Two Experiments on Using a Scintillometer to Infer the Surface Fluxes of Momentum and Sensible Heat

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ABSTRACT

A traditional use of scintillometry is to infer path-averaged values of the turbulent surface fluxes of sensible heat \( H_s \) and momentum \( u^* \), where \( \rho \) is air density and \( u \) is the friction velocity. Many scintillometer setups, however, measure only the path-averaged refractive-index structure parameter \( C_n^2 \); the wind information necessary for inferring \( u^* \) and \( H_s \) comes from point measurements or is absent. The Scintec AG SLS20 surface-layer scintillometer system, however, measures both \( C_n^2 \) and the inner scale of turbulence \( l_0 \), where \( l_0 \) is related to the dissipation rate of turbulent kinetic energy \( \epsilon \). The SLS20 is thus presumed to provide path-averaged estimates of both \( u^* \) and \( H_s \). This paper describes comparisons between SLS20-derived estimates of \( u^* \) and \( H_s \) and simultaneous eddy-covariance measurements of these quantities during two experiments: one, over Arctic sea ice; and a second, over a midlatitude land site during spring. For both experiments, the correlation between scintillometer and eddy-covariance fluxes is reasonable: correlation coefficients are typically above 0.7 for the better-quality data. For both experiments, though, the scintillometer usually underestimates \( u^* \) and underestimates the magnitude of \( H_s \) when compared with the corresponding eddy-covariance values. The data also tend to be more scattered when \( C_n^2 \), \( 10^{-2} \) m\(^{-2/3} \): the signal-to-noise ratio for scintillometer-derived fluxes decreases as \( C_n^2 \) decreases. An essential question that arises during these comparisons is what similarity functions to use for inferring fluxes from the scintillometer \( C_n^2 \) and \( l_0 \) measurements. The paper thus closes by evaluating whether any of four candidate sets of similarity functions is consistent with the scintillometer data.

1. Introduction

A twinkling star is the classic example of scintillation. As twinkling stars give information on the thermal structure of the atmosphere, so too does the scintillation of electromagnetic waves provide information on the turbulent fluxes of momentum and sensible and latent heat in the atmospheric surface layer. This idea goes back to at least Wesely (1976) and Wyngaard and Clifford (1978), and its implementation is another application of the so-called inertial-dissipation method (e.g., Taylor 1961; Fairall and Larsen 1986). Andreas (1990) reviews the theoretical and experimental foundation for this use of scintillometry.

Deriving the surface fluxes from scintillation measurements has presumed advantages over measuring the fluxes at a point with eddy-covariance instruments. Foremost is the notion that, because scintillation measurements yield path-averaged statistics, the derived fluxes might be representative area averages, even in nonhomogeneous terrain (Wyngaard and Clifford 1978; Coulter and Wesely 1980; Andreas 1989; Green et al. 1994; Beyrich et al. 2002). Such path-averaged fluxes could then provide appropriate validation data for remotely sensed fluxes and for weather forecast or general circulation models.

The scintillation method for obtaining the turbulent surface fluxes has been tested sporadically with various instrument configurations; validation has sometimes been promising (Hill et al. 1992; De Bruin et al. 1995; Green et al. 1997; Chehbouni et al. 1999), but few validation tests have used only path-averaging instruments to deduce the turbulent fluxes. In particular, the wind information needed to estimate the sensible and latent heat fluxes in other than free convection has often come from point instruments (e.g., Green et al. 1994, 2001; Hoedjes et al. 2002; Kleissl et al. 2008). As a result, these tests produced no path-averaged measurement of the momentum flux.

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The Scintec AG (Rottenburg, Germany) SLS20 surface-layer scintillometer system is designed to fill this need for path-averaged measurements of both the momentum and sensible heat fluxes (Thiermann 1992). The SLS20 is a displaced-beam scintillometer. The transmitter splits a single laser into two beams. The intensity fluctuations in either beam at the receiver give the refractive-index structure parameter $C_n^2$. The correlation in intensity of the two beams at the receiver is a measure of the inner scale of turbulence $\ell_0$, which is, in turn, related to the dissipation rate of turbulent kinetic energy $\varepsilon$. From $C_n^2$ and $\varepsilon$, we can iteratively solve equations that are based on Monin–Obukhov similarity theory for the surface stress or momentum flux $\tau$ and the sensible heat flux $H_s$. Both are, thus, path-averaged estimates of these fluxes.

I used the Scintec SLS20 to measure $\tau$ and $H_s$ in two experiments. One was the 1997–98 experiment to study the Surface Heat Budget of the Arctic Ocean (SHEBA; Andreas et al. 1999; Uttal et al. 2002). The other was the 2005 Rapid Forcing Experiment (Andreas et al. 2006b, 2008a). Both experiments also included instruments near the scintillometer path that simultaneously measured $\tau$ and $H_s$ by eddy covariance. Here, I compare the scintillometer-derived $H_s$ and $\tau$ (actually $u_\ast$, the friction velocity) with the eddy-covariance measurements of $H_s$ and $u_\ast$.

I cannot conclude that the Scintec SLS20 made accurate measurements of $u_\ast$ and $H_s$ during these two diverse experiments. The correlation between scintillometer $H_s$ and eddy-covariance $H_s$ was good in both experiments, but the magnitude of the scintillometer $H_s$ was biased low. When $C_n^2$ was at least $10^{-14}$ m$^{-2/3}$, the scintillometer $u_\ast$ and the eddy-covariance $u_\ast$ also had good correlation; the scintillometer $u_\ast$, though, was biased low. When $C_n^2$ was less than $10^{-14}$ m$^{-2/3}$, the correlation between scintillometer and eddy-covariance measurements of $u_\ast$ was only 0.33, and now the scintillometer $u_\ast$ was biased high.

A key assumption for obtaining fluxes from scintillation data is that the path-averaged $C_n^2$ and $\ell_0$ (or $\varepsilon$) obey Monin–Obukhov similarity theory. A related issue is what similarity functions to use for converting $C_n^2$ and $\varepsilon$ to fluxes. Here, I have enough independent information to study what the Scintec SLS20 says about Monin–Obukhov similarity in both stable and unstable stratification. I use four distinct sets of functions for inferring $u_\ast$ and $H_s$ from scintillation data. None of these functions stand out, however, as the best set for obtaining fluxes from scintillation measurements.

When, alternatively, I determine the similarity functions for $C_n^2$ and $\varepsilon$ from the scintillometer and eddy-covariance data, the derived values are so scattered that I cannot conclude that the path-averaged $C_n^2$ and $\varepsilon$ actually obey similarity theory. For example, the scintillometer-derived similarity function for $\varepsilon$ is not 1 at neutral stratification, as required by similarity theory, but, rather, is about 0.3. That is, the scintillometer-based estimate of $\varepsilon$ is biased very low. De Bruin et al. (2002) and Hartogensis et al. (2002) observed this same behavior in the SLS20.

### 2. Mathematical framework

In turbulence notation, the fluxes of interest are the surface momentum flux (or surface stress) $\tau$ and the surface sensible heat flux $H_s$: 

$$\tau = \rho u_\ast^2 = -\rho u\bar{v} \quad \text{and} \quad (2.1a)$$

$$H_s = -\rho c_p u_\ast \bar{\theta} = \rho c_p \bar{\theta} \bar{\theta}. \quad (2.1b)$$

Here, $u$ and $w$ are the turbulent fluctuations in the along-wind and vertical components of the wind vector, $\theta$ is the turbulent fluctuation in temperature, $\rho$ is the air density, and $c_p$ is the specific heat of air at constant pressure. The overbars indicate time averages. Equation (2.1a) also defines the friction velocity $u_\ast$, and (2.1b) defines the temperature flux scale $\bar{\theta}$. These two equations represent the so-called Reynolds fluxes measured by eddy-covariance instruments.

