assumed to be equal to the restoring force R in still air. The air density is calculated from a knowledge of temperature and pressure at the balloon altitude. C_d may be calculated at several points as the balloon approaches its H_0. A sufficient number of measurements should yield a valid C_d.

Methods for determining the elastic stretch of superpressure balloons have been given by Booker and Cooper (1965). These figures should be obtained before the calculations in the previous sections are conducted.

5. Results

Over eighty different combinations of wind speed and wave amplitude covering all usual lee wave conditions have been used in a study of balloon response. An air parcel trajectory and the resulting balloon trajectory are shown in Fig. 1. The results show that the balloon will underestimate the amplitude and indicate the maxima and minima too far upstream. A second notable result is that the amplitude and measured wavelength of the first wave are in serious error but they become more accurate for later waves. The curves presented in this and later sections are not necessarily accurate for balloons with different characteristics but similar curves can be easily constructed.

6. First wave error

We have shown in the previous section that there are certain incongruities between the trajectory of an air parcel and that of a superpressure balloon. The most prominant deviation occurs within the first wave. To examine this deviation in more detail, solutions

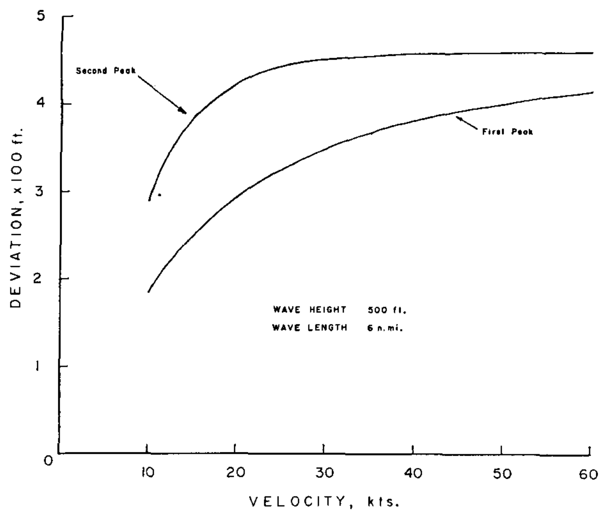

FIG. 2. Wave height error vs. wind speed.

were obtained for various wind speeds while amplitude and wavelength are held constant at 500 ft and 6 n mi, respectively. Fig. 2 shows these results. For a 60 kt wind speed the balloon will attain an altitude of 417 ft at the first peak, then reach 462 ft at the second. However, if a 10 kt wind exists, the balloon will only achieve an altitude of 184 ft in the first wave and then reach 289 ft in the second wave.

This indicates that the assumption that the balloon will follow the air trajectory is more in error for low wind speeds than for high. This error is also greater for longer wavelengths and smaller amplitudes. It is worthwhile at this point to mention again that we are dealing only with a steady state sinusoidal wave.

Although it is not within the scope of this article to study non-steady state or turbulent flow, it is our opinion that trajectory discrepancies under these conditions would be similar to those found in the first wave of steady state flow.

7. Error nomogram

The errors that are presented in the above two sections can now be incorporated into a nomogram for ease in determining the position of an air parcel with respect to the position of a superpressure balloon. When the

