On the Use of Court's Versus Durst's Techniques for Computing Vector Correlation Coefficients

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Vector correlation coefficients relating the wind at one time, place or height to the wind at other times, places or heights, have been presented by several investigators (Durst, 1954, 1957; Charles, 1959a, b; Kochanski, 1961). The Durst stretch correlation coefficient or some of its variations have been most widely used, although Charles (1959c) used Court's total vector correlation as well as Durst's stretch correlation in at least one case and compared the two techniques. It is the purpose of this note to describe a second use of both of these techniques (Lamberth and Veith, 1963), and to discuss the results.

In the last named paper vector correlation coefficients were computed between the wind at El Paso, Tex., as a base, and Tucson, Ariz., Albuquerque, N. Mex., and Amarillo and Midland, Tex. Computations were also made at each of these locations individually. Interlevel combinations were computed and lags up to 48 hr were used.

The data used in this study (furnished by the U. S. Weather Bureau) consisted of wind direction and speed from twice-daily rawin observations. The data were continuous and complete for the five stations named, missing data having been supplied by suitable techniques for interpolation or extrapolation.

The total vector correlation coefficient, Court's R_{WZ}, is an extension of simple linear correlation to multiple correlation applied to vectors and is the correlation coefficient of two samples of winds,

$$\sum_{k=1}^{n} W_k \text{ and } \sum_{k=1}^{n} Z_k,$$

where

$$\mathbf{W} = U\mathbf{i} + V\mathbf{j} \quad \text{and} \quad \mathbf{Z} = X\mathbf{i} + Y\mathbf{j}$$

are wind vectors separated in space, in time, or in both

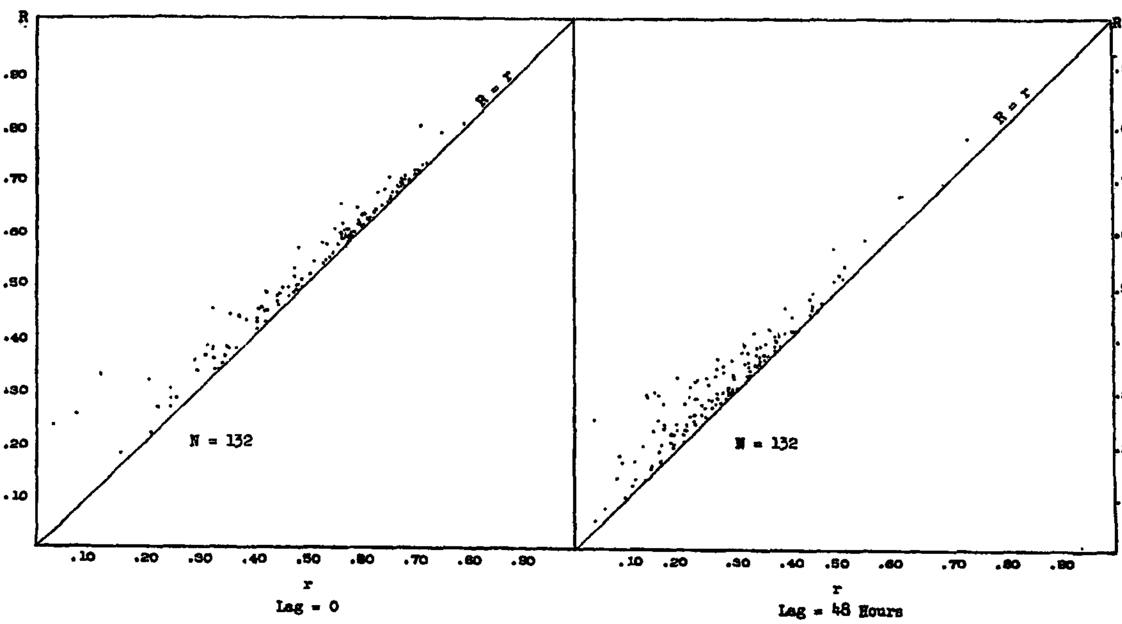

FIG. 1. Comparison of Court's R and Durst's r between El Paso and Tucson.

space and time. The expression for R_{WZ} contains the simple linear correlation coefficients, r_{ux}, r_{uy}, r_{vx} and r_{xy}, where u, v, x and y are deviations from the means $\bar{U}$, $\bar{V}$, $\bar{X}$ and $\bar{Y}$, i.e., $u=U-\bar{U}$, etc., as well as s_u^2 and s_v^2, the sample variances of u and v, respectively. It is written as

$$R_{WZ}^2 = \frac{s_u^2(r_{ux}^2+r_{uy}^2-2r_{ux}r_{uy}r_{xy})+s_v^2(r_{vx}^2+r_{vy}^2-2r_{vx}r_{vy}r_{xy})}{(s_u^2+s_v^2)(1-r_{xy}^2)}. \quad (1)$$

By definition of the simple linear correlation coefficients

$$r_{ux}^2=\frac{s_{ux}^2}{s_u^2 s_x^2},\ r_{vx}^2=\frac{s_{vx}^2}{s_v^2 s_x^2},$$

where s_{ux}^2 and s_{vx}^2 are covariances, etc., Eq. (1) becomes

$$R_{WZ}^2 = \frac{s_y^2(s_{ux}^2+s_{vx}^2)+s_x^2(s_{uy}^2+s_{vy}^2)-2s_{xy}(s_{ux}s_{uy}+s_{vx}s_{vy})}{(s_u^2+s_v^2)(s_x^2 s_y^2-s_{xy}^2)}. \quad (2)$$

Durst's r_{WZ} is defined by

$$r_{WZ}^2=\frac{(\Sigma ux+\Sigma vy)^2}{(\Sigma u^2+\Sigma v^2)(\Sigma x^2+\Sigma y^2)}, \quad (3)$$

where the notation is the same as that in the preceding paragraph. Note that the above expression includes only the "stretch" portion of the "stretch and turn" coefficient, the "turn" portion having been neglected here as in most studies using Durst's technique.

The basic difference between Court's R and Durst's r is that R employs multiple correlation of three variables, while r parallels more the simple correlation of two variables. Since R is substantially more difficult to compute than r, it is appropriate to inquire whether it is more useful or representative.

For the comparisons shown here R and r were computed separately for each season of each of the three years for a total of 12 periods of three months each. Since both of the daily rawin observations were used, it follows that each R or r was computed from 180 to 184 observations. The coefficients compared were those which were computed between El Paso and Tucson at the following pressure-height pairs, shown in kilometers MSL: 3–3, 6–3, 6–6, 9–3, 9–6, 9–9, 12–6, 12–9, 12–12, 24–12 and 24–24. Only the difference between R and r was being studied here and since this quantity did not vary much between the seasons, the years, or the eleven height-pairs, all of them were combined, resulting in 132 values of the correlation coefficient computed by each of the techniques. The results of this comparison are shown in Fig. 1 where the scattergram on the left is for synchronous data (separation in space only), while that on the right is for data lagged 48 hours (separation with both space and time).

As found by Charles (1959c), R was always $\geq r$. The difference between the two increased with the lag; as the time increased, Durst's r decreased faster than did Court's R. For R less than about 0.30, r appears questionable. Charles used 450 data pairs in his study and surmised that, with fewer pairs, the threshold value of r for important discrepancy with R would be larger than 0.30. The two scattergrams shown here appear very similar to his and infer that even for as few as 180 data pairs, r remains close to R for values greater than 0.30.

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