The fundamental meteorological variables that the Scintec scintillometer derives from the measured laser intensity fluctuations are the refractive-index structure parameter $C_n^2$ and the inner scale of turbulence $\ell_0$. The inner scale is related to the dissipation rate of turbulent kinetic energy $\varepsilon$ through (Hill and Clifford 1978; Andreas 1992; Hill 1997)

$$\ell_0 = [9\Gamma(1/3)\beta D]^{3/4} e^{-1/4}. \quad (2.2)$$

Here, $\Gamma$ is the gamma function of the indicated argument $(1/3)$; $\beta$ is the one-dimensional Obukhov–Corssin constant, taken as 0.40 (Andreas 1987; Sreenivasan 1996); and $D$ is the thermal diffusivity of air.

The refractive-index structure parameter is related to quantities with meteorological significance (e.g., Hill 1978; Andreas 1988):

$$C_n^2 = A^2 C_T^2 + 2A B C_{TQ} + B^2 C_Q^2. \quad (2.3)$$

In this, $C_T^2$ is the temperature structure parameter, $C_Q^2$ is the humidity structure parameter, and $C_{TQ}$ is the temperature–humidity structure parameter. These are related to the spectra of temperature and humidity, to the temperature–humidity cospectrum, and also to the
surface fluxes of sensible and latent heat (e.g., Davidson et al. 1978; Fairall et al. 1980; Kohsiek 1982). The \( A \) and \( B \) in (2.3) are coefficients that depend on the wavelength \( \lambda \) of the electromagnetic wave and on mean pressure \( P \), temperature \( T \), and humidity (Andreas 1988).

For the wavelength of the Scintec SLS20 \( (\lambda = 0.685 \, \mu m) \), \( C_n^2 \) depends only weakly on \( C_{TQ} \) and \( C_{Q}^2 \) (Andreas 1988); I therefore ignore these terms in my analysis. That is,

\[
C_n^2 = A^2 C_T^2, (2.4)
\]

and (Andreas 1988)

\[
A = -78.44 \times 10^{-6} (P/T^2). (2.5)
\]

Here, \( A \) has units of inverse kelvins, \( P \) must be in hectopascals, and \( T \) must be in kelvins (Hill et al. 1992).

This \( C_T^2 \) is a variable that obeys Monin–Obukhov similarity theory (Wyngaard et al. 1971; Panofsky and Dutton 1984, p. 183; Hill 1989):

\[
g(z/L) = \frac{z^{2/3} C_T^2}{\theta_0^2} = \frac{z^{2/3} C_n^2}{A^2 \theta_0^2}. (2.6)
\]

Here, \( g \) is a presumed universal function of both \( z \), the height at which \( C_T^2 \) (or \( C_n^2 \)) is measured, and \( L \), the Obukhov length, defined as

\[
L^{-1} = \frac{\gamma k \theta_0}{T u_0}, (2.7)
\]

where \( k = 0.40 \) is the von Kármán constant and \( \gamma \) is the acceleration of gravity. The rightmost term in (2.6) follows from (2.4).

The dissipation rate \( \varepsilon \) is likewise a similarity variable (e.g., Wyngaard and Coté 1971; Kaimal and Finnigan 1994, p. 16):

\[
\phi_e(z/L) = \frac{\kappa z \varepsilon}{u'_e}, (2.8)
\]

where \( \phi_e \) is another presumed universal function of \( \zeta = z/L \).

With scintillometer measurements of \( C_n^2 \) and \( \ell_0 \), the computational procedure is first to calculate \( \varepsilon \) from (2.2). Then one iteratively solves the coupled equations (2.6)–(2.8) for \( u'_e \), \( \theta_0 \), and \( L \). The solution usually converges in three–six iterations. Last, one calculates \( \tau \) and \( H_s \) from (2.1).

The crux of this analysis involves specifying \( g(\zeta) \) and \( \phi_e(\zeta) \).

3. Similarity functions

Because the boundary layer community has not converged on the best equations to use for \( g(\zeta) \) and \( \phi_e(\zeta) \) when analyzing scintillometer data, I try four distinct pairs of functions here.

a. Wyngaard

Wyngaard et al. (1971) were the first to write expressions for \( g(\zeta) \); Wyngaard (1973) offered slightly modified versions. Andreas (1988) further modified these functions for compatibility with a von Kármán constant of 0.40; Wyngaard et al. (1971) and Wyngaard (1973) had used 0.35 on the basis of the Kansas data. The functions for \( g(\zeta) \) that I will henceforth identify as the “Wyngaard” functions are, thus,

\[
g(\zeta) = 4.9(1 - 6.1\zeta)^{-2/3} \text{ for } \zeta \leq 0 \quad \text{and} \quad (3.1a)
\]

\[
g(\zeta) = 4.9(1 + 2.2\zeta)^{2/3} \text{ for } \zeta \geq 0. \quad (3.1b)
\]

During this same period, Wyngaard and Coté (1971) were the first to derive expressions for \( \phi_e(\zeta) \). Again, Andreas (1988) modified these to reflect a von Kármán constant of 0.40:

\[
\phi_e(\zeta) = [1 + 0.46(-\zeta)^{2/3}]^{3/2} \text{ for } \zeta \leq 0 \quad \text{and} \quad (3.2a)
\]

\[
\phi_e(\zeta) = [1 + 2.3\zeta^{3/5}]^{3/2} \text{ for } \zeta \geq 0 \quad \text{and} \quad (3.2b)
\]

I will refer to these as the Wyngaard functions for \( \phi_e \).

b. Thiermann–Grassl

Thiermann and Grassl (1992) developed the first set of similarity functions specifically for obtaining the surface fluxes from scintillometer measurements of \( C_n^2 \) and \( \ell_0 \). I will henceforth refer to these as the “Thiermann–Grassl” functions:

\[
g(\zeta) = 6.34(1 - 7\zeta + 75\zeta^2)^{-1/3} \text{ for } \zeta \leq 0 \quad \text{and} \quad (3.3a)
\]

\[
g(\zeta) = 6.34(1 + 7\zeta + 20\zeta^2)^{1/3} \text{ for } \zeta \geq 0 \quad \text{and} \quad (3.3b)
\]

and

\[
\phi_e(\zeta) = (1 - 3\zeta)^{-1} - \zeta \text{ for } \zeta \leq 0 \quad \text{and} \quad (3.4a)
\]

\[
\phi_e(\zeta) = (1 + 4\zeta + 16\zeta^2)^{1/2} \text{ for } \zeta \geq 0. \quad (3.4b)
\]

Equations (3.3) are slightly different than what Thiermann and Grassl (1992) gave. Their definition of \( g(\zeta) \) was unusual in that, instead of the standard form (2.6), it had the von Kármán constant multiplying \( z \). I removed this \( k \) and modified their given similarity functions accordingly to obtain (3.3).
c. Edson–Fairall

From eddy-covariance measurements over the ocean, Edson and Fairall (1998) deduced the following similarity functions:

\[ g(\xi) = 5.9(1 - 8\xi)^{-2/3} \quad \text{for} \quad \xi \leq 0 \quad \text{and} \quad (3.5a) \]

\[ g(\xi) = \frac{5.9(1 + 6\xi)}{(1 + 5\xi)^{1/3}} \quad \text{for} \quad \xi \geq 0 \quad \text{(3.5b)} \]

and

\[ \phi_e(\xi) = \frac{1}{1 - 7\xi} - \xi \quad \text{for} \quad \xi \leq 0 \quad \text{(3.6a)} \]

\[ \phi_e(\xi) = 1 + 5\xi \quad \text{for} \quad \xi \geq 0. \quad \text{(3.6b)} \]

I will refer to these as the “Edson–Fairall” functions.

d. Budget method

For steady-state conditions, the budget equation for temperature variance in the atmospheric surface layer simplifies to a balance between production and dissipation (e.g., Large and Pond 1982; Panofsky and Dutton 1984, pp. 94, 184; Andreas 1987):

\[ 2u_0\theta_0 \frac{\partial \Theta}{\partial z} = N_0. \quad \text{(3.7)} \]

In this, \( N_0 \) is the dissipation rate of temperature variance, and \( \Theta \) is the average potential temperature; thus, \( \partial \Theta / \partial z \) is the vertical gradient in potential temperature. The left term is the production of temperature variance; the right term, the dissipation of it.