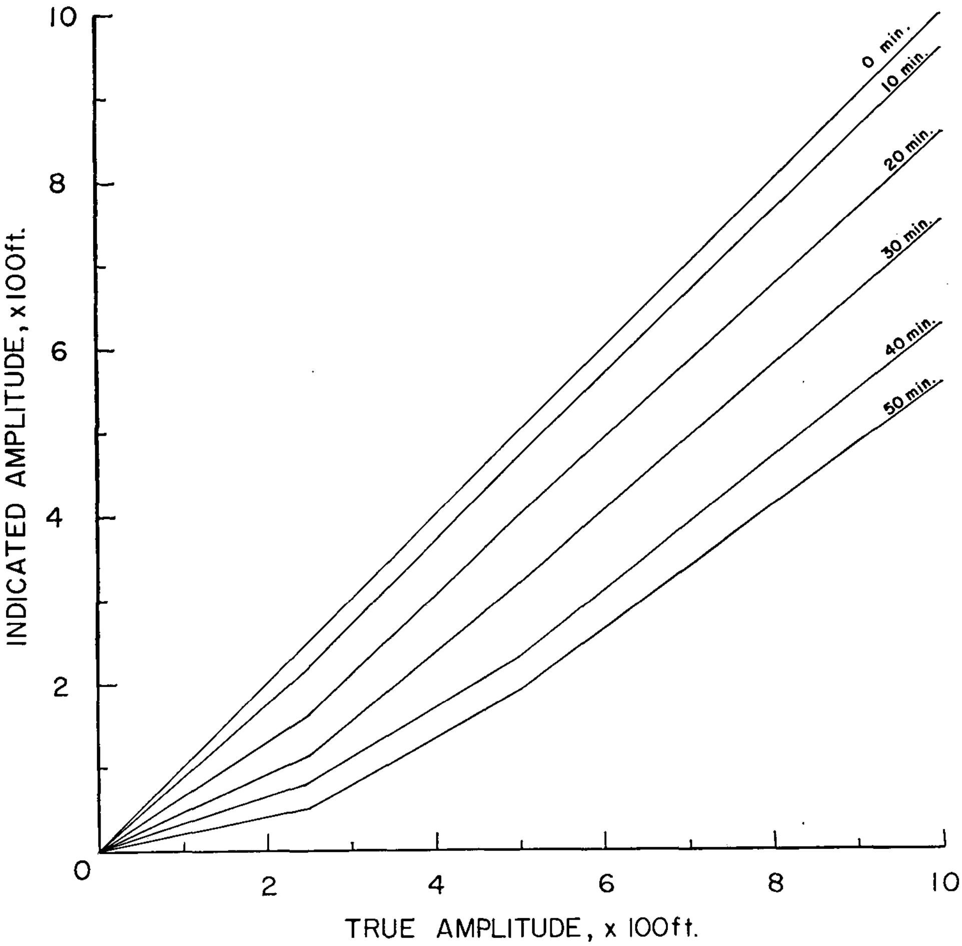

FIG. 3. Error nomogram.

horizontal wavelength is divided by the horizontal wind velocity, the result yields the time that the balloon will travel through a given wave. When the indicated wave amplitude of the balloon is plotted along the ordinate and the actual air parcel amplitude is plotted along the abscissa for the above times of travel through the wave, a nomogram can be constructed as shown in Fig. 3.

To use this chart, one enters the ordinate with the indicated amplitudes of the balloon and proceeds horizontally to the time of travel through the wave, then downward to the actual streamline amplitude. For example, if the radar plot shows a 500-ft wave amplitude and the time of travel through the wave is 20 min, the actual streamline amplitude is 610 ft.

8. Obtaining the complete air trajectory

The above procedure can be used to obtain a graphical means of correcting balloon measurements to obtain streamline amplitude. This procedure can be used directly on a balloon trajectory plot to obtain streamline amplitude without further computations. However, errors may still be present if the wave amplitude varies along the streamline. The following procedure may be used to compute the air parcel trajectory in all cases but this requires a machine to process the data.

The restoring force R may be expressed as a function of displacement of a given balloon from its equilibrium level H_0. The restoring force and the appropriate balloon constants can then be used in Eq. (1) to yield W_d, the flow of air relative to the balloon. Since W_b would be available from a measurement of the balloon trajectory, W_a may be obtained from

$$W_a = W_b + W_d.$$

Execution of the above steps at a sufficient number of points along the balloon trajectory would yield a vertical air velocity profile. Integration of these velocities would yield an approximate air streamline.

9. Further development

The foregoing procedure allows accurate measurement of vertical (and horizontal) air motions by correcting for the errors caused by a restoring force. This is a simple procedure which is applicable and, we feel, sufficient for mountain waves.

The procedure does not consider errors caused by inertia effects on the balloon trajectory and variations of air density on the scale of the balloon, which would be further complicating factors in the use of superpressure balloons in a convective atmosphere. Further development of our model to include these effects would be straightforward.

REFERENCES

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