NOTES AND CORRESPONDENCE

Note on the Relationship Between Total Precipitable Water and Surface Dew Point

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1. Introduction

Recently Reitan (1963) achieved a correlation of 0.98 between mean monthly total precipitable water and mean monthly surface dew point from a total of 540 observations. He found that the line of regression relating these mean values was of the form

$$\ln U = A + Bt_d, \tag{1}$$

where U is precipitable water (cm), t_d is the dew point temperature (°F), $A=-0.981$ and $B=0.0341$. Reitan estimates the standard error from regression at 10 per cent. The above result prompted Bolsenga (1965) to follow the same procedure to determine the regressional relations between mean daily and hourly observations of total water vapor content and surface dew point. Although the correlations he estimates are lower (0.85 to 0.80 for the mean daily and hourly observations, respectively), he found that the same basic relationship given by Eq. (1) exists. Bolsenga, however, estimates the coefficients of Eq. (1) to be $A=-1.249$ and $B=0.0427$ for the mean daily observations, and $A=-1.288$ and $B=0.0384$ for the hourly observations. Estimates of explained variance reveal that about 96 per cent of the differences in total water vapor are related to differences in dew point for the mean monthly values, 72 per cent for mean daily values and 64 per cent for hourly values. Since a unique relationship between total water content and surface dew point depends on the variability of the moisture profile, the tendency towards lower correlations with decreasing time intervals is to be expected.

In this note, an equation is derived which is of the same form as that obtained statistically by Reitan and Bolsenga with the exception that the A coefficient is not a constant, but a variable which is clearly dependent upon the actual moisture profile. Climatic values of the moisture profile are obtained which will enable good estimates of the total water vapor content to be obtained from the surface dew point regardless of latitude or season.

2. Development

For most practical purposes, the relation between vapor pressure e and dew point may be given by the empirical formula of Tetens (1930),

$$e = E_0 \times 10^{(\alpha t_d-\beta)/(t_d+\gamma)}, \tag{2}$$

where $E_0=6108$ dyne cm^{-2}, $\alpha=7.5$, $\beta=238.1$F and $\gamma=395.1$F when t_d is expressed in °F. Since the surface mixing ratio w_0 is given to a good approximation by

$$w_0=\epsilon\frac{e_0}{p_0}, \tag{3}$$

where p_0 is the pressure at the earth's surface and $\epsilon=0.622$, it follows from Tetens' relation that

$$w_0=\frac{\epsilon E_0}{p_0}\times 10^{(\alpha t_d-\beta)/(t_d+\gamma)}. \tag{4}$$

Ordinarily the moisture content is greatest at the earth's surface and decreases to a value of zero at the top of the atmosphere. Regardless of the exact moisture profile, with the proper choice of a power λ for a given atmospheric situation, the average decrease of moisture through the entire atmospheric column may be described by the power law

$$w=w_0\left(\frac{p}{p_0}\right)^{\lambda}. \tag{5}$$

From the definition of the vertical mean mixing ratio $\bar{w}$, it follows that

$$\bar{w}=\frac{1}{p_0}\int_0^{p_0} w_0\left(\frac{p}{p_0}\right)^{\lambda}dp=\frac{w_0}{\lambda+1}. \tag{6}$$

Since it can be shown that the total precipitable water U is related to the mean mixing ratio by

$$U=\frac{p_0}{g}\bar{w},$$

where g is the acceleration of gravity, then

$$U=\frac{p_0 w_0}{g(\lambda+1)}. \tag{7}$$

Eq. (7) illustrates the relationship between the total water vapor content and surface moisture conditions for a particular moisture profile as described by λ.

Substituting (4) into (7) yields

$$U=\frac{\epsilon E_0}{g(\lambda+1)}\times 10^{(\alpha t_d-\beta)/(t_d+\gamma)}, \qquad (8)$$

or

$$\ln U=\ln\left(\frac{\epsilon E_0}{g}\right)-\ln(\lambda+1)+\left(\frac{\alpha t_d-\beta}{t_d+\gamma}\right)\ln 10. \qquad (9)$$

Considering the normal range of surface dew points and the magnitude of t_d with respect to the magnitude of γ, little error will result if the denominator $(t_d+\gamma)$ is assigned the mean value of 440F. Evaluation of Eq. (9) then yields

$$\ln U=[0.1133-\ln(\lambda+1)]+0.0393t_d, \qquad (10)$$

which is of the same form as the regression equation obtained by Reitan and Bolsenga. In (10), however, the "A" coefficient is not necessarily a constant, but depends on the vertical distribution of moisture. It is interesting to note that B has very nearly the values obtained by Reitan and Bolsenga.

As indicated earlier from the results of Reitan and Bolsenga, the longer the time period over which the mean values of water vapor content and surface dew point are formed, the more unique the relation is between these two variables; or, as depicted by (10), the better the approximation becomes that the moisture profile is constant. The scatter about the statistically determined regression lines of Reitan and Bolsenga merely results from variations of λ.

3. Improvements

Table 1 illustrates the dependence of λ on latitude and season. These values of λ were obtained from the mean Northern Hemisphere soundings tabulated by London (1957). It should be noted that the Northern Hemisphere averages are derived by weighting each latitude band equally. For a given surface dew point, it follows from (10) that the percentage error resulting in an estimate of U from an incorrect estimate of λ is given by

$$\text{Error }(U)\equiv 100\frac{\hat{U}-U}{U}=100\frac{\hat{\lambda}-\lambda}{\hat{\lambda}-1},$$

TABLE 1. Seasonal and latitudinal mean values of λ.

Season latitudinal zone (deg N)	Winter	Spring	Summer	Fall	Annual average
0–10	3.37	2.85	2.80	2.64	2.91
10–20	2.99	3.02	2.70	2.93	2.91
20–30	3.60	3.00	2.98	2.93	3.12
30–40	3.04	3.11	2.92	2.94	3.00
40–50	2.70	2.95	2.77	2.71	2.78
50–60	2.52	3.07	2.67	2.93	2.79
60–70	1.76	2.69	2.61	2.61	2.41
70–80	1.60	1.67	2.24	2.63	2.03
80–90	1.11	1.44	1.94	2.02	1.62
Northern Hemisphere average	2.52	2.64	2.62	2.70	2.61

where U, λ are the true values and $\hat{U}$, $\hat{\lambda}$ are the estimated values. Since the "A" coefficients obtained by Reitan and Bolsenga imply $\hat{\lambda}$ values of about 2.0 and 3.0, respectively, it is evident from Table 1 that errors as large as 50 per cent may result even for mean monthly conditions. Hence it seems necessary to consider the latitudinal and seasonal dependence of λ when relating the surface dew point to the total water vapor content. This may now be done by utilizing Table 1 with Eq. (10). More characteristic relationships for individual stations could be obtained in a similar manner from tabulated values of λ for those stations. This type of information would indeed be a useful supplement to regions where few or no balloon soundings are available.

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