The Monin–Obukhov similarity function for the non-dimensional temperature gradient is

\[ \phi_h(\xi) = \frac{kz}{\theta_0} \frac{\partial \Theta}{\partial z}. \quad \text{(3.8)} \]

The dissipation rate of temperature variance is also related to \( C_f^2 \) through the temperature spectrum in the inertial-convective subrange such that (Hill and Clifford 1978; Andreas 1988)

\[ \beta N_0 \nu^{-1/3} = 0.249C_f^2. \quad \text{(3.9)} \]

On inserting (3.8) and (3.9) into (3.7) and using the definition of \( \phi_e, \) (2.8), we obtain (Andreas 1988; Panofsky and Dutton 1984, p. 184)

\[ \frac{2\beta \phi_h(\xi)}{0.249k^{2/3} \phi_e(\xi)^{1/3}} = \frac{\nu^{-1/3}}{\theta_0^{2/3}} \frac{\partial U}{\partial z} = g(\xi). \quad \text{(3.10)} \]

That is, the temperature variance budget also yields an expression for \( g(\xi) \).

Equation (3.10) has two advantages over my earlier expressions for \( g(\xi) \). Notice that (3.1), (3.3), and (3.5) offer several opinions as to the value of \( g(\xi) \) at neutral stratification. Equation (3.10) provides a theoretical expression for that coefficient. On evaluating the constants on the left side of (3.10), I get

\[ g(\xi) = \frac{5.92\phi_h(\xi)}{\phi_e(\xi)^{1/3}}. \quad \text{(3.11)} \]

\( g(\xi) \) is thus predicted to be 5.92 at neutral stratification since \( \phi_h(\xi = 0) = \phi_e(\xi = 0) = 1 \).

The second advantage of (3.10) is that I can use functions of my own choosing for \( \phi_h \) and \( \phi_e \). For compatibility with my other algorithms (i.e., Andreas et al. 2008b, 2010a,b), I use the function from Paulson (1970) for \( \phi_h \) in unstable stratification,

\[ \phi_h(\xi) = (1 - 16\xi)^{-1/2}. \quad \text{(3.12a)} \]

and the new function from Grachev et al. (2007) in stable stratification,

\[ \phi_h(\xi) = 1 + \frac{5\xi + 5\xi^2}{1 + 3\xi + \xi^2}. \quad \text{(3.12b)} \]

For \( \phi_e \) in (3.11), I use the functions that I derive next from the turbulent kinetic energy equation.

Again, for steady-state conditions, the turbulent kinetic energy equation also reflects a near balance between production and dissipation (Large and Pond 1981; Panofsky and Dutton 1984, p. 93f; Wyngaard 2010, p. 231):

\[ u_0^2 \frac{\partial U}{\partial z} - \frac{\gamma}{T} u_0 \theta_0 = \varepsilon. \quad \text{(3.13)} \]

Here, \( U \) is the mean wind speed; thus, \( \partial U / \partial z \) is the vertical gradient in wind speed. As with (3.7), the terms on the left are the surface-layer production of turbulent kinetic energy (respectively, mechanical production and buoyancy production); the term on the right is the dissipation.

On introducing the nondimensional gradient in wind speed,

\[ \phi_m(\xi) = \frac{kz}{\theta_0} \frac{\partial U}{\partial z}, \quad \text{(3.14)} \]

and using (2.7) and (2.8), I can rewrite (3.13) as

\[ \phi_m(\xi) - \xi = \phi_e(\xi). \quad \text{(3.15)} \]
This is a budget-based expression for $\phi_m$. For consistency in my terminology, I also use this expression for $\phi_m$ in my budget-based expression for $g(\zeta)$, (3.11).

As with (3.11), I now have the option of choosing $\phi_m$ for compatibility with my other algorithms. Hence, for $\phi_m$ in unstable stratification, I use the function from Paulson (1970):

$$\phi_m(\zeta) = (1 - 16\zeta)^{-1/4}. \quad (3.16a)$$

In stable stratification, I use the new result from Grachev et al. (2007):

$$\phi_m(\zeta) = 1 + \frac{6.5\zeta(1 + \zeta)^{1/3}}{1.3 + \zeta}. \quad (3.16b)$$

Another advantage of these budget results, (3.11) and (3.15), is that they provide a way to incorporate into $g(\zeta)$ and $\phi_m(\zeta)$ new similarity functions to treat the very stable stratification that was observed over sea ice—namely, those from Grachev et al. (2007).

e. System of equations

In summary, three coupled equations are necessary for estimating $u_*$ and $H_s (= -\rho c_p u_* T_a)$ from scintillometer measurements of $C_n^2$ and $t_0$. Once $t_0$ is converted to $\varepsilon$ through (2.2), the three coupled equations derive from (2.6), (2.7), and (2.8):

$$\theta_e = \left[ \frac{z^{2/3} C_n^2}{A^2 g(\zeta)} \right]^{1/2}, \quad (3.17a)$$

$$u_* = \frac{k z \varepsilon}{\phi_e(\zeta)} \left[ \frac{g(\zeta)}{\phi_e(\zeta)} \right]^{1/3}, \quad (3.17b)$$

$$\zeta = \frac{z}{L} = \frac{\gamma k z \theta_e}{T u_*^2}. \quad (3.17c)$$

Here, $z$ is the path height of the scintillometer beam. Clearly, we also need average pressure and temperature for calculating $A$ and $\zeta$.

Notice the square root in (3.17a): scintillometer data do not tell us the sign of the sensible heat flux or the sign of the stratification. Often that sign is inferred from the diurnal cycle—negative heat flux at night, and positive heat flux during the day (e.g., Thiermann and Grassl 1992; Green et al. 1994, 1997; Hartogensis et al. 2002; Kleissl et al. 2008). Over sea ice, however, stable stratification—and, thus, a negative heat flux—are usual, even during daylight.

After measuring the surface–air potential temperature difference, we could assume that the sign of $H_s$ reflects a flux down the temperature gradient. Measuring the surface temperature of sea ice is fairly routine, and the SHEBA Atmospheric Surface Flux Group measured this variable hourly during the SHEBA scintillometer measurements reported here (Andreas et al. 2010a,b). In an earlier analysis, however, when I used the SHEBA surface–air temperature difference to deduce the sign of $H_s$, I found many heat flux data points in the wrong quadrants of scatterplots: that is, a positive scintillometer flux associated with a negative eddy-covariance flux; and a negative scintillometer flux with a positive eddy-covariance flux. The simple explanation is that, over sea ice, the surface–air temperature difference is usually small; hence, inherent uncertainty in the measurement of surface temperature, which is typically ±0.5°C (Andreas et al. 2010b), can imply the wrong sign for the scintillometer heat flux.

For my analysis here, I thus chose the sign of the scintillometer-derived $H_s$ to agree with the sign of the eddy-covariance $H_s$, as did De Bruin et al. (2002). In later scatterplots, this choice (perhaps artificially) improves the apparent correlation between scintillometer and eddy-covariance sensible heat fluxes by eliminating errant points in the second and fourth quadrants.

With the four sets of $g(\zeta)$ and $\phi_m(\zeta)$ functions, I invoke (3.17) four times for each set of $C_n^2$ and $t_0$ measurements and analyze all of the resulting flux estimates.

4. Datasets

a. The two sites

The Scintec scintillometer used during SHEBA and the Rapid Forcing Experiment was an upgraded version of the standard SLS20 for use over longer propagation paths. It thus had a slightly more powerful laser that operated at a wavelength of 0.685 $\mu$m instead of the 0.670-$\mu$m wavelength of the standard SLS20.

During the year-long SHEBA experiment, the scintillometer was placed on multiyear Arctic sea ice (Andreas et al. 1999, 2003). Because of instrument problems, the scintillometer did not run continuously as the other SHEBA instruments did. It yielded “winter” data from 31 October to 2 December 1997 and “summer” data from 20 May to 2 June 1998. During the winter measurements, the propagation path was 350 m at a height of 2.88 m. During summer, the propagation path was 300 m at a height of 2.60 m.

The SHEBA site was nearly ideal for a micrometeorological study. The surface was snow-covered sea ice in all directions for hundreds of kilometers. The ice itself was ridged, as is typical of Arctic sea ice. These ridges affected the aerodynamic roughness of the surface but were randomly distributed and, thus, did not alter the
overall horizontal homogeneity of the surface. Occasional leads—cracks in the sea ice that expose relatively warm ocean water—appeared in the vicinity of the ice camp, but none opened near the scintillometer. Because sea ice is horizontal, the SHEBA site was unaffected by density currents that are outside the scope of Monin–Obukhov similarity theory.

The site for the Rapid Forcing Experiment, in contrast, was not as ideal. That experiment took place in a 15-acre field in rural Lebanon, New Hampshire (Andreas et al. 2006b, 2008a). Trees 5–10 m tall bordered the field to the west and north, and the field sloped upward about 6% from west to east. The scintillometer path and the eddy-covariance instruments were set up to take best advantage of the dominant wind direction, which was westerly and northwesterly. The 182-m scintillometer propagation path ran south-southeasterly from the transmitter to the receiver at a height of 2.44 m. This setup gave a couple hundred meters of fetch over the field to the scintillometer path and to the point instruments.

The experiment ran from 11 to 27 April 2005. The field had been mowed the previous autumn. The experiment thus began with the field covered in grass stubble and clippings a few tens of millimeters thick; the grass was dormant, and the ground was near freezing. The ground warmed and the grass greened up by the end of the experiment but was still short. Because the Rapid Forcing Experiment was not ideal, its data provide an opportunity to evaluate the notion that scintillometer-derived fluxes are representative values even in complex terrain.

b. Scintillometer data

The fundamental data that the Scintec SLS20 yields are minute averages of $C_n^2$, $\ell_0$, and a data-quality number designated NOK. To obtain these averages, the system software divides each minute into ten 6-s blocks. For the first block of each minute, the software turns off the laser and measures the background. This background calibration is applied to the nine subsequent blocks for that minute. Each of these nine blocks yields measurements of $C_n^2$ and $\ell_0$; the software also evaluates the quality of each data block. If the block passes the quality-control criteria, its $C_n^2$ and $\ell_0$ values are used for computing the minute averages; otherwise, the block is ignored. NOK reports how many of the nine available data blocks are used in finding the minute averages of $C_n^2$ and $\ell_0$.

For both SHEBA and the Rapid Forcing Experiment, I further averaged these minute data into hourly values to coincide with the simultaneous hourly averages from the eddy-covariance instruments. I weighted each minute average of $C_n^2$ and $\ell_0$ in these hourly averages according to NOK and further used NOK to calculate a “quality” for the hourly averages (same for both $C_n^2$ and $\ell_0$). In essence, this quality metric reflects the percentage of good measurements during the hour.

In my analysis here, I ignored any hourly scintillometer data for which the computed quality metric was less than 25%. These were cases in which fewer than 25% of the scintillometer measurements during an hour passed quality controls. From 624 h of available SHEBA scintillometer data, this screening eliminated 97 h of data. Falling and blowing snow led to most of the rejections. For the Rapid Forcing Experiment, this screening eliminated only 8 h from 273 h of available data. Rain and drizzle caused these rejections.

c. Eddy-covariance data

My purpose in this paper is to compare the hourly scintillometer-derived estimates of $u_*$ and $H_s$ with simultaneous eddy-covariance measurements of these quantities. Both SHEBA and the Rapid Forcing Experiment included eddy-covariance measurement of $u_*$ and $H_s$ [see (2.1)] with sonic anemometers (referred to hereinafter as “sonics”) made by Applied Technologies, Inc. (ATI; Kaimal et al. 1990; Kaimal and Gaynor 1991; Kaimal and Finnigan 1994, p. 218f.). These sonics sampled at 10 Hz. For SHEBA, a 20-m tower with ATI sonics at five levels was near the receiver end of the scintillometer path. The Rapid Forcing Experiment had a single sonic positioned near the center of the scintillometer path at a height of 3.6 m.

Persson et al. (2002), Grachev et al. (2005, 2007), and Andreas et al. (2006a, 2010a,b) provide many more details of the SHEBA eddy-covariance measurements. The only point I need to make is that the SHEBA tower yielded up to five independent measurements of $u_*$ and $H_s$ each hour. As the SHEBA eddy-covariance values of $u_*$ and $H_s$ for the subsequent comparisons, I use the median values of all reported hourly $u_*$ and $H_s$ measurements.

Similarly, Andreas et al. (2006b, 2008a) provide details on the point turbulence measurements during the Rapid Forcing Experiment and on how we computed the hourly averaged values of $u_*$ and $H_s$. Briefly, because these measurements were on a slope, we could not base coordinate rotations on the assumption that the average vertical velocity was zero. Rather, we assumed that the sonic was properly leveled and, thus, made only one coordinate rotation—aligning $\mathbf{m}\mathbf{w}$ with the mean wind direction.

5. Flux comparisons

a. SHEBA

It is useful to define a $C_n^2$ limit, $C_{n,\text{lim}}^2$, to separate high-quality and lower-quality data. Because the basic
scintillometer signal is intensity fluctuations, which are monotonically related to \( C_n^2 \), small \( C_n^2 \) means a small scintillometer signal. I take \( C_{n,\text{lim}}^2 \) as the approximate lower limit for which the signal-to-noise ratio of the scintillometer is high enough to yield precise measurements of \( C_n^2 \) and \( \theta_0 \) (Thiermann and Grassl 1992).

About one-half of the SHEBA \( C_n^2 \) values are less than \( 10^{-14} \) m\(^{-2/3}\). Moreover, the SHEBA scatterplots of scintillometer versus eddy-covariance data for both \( u^* \) and \( H_s \) have different characteristics when \( C_n^2 \) is less than \( 10^{-14} \) m\(^{-2/3}\) and when \( C_n^2 \) is above this value (Figs. 1 and 2). This \( 10^{-14} \) m\(^{-2/3}\) is a tentative estimate for \( C_{n,\text{lim}}^2 \) and, thus, the dividing line between data with poorer and better signal-to-noise ratio.

For example, if, instead, I set \( C_{n,\text{lim}}^2 = 5 \times 10^{-15} \) m\(^{-2/3}\), the quality metrics that I will define shortly tend to be worse for the \( C_n^2 \geq C_{n,\text{lim}}^2 \) range than with \( C_{n,\text{lim}}^2 = 10^{-14} \) m\(^{-2/3}\). In effect, both \( C_n^2 \) and \( \theta_0 \) were measured more precisely when \( C_n^2 \approx 10^{-14} \) m\(^{-2/3}\).

A key feature of the sensible heat flux plots in Fig. 1 is how small the fluxes are: \( |H_s| \) is rarely larger than 20 W m\(^{-2}\). Such small fluxes are typical over sea ice (Persson et al. 2002; Andreas et al. 2010a,b) and explain why \( C_n^2 \) was often so small.

Neither Fig. 1 nor Fig. 2 provides obvious guidance for choosing among the four sets of similarity functions. For sensible heat flux (Fig. 1), the Wyngaard functions stand out as giving larger fluxes than the other functions. For friction velocity (Fig. 2), the Wyngaard functions tend to produce lower \( u^* \) than the other functions.

To quantify the performance of the various similarity functions, I computed the mean bias error (MBE) and the root-mean-square error (RMSE) (Willmott 1982) for the scintillometer-derived fluxes. Let \( S_i \) be an hourly scintillometer-derived flux (either \( H_s \) or \( u^* \)) and let \( M_i \) be the corresponding eddy-covariance value. Then,

\[
\text{MBE} = \frac{1}{N} \sum_{i=1}^{N} (S_i - M_i) \quad \text{and} \quad (5.1)
\]

\[
\text{RMSE} = \left[ \frac{1}{N} \sum_{i=1}^{N} (S_i - M_i)^2 \right]^{1/2}, \quad (5.2)
\]

where \( N \) is the number of data pairs. Table 1 shows these overall metrics for the data depicted in Figs. 1 and 2. Table 1 also lists the correlation coefficients for the comparisons for each set of similarity functions.

Last, the table lists the number of data pairs. These numbers are not all the same because, sometimes, the scintillometer iteration did not converge. If my analysis routine made 20 iterations without \( u^* \) and \( \theta_0 \) converging.

---

**FIG. 1.** The hourly SHEBA scintillometer measurements of sensible heat flux are compared with simultaneous eddy-covariance measurements of \( H_s \) for cases in which (left) \( C_n^2 < 10^{-14} \) m\(^{-2/3}\) and (right) \( C_n^2 \geq 10^{-14} \) m\(^{-2/3}\). Four different sets of similarity functions, as described in sections 3a–d, are used for evaluating the scintillometer \( H_s \). The heavy line is 1:1.
to within 0.1%, I assumed that the iteration failed. The Wyngaard equations were better at providing solutions than were the other similarity functions. Most of the failures to converge occurred in unstable stratification. Most of the failures in the Thiermann–Grassl, Edson–Fairall, and budget functions were for the same $C_n^2$ and $\ell_0$ pairs.

Table 1 quantifies some of the visual observations. The Wyngaard functions produce the largest MBE and RMSE for the sensible heat flux data. The Thiermann–Grassl and budget functions do best for predicting sensible heat flux. The Thiermann–Grassl functions always give the smallest RMSE, but the budget functions yield a very small MBE when $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$.

The $u_*$ metrics in Table 1 are much better for the cases in which $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$ than when $C_n^2 < 10^{-14} \text{ m}^{-2/3}$. Furthermore, MBE changes sign between the two cases. MBE is positive when $C_n^2 < 10^{-14} \text{ m}^{-2/3}$—the scintillometer $u_*$ is larger than the eddy-covariance $u_*$ because of some very large scintillometer $u_*$ values. When $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$, on the other hand, MBE is negative—the scintillometer $u_*$ tends to be smaller than the eddy-covariance $u_*$. RMSE is also larger by a factor of $\sim 3$ when

**Fig. 2.** Similar to Fig. 1 but for friction velocity, shown for cases in which the stratification measured by eddy covariance was (left) stable and (right) unstable.
C_n^2 < 10^{-14} \text{ m}^{-2/3} than when C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$. Again, these results seem related to the signal-to-noise ratio.

The Thiermann–Grassl, Edson–Fairall, and budget functions produced similar values of MBE and RMSE for $u*$ for the two $C_n^2$ cases: when $C_n^2 < 10^{-14} \text{ m}^{-2/3}$ and $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$. These metrics for the Wyngaard $u*$ values were better than for these other three functions when $C_n^2 < 10^{-14} \text{ m}^{-2/3}$ but worse when $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$. The Wyngaard functions again had better convergence success than the other three functions, however, and these additional successes may partly explain the poorer metrics, especially when $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$.

### b. Rapid Forcing Experiment

During the Rapid Forcing Experiment, only $9$ h in over $260$ h of useful measurements had $C_n^2 < 10^{-14} \text{ m}^{-2/3}$. Hence, in creating Figs. 3 and 4, I do not distinguish between large and small $C_n^2$.

The $H_s$ and $u*$ scatterplots from the Rapid Forcing Experiment (Figs. 3 and 4) have much different character than the comparable plots from SHEBA (Figs. 1 and 2). The first obvious difference is how much larger $H_s$ was over this midlatitude, terrestrial site than at SHEBA—$H_s$ was up to $350 \text{ W m}^{-2}$. The negative sensible heat fluxes, however, have comparable magnitude to the SHEBA fluxes and similar clustering and bias—at least when compared with the SHEBA fluxes when $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$.

The $u*$ plots are also much different. The $u*$ values from the Rapid Forcing Experiment show much more scatter (cf. RMSE values in Tables 1 and 2) and, curiously, range up to almost $0.9 \text{ m s}^{-1}$, whereas the SHEBA $u*$ values barely reach $0.4 \text{ m s}^{-1}$. Andreas (2011) noticed this limit in SHEBA scintillometer scatterplots of $u*$ and explained it as an effect of blowing snow.

In brief, the Scintec scintillometer software does quality checking to decide whether the changes in received laser intensity look like turbulence or like non-turbulent disturbances such as insects flying through the beam. If the intensity changes appear to be insect induced, the software downgrades NOK. Coincidentally, we noticed during our SHEBA operations that the Scintec software interpreted drifting and blowing snow in the laser beam as insects and, through NOK, flagged the data as bad. Because snow on sea ice begins drifting when $u*$ is approximately $0.30 \text{ m s}^{-1}$ (Andreas 2011), we rarely obtained good scintillometer data from SHEBA when the eddy-covariance $u*$ was above $0.30 \text{ m s}^{-1}$ (Fig. 2).

For the positive $H_s$ values in Fig. 3 (unstable stratification), the Edson–Fairall functions generally provide the largest scintillometer $H_s$ values. Meanwhile, the Thiermann–Grassl functions provide obviously low values. In Fig. 4, the Edson–Fairall functions also provide the largest scintillometer $u*$ values for unstable stratification; the Wyngaard functions generally give the smallest $u*$ values for both stable and unstable stratification. Table 2 summarizes the metrics for the Rapid Forcing Experiment.

### Table 1. The correlation coefficient (Corr coef), mean bias error, and root-mean-square error for comparisons of scintillometer-derived and eddy-covariance sensible heat flux and friction velocity for the SHEBA data. The scintillometer-derived values are based on the four sets of similarity functions described in section 3; “Number” is the number of data pairs in a given category. As explained in the text, I made these comparisons for cases with $C_n^2 < 10^{-14} \text{ m}^{-2/3}$ and with $C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$.

<table>
<thead>
<tr>
<th>Function</th>
<th>$C_n^2 &lt; 10^{-14} \text{ m}^{-2/3}$</th>
<th>$C_n^2 \geq 10^{-14} \text{ m}^{-2/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Corr coef</td>
</tr>
<tr>
<td>Wyngaard</td>
<td>234</td>
<td>0.777</td>
</tr>
<tr>
<td>Thiermann–Grassl</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Edson–Fairall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Budget</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The correlation coefficients reflected in Figs. 3 and 4 are similar for all four sets of functions (Table 2). The mean bias errors and the root-mean-square errors for the $H_s$ scatterplot, however, are varied. The Thiermann–Grassl functions badly underestimate $H_s$, as could be seen from visual inspection of Fig. 3, and also have the largest RMSE. The MBE for the budget equations is over 2 times as large as for the Wyngaard and Edson–Fairall functions, but the Wyngaard functions lead to more scatter (i.e., a large RMSE).

For the $u_*$ comparisons, the Thiermann–Grassl, Edson–Fairall, and budget functions produce comparable metrics, with the budget functions yielding slightly poorer fits. The Wyngaard functions produce the most deviant MBE for $u_*$ and the most scatter (largest RMSE).

As in Figs. 1 and 2 and Table 1, however, the Wyngaard functions converged to solutions for $H_s$ and $u_*$ 10–13 times more than did the other three sets of functions for the same input data.

c. Stratification dependence

Rather than just the four sets of similarity functions represented in Figs. 1–4, there are actually eight sets: each set of the four has unique functions for stable and unstable stratification. Besides the metrics tabulated in Tables 1 and 2, it is therefore worthwhile to further present fitting metrics distinguished by stratification, which I do in Tables 3–5.

In the $H_s$ scatterplots, Figs. 1 and 3, the data with positive $H_s$ were obtained in unstable stratification, and the data with negative $H_s$ were obtained in stable stratification. To distinguish the stability regime of the $u_*$ data, Figs. 2 and 4 show separate $u_*$ scatterplots for stable and unstable stratification.

For SHEBA $H_s$ data (Table 3), the functions generally behave better (i.e., smaller MBE and RMSE and
higher correlation) when \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \) than when \( C_n^2 < 10^{-14} \text{ m}^{-2/3} \). The one exception is the Wyngaard functions in unstable stratification. Here, the correlation coefficient is actually negative because the Wyngaard functions reached a solution three more times than did the other functions; but the resulting \( H_s \) values were large while the eddy-covariance \( H_s \) values were small. See the red circles above \( H_s = 40 \text{ W m}^{-2} \) in the \( \langle C_n^2 \rangle \geq 10^{-14} \text{ m}^{-2/3} \rangle \), panel in Fig. 1. These three points probably also explain why the Wyngaard metrics in Table 1 are poorer for \( H_s \) when \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \) than for the other three sets of functions.

Another curious feature of the \( H_s \) metrics in Table 3 is that the MBE in stable stratification changes sign between the cases with \( C_n^2 < 10^{-14} \text{ m}^{-2/3} \) (when it is negative) and with \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \) (when it is positive). We can see this result by comparing the \( \langle C_n^2 \rangle < 10^{-14} \text{ m}^{-2/3} \rangle \) and \( \langle C_n^2 \rangle \geq 10^{-14} \text{ m}^{-2/3} \rangle \) panels in Fig. 1. When \( H_s \) is negative, the scintillometer \( H_s \) values tend to be below the 1:1 line when \( C_n^2 < 10^{-14} \text{ m}^{-2/3} \) but above it when \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \).

For the best data in Table 3—the cases with \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \rangle \)—but excluding the anomalous Wyngaard functions, the other three sets of functions perform better in unstable stratification than in stable stratification. The one exception is the budget functions, which produce a very small mean bias error in stable stratification.

The data from the Rapid Forcing Experiment contrast with these results. In Table 4, the \( H_s \) metrics for MBE and RMSE are markedly smaller in stable stratification than in unstable stratification. Moreover, the bias is positive in stable stratification—the scintillometer overpredicts the eddy-covariance \( H_s \)—but negative in stable stratification—the scintillometer underpredicts the eddy-covariance \( H_s \). The same behavior was observed by P. Klein (2011, personal communication) in her Scintec scintillometer.

### Table 2. As in Table 1, except that these are comparisons for data from the Rapid Forcing Experiment for all ranges of \( C_n^2 \).

<table>
<thead>
<tr>
<th></th>
<th>Wyngaard</th>
<th>Thiermann–Grassl</th>
<th>Edson–Fairall</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_s ) comparison</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>259</td>
<td>249</td>
<td>247</td>
<td>246</td>
</tr>
<tr>
<td>Corr coef</td>
<td>0.968</td>
<td>0.970</td>
<td>0.970</td>
<td>0.970</td>
</tr>
<tr>
<td>MBE (W m(^{-2}))</td>
<td>–1.29</td>
<td>–8.61</td>
<td>–0.95</td>
<td>–3.47</td>
</tr>
<tr>
<td>RMSE (W m(^{-2}))</td>
<td>29.55</td>
<td>33.96</td>
<td>28.50</td>
<td>28.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u_\ast ) comparison</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>260</td>
<td>250</td>
<td>248</td>
<td>247</td>
</tr>
<tr>
<td>Corr coef</td>
<td>0.828</td>
<td>0.836</td>
<td>0.836</td>
<td>0.833</td>
</tr>
<tr>
<td>MBE (m s(^{-1}))</td>
<td>–0.173</td>
<td>–0.154</td>
<td>–0.155</td>
<td>–0.158</td>
</tr>
<tr>
<td>RMSE (m s(^{-1}))</td>
<td>0.220</td>
<td>0.203</td>
<td>0.202</td>
<td>0.207</td>
</tr>
</tbody>
</table>

### Table 3. Similar to Table 1, except that now the SHEBA \( H_s \) comparisons are also sorted by stratification.

<table>
<thead>
<tr>
<th></th>
<th>Wyngaard</th>
<th>Thiermann–Grassl</th>
<th>Edson–Fairall</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstable stratification and ( C_n^2 &lt; 10^{-14} \text{ m}^{-2/3} \rangle</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Number</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
</tr>
<tr>
<td>Corr coef</td>
<td>0.432</td>
<td>0.440</td>
<td>0.462</td>
<td>0.461</td>
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<tr>
<td>MBE (W m(^{-2}))</td>
<td>4.66</td>
<td>3.59</td>
<td>4.25</td>
<td>4.22</td>
</tr>
<tr>
<td>RMSE (W m(^{-2}))</td>
<td>8.12</td>
<td>6.88</td>
<td>7.36</td>
<td>7.34</td>
</tr>
<tr>
<td>Unstable stratification and ( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \rangle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>11</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Corr coef</td>
<td>–0.683</td>
<td>0.743</td>
<td>0.774</td>
<td>0.764</td>
</tr>
<tr>
<td>MBE (W m(^{-2}))</td>
<td>9.34</td>
<td>0.55</td>
<td>1.92</td>
<td>1.74</td>
</tr>
<tr>
<td>RMSE (W m(^{-2}))</td>
<td>19.66</td>
<td>2.93</td>
<td>3.34</td>
<td>3.27</td>
</tr>
<tr>
<td>Stable stratification and ( C_n^2 &lt; 10^{-14} \text{ m}^{-2/3} \rangle</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Number</td>
<td>117</td>
<td>117</td>
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</tr>
<tr>
<td>Corr coef</td>
<td>0.475</td>
<td>0.545</td>
<td>0.517</td>
<td>0.529</td>
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<tr>
<td>MBE (W m(^{-2}))</td>
<td>–0.95</td>
<td>–1.30</td>
<td>–1.13</td>
<td>–1.35</td>
</tr>
<tr>
<td>RMSE (W m(^{-2}))</td>
<td>4.98</td>
<td>4.63</td>
<td>4.78</td>
<td>4.76</td>
</tr>
<tr>
<td>Stable stratification and ( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \rangle</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Number</td>
<td>214</td>
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<td>214</td>
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</tr>
<tr>
<td>Corr coef</td>
<td>0.698</td>
<td>0.766</td>
<td>0.735</td>
<td>0.699</td>
</tr>
<tr>
<td>MBE (W m(^{-2}))</td>
<td>2.49</td>
<td>1.02</td>
<td>2.27</td>
<td>0.41</td>
</tr>
<tr>
<td>RMSE (W m(^{-2}))</td>
<td>5.81</td>
<td>4.84</td>
<td>5.49</td>
<td>5.32</td>
</tr>
</tbody>
</table>
The serious differences in MBE and RMSE, however, are likely an artifact of the magnitudes of the fluxes in stable and unstable stratification in the Rapid Forcing set. The range of $H_s$ in unstable stratification is 6 times that in stable stratification. This fact certainly accounts for the larger magnitudes of MBE and RMSE in unstable stratification.

In terms of $u^*$ estimates for the two datasets, the metrics are, at first glance, confusing. The Rapid Forcing Experiment (Table 4) is unambiguous: the scintillometer underestimates $u^*$ regardless of the stratification or which similarity functions I use. For the SHEBA data (Table 5), the scintillometer underestimates $u^*$ in stable stratification but overestimates it in unstable stratification. All four sets of similarity functions agree on these points, and the MBE and RMSE values are similar.

Figures 2 and 4 provide context for these numerical revelations. For both “stable” panels in Fig. 2, the majority of points are below the 1:1 line; the mean bias errors are thus both negative. On the other hand, for the “unstable” panels in Fig. 2, most of the points are above the 1:1 line; and the mean bias errors are computed as positive.

**Table 4.** As in Table 2, except that these are the comparisons for data from the Rapid Forcing Experiment sorted by stratification.

<table>
<thead>
<tr>
<th></th>
<th>Wyngaard</th>
<th>Thiermann–Grassl</th>
<th>Edson–Fairall</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$ comparison for unstable stratification</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>116</td>
<td>107</td>
<td>105</td>
<td>104</td>
</tr>
<tr>
<td>Corr coef</td>
<td>0.931</td>
<td>0.925</td>
<td>0.925</td>
<td>0.926</td>
</tr>
<tr>
<td>MBE (W m$^{-2}$)</td>
<td>-13.75</td>
<td>-28.30</td>
<td>-13.48</td>
<td>-14.83</td>
</tr>
<tr>
<td>RMSE (W m$^{-2}$)</td>
<td>41.17</td>
<td>50.03</td>
<td>40.91</td>
<td>41.51</td>
</tr>
<tr>
<td>$H_s$ comparison for stable stratification</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Number</td>
<td>143</td>
<td>142</td>
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</tr>
<tr>
<td>Corr coef</td>
<td>0.656</td>
<td>0.785</td>
<td>0.782</td>
<td>0.750</td>
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<tr>
<td>MBE (W m$^{-2}$)</td>
<td>8.81</td>
<td>6.23</td>
<td>8.32</td>
<td>4.86</td>
</tr>
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<td>RMSE (W m$^{-2}$)</td>
<td>14.37</td>
<td>11.66</td>
<td>13.24</td>
<td>11.50</td>
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</tbody>
</table>

**Table 5.** Similar to Table 1, except that now the SHEBA $u^*$ comparisons are also sorted by stratification.

<table>
<thead>
<tr>
<th></th>
<th>Wyngaard</th>
<th>Thiermann–Grassl</th>
<th>Edson–Fairall</th>
<th>Budget</th>
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<tbody>
<tr>
<td>$u^*$ comparison for unstable stratification</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>116</td>
<td>107</td>
<td>105</td>
<td>104</td>
</tr>
<tr>
<td>Corr coef</td>
<td>0.715</td>
<td>0.629</td>
<td>0.611</td>
<td>0.614</td>
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<tr>
<td>MBE (m s$^{-1}$)</td>
<td>-0.224</td>
<td>-0.208</td>
<td>-0.197</td>
<td>-0.216</td>
</tr>
<tr>
<td>RMSE (m s$^{-1}$)</td>
<td>0.269</td>
<td>0.254</td>
<td>0.246</td>
<td>0.260</td>
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<tr>
<td>$u^*$ comparison for stable stratification</td>
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<tr>
<td>Number</td>
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<td>0.884</td>
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<tr>
<td>MBE (m s$^{-1}$)</td>
<td>-0.130</td>
<td>-0.112</td>
<td>-0.122</td>
<td>-0.113</td>
</tr>
<tr>
<td>RMSE (m s$^{-1}$)</td>
<td>0.167</td>
<td>0.150</td>
<td>0.158</td>
<td>0.152</td>
</tr>
</tbody>
</table>

The serious differences in MBE and RMSE, however, are likely an artifact of the magnitudes of the fluxes in stable and unstable stratification in the Rapid Forcing set. The range of $H_s$ in unstable stratification is 6 times that in stable stratification. This fact certainly accounts for the larger magnitudes of MBE and RMSE in unstable stratification.

In terms of $u^*$ estimates for the two datasets, the metrics are, at first glance, confusing. The Rapid Forcing Experiment (Table 4) is unambiguous: the scintillometer underestimates $u^*$ regardless of the stratification or which similarity functions I use. For the SHEBA data (Table 5), the scintillometer underestimates $u^*$ in stable stratification but overestimates it in unstable stratification. All four sets of similarity functions agree on these points, and the MBE and RMSE values are similar.

Figures 2 and 4 provide context for these numerical revelations. For both “stable” panels in Fig. 2, the majority of points are below the 1:1 line; the mean bias errors are thus both negative. On the other hand, for the “unstable” panels in Fig. 2, most of the points are above the 1:1 line; and the mean bias errors are computed as positive.
As I will explain shortly, the Scintec scintillometer tends to overestimate $u_a$ when the eddy-covariance $u_a$ is less than roughly 0.1 m s$^{-1}$ but to underestimate $u_a$ when the eddy-covariance $u_a$ is greater than 0.1 m s$^{-1}$. The “$C_n^2 \geq 10^{-14}$ m$^{-2/3}$/stable” panel in Fig. 2 shows this behavior. Because the few points in the “$C_n^2 \geq 10^{-14}$ m$^{-2/3}$/unstable” panel in Fig. 2 mostly reflect eddy-covariance $u_a$ values near 0.1 m s$^{-1}$, they consequently cluster near the 1:1 line and produce a small MBE.

In the Rapid Forcing set (Fig. 4 and Table 4), the eddy-covariance $u_a$ values are much larger, and few measurements show the very small values that SHEBA did. Hence, the mean bias errors for $u_a$ from this experiment are negative regardless of stratification. We can see, though, in Fig. 4 that the points with small eddy-covariance $u_a$ in the stable panel are well correlated, as in Fig. 2 (“$C_n^2 \geq 10^{-14}$ m$^{-2/3}$/stable” panel), and seem to be tending above the 1:1 line for eddy-covariance $u_a$ values less than roughly 0.1 m s$^{-1}$.

Coincidentally, De Bruin et al. (2002) and Hartogensis et al. (2002) observed similar behavior in a Scintec SLS20. In their comparisons, Scintec scintillometers also overestimated $u_a$ for small $u_a$ and underestimated it for large $u_a$ when compared with eddy-covariance measurements. De Bruin et al. speculated that the overestimation at small $u_a$ could result from random noise in the scintillometer and that the underestimation at large $u_a$ might be explained by inactive turbulence, which affects turbulence variances but does no transport and, thus, does not influence fluxes.

Hartogensis et al. (2002) investigated the hypothesis that the separation between transmitted beams in the Scintec SLS20 was not as specified by the manufacturer (and, thus, as used in the processing software), $d = 2.7$ mm. When they recalculated $l_0$ from their raw Scintec data using $d = 2.6$ mm, the scintillometer-derived $u_a$ agreed better with the eddy-covariance $u_a$ for $u_a$ above 0.2 m s$^{-1}$; but the scintillometer still overestimated $u_a$ for small eddy-covariance $u_a$.

Although it should be possible to incorporate the ideas from De Bruin et al. (2002) into a revised theory of scintillometer behavior, the solution that Hartogensis et al. (2002) tried is not generally practical. If each Scintec scintillometer is built with a slightly different laser beam separation $d$, these instruments will not be useful off the shelf. Each would have to be calibrated against eddy-covariance data to determine $d$ before permanent deployment, as Hartogensis et al. have done.

6. Scintillometer similarity functions

Although the Monin–Obukhov similarity functions $g(\xi)$ and $\phi_\epsilon(\xi)$ together are the cornerstone of methods for obtaining surface fluxes from scintillation data, to my knowledge, only Hoedjes et al. (2002) have calculated $g(\xi)$ from scintillometer data. No scintillometer data have been used to study $\phi_\epsilon(\xi)$. Although their results appear encouraging, Hoedjes et al. had hardly any data with $|z/L| > 1$. Between the SHEBA and Rapid Forcing datasets, I have enough data to calculate both $g(\xi)$ and $\phi_\epsilon(\xi)$ from near-neutral stratification to $z/L \sim 5$ and to $z/L < -1$.

Here, I will evaluate $g(z/L)$ from (2.6) and $\phi_\epsilon(z/L)$ from (2.8) by combining the scintillometer measurements of $C_n^2$ and $\epsilon$ with the corresponding eddy-covariance measurements of $u_a$, $\theta_a$, and $\xi = z/L$, where $L$ comes from the eddy-covariance measurements according to (2.7). That is, $g(\xi)$ and $\phi_\epsilon(\xi)$ derive from both scintillometer and eddy-covariance measurements while $L$ comes strictly from eddy-covariance measurements.

Figure 5 shows my calculations of $g(z/L)$. The plots also show the four sets of similarity functions I have been using to derive fluxes: (3.1), (3.3), (3.5), and (3.11). Figure 6 shows $\phi_\epsilon(z/L)$. Both figures have two panels: a large-scale panel that covers the entire $z/L$ range of the data and a panel that focuses on near-neutral stratification, where the scintillometer-derived $\phi_\epsilon$ behaves oddly.

Neither set of figures provides any compelling evidence to help us to choose among the candidate similarity functions. In fact, from the scatter in these plots, it is not even obvious that the scintillometer data obey Monin–Obukhov similarity. Only for unstable stratification in Fig. 5 [the $g(\xi)$ plot] do the scintillometer data show any tendency to collapse to a consistent behavior.

Still with regard to Fig. 5, in stable stratification, the scintillometer $g(\xi)$ values are much smaller for $\xi > 1$ than suggested by the Wyngaard, Thiermann–Grassl, and Edson–Fairall functions. The budget $g(\xi)$ function seems to capture the tendency of the data better here; but the data are so few and scattered that they cannot provide definitive support for this particular budget function, which relies on the new Grachev et al. (2007) functions, (3.12b) and (3.16b).

In contrast to Fig. 5, Hoedjes et al. (2002) found that $g(\xi)$ values inferred from $C_n^2$ measurements with a large-aperture scintillometer collapsed reasonably well for $|z/L| < 1$, the stratification region where they obtained most of their data. The difference between their observations and mine raises the issue of whether small-aperture and large-aperture scintillometers measure similar values of $C_n^2$.

Figure 6 shows $\phi_\epsilon(\xi)$ as deduced from both the SHEBA and Rapid Forcing datasets. Again, the data are too scattered to let us choose among the similarity functions. If anything, we can say that, for stable stratification with
\( \zeta > 1 \), the budget \( \phi_v \) is the poorest choice among the candidate functions. Again, though, the data offer no evidence that the scintillometer measurements follow Monin–Obukhov similarity.

The right panel in Fig. 6, which focuses on near-neutral stratification, highlights a troubling feature of the scintillometer data and, thus, explains why the Scintec scintillometer generally underestimates \( u^* \) (Figs. 2 and 4).

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**Fig. 5.** The similarity function \( g(z/L) \) is evaluated according to (2.6) from scintillometer measurements of \( C_n^2 \) and from eddy-covariance measurements of \( \theta_u \) and \( L \) during SHEBA (SH) and the Rapid Forcing Experiment (RF). The data are distinguished by whether \( C_n^2 < 10^{-14} \text{ m}^{-2/3} \) or \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \). The four candidate expressions for \( g(z/L) \) are also shown: (3.1), (3.3), (3.5), and (3.11). (left) All of the data; (right) the focus is on near-neutral stratification.

**Fig. 6.** The similarity function \( \phi_v(z/L) \) is evaluated according to (2.2) and (2.8) from scintillometer measurements of inner scale \( \ell_0 \) and from eddy-covariance measurements of \( u^* \) and \( L \) during SHEBA and the Rapid Forcing Experiment. The data are distinguished by whether the scintillometer also measured \( C_n^2 < 10^{-14} \text{ m}^{-2/3} \) or \( C_n^2 \geq 10^{-14} \text{ m}^{-2/3} \). The four candidate expressions for \( \phi_v(z/L) \) are also shown: (3.2), (3.4), (3.6), and (3.15). (left) All of the data; (right) the focus is on near-neutral stratification.
The scintillometer-derived $\epsilon$ values produce $\phi_e$ values that, on average, are well below 1 at neutral stratification. Although all of the similarity functions I consider—(3.2), (3.4), (3.6), and (3.15)—predict $\phi_e = 1$ for $\zeta = 0$, Fig. 6 suggests that $\phi_e \approx 0.3$ for $\zeta = 0$. Figure 6 displays so many small $\phi_e$ values because the scintillometer underestimates $\epsilon$ (cf. Hartogensis et al. 2002); as a result, (3.17b) underestimates $u_\ast$. 

While sporadic suggestions have appeared in the literature that $\phi_e$ at neutral stratification does not equal 1 because production does not perfectly balance dissipation, the imbalance is typically only about 15%: that is, $\phi_e \approx 0.85$ near $\zeta = 0$ (e.g., Frenzen and Vogel 1992, 2001). No reliable measurements with traditional turbulence instruments have ever reported $\phi_e \sim 0.3$ at neutral stratification. Moreover, both the SHEBA and Rapid Forcing data agree that $\phi_e$ is much smaller than 1 near neutral stratification. In other words, this result cannot be caused by the nonideal geography of the Rapid Forcing site.

7. Conclusions

Simultaneous scintillometer and eddy-covariance data from two diverse sites provide a generally consistent picture of how well the Scintec surface-layer scintillometer system SLS20 does in providing path-averaged values of the surface sensible heat flux and momentum flux (represented here as the friction velocity). One site, over Arctic sea ice (the SHEBA experiment), was ideal for micrometeorological research but featured small values of $H_s$. The second site, a mowed, midlatitude field in spring (the Rapid Forcing Experiment), provided much larger heat and momentum fluxes but was complex.

The SHEBA data were nearly evenly distributed between cases with $C_n^2 < 10^{-14}$ $m^{-2/3}$ and $C_n^2 \geq 10^{-14}$ $m^{-2/3}$. In general, scatterplots of scintillometer-derived and eddy-covariance measurements of $H_s$ and $u_\ast$ showed larger mean bias errors and root-mean-square errors when $C_n^2 < 10^{-14}$ $m^{-2/3}$ than when $C_n^2 \geq 10^{-14}$ $m^{-2/3}$. I attribute this effect to poorer signal-to-noise ratio when $C_n^2 < 10^{-14}$ $m^{-2/3}$. This dependence on signal strength probably limits the Scintec’s utility for inferring fluxes to situations when the surface layer stratification is not near neutral.

During the Rapid Forcing Experiment, almost all $C_n^2$ values were above $10^{-14}$ $m^{-2/3}$, but the data metrics corroborated what I saw during SHEBA despite the complexity of the Rapid Forcing site. When the eddy-covariance $u_\ast$ is small ($\sim 0.1$ $m$ $s^{-1}$ or less), the scintillometer tends to overestimate $u_\ast$. When the eddy-covariance $u_\ast$ is larger, the scintillometer underestimates $u_\ast$. De Bruin et al. (2002) and Hartogensis et al. (2002) observed this same behavior in their Scintec SLS20s.

For the most reliable data—that is, when $C_n^2 \geq 10^{-14}$ $m^{-2/3}$—the scintillometer tended to underestimate the magnitude of the $H_s$ when compared with the eddy-covariance value. That is, in stable stratification, when $H_s$ is negative, the scintillometer estimate of $H_s$ was larger (less negative) than the eddy-covariance measurement. In unstable stratification, when $H_s$ was positive, the scintillometer estimate of $H_s$ was smaller (less positive) than the eddy-covariance measurement. This result is explained, at least in part, by the scintillometer’s tendency to underestimate $u_\ast$ because the scintillometer $H_s$ is calculated as $-pc_p u_\ast \theta_w$.

I used four distinct sets of similarity functions, $g(\zeta)$ and $\phi_e(\zeta)$, for inferring $u_\ast$ and $H_s$ from the scintillometer measurements of $C_n^2$ and $\epsilon$, designated the Wyngaard, Thiermann–Grassl, Edson–Fairall, and budget functions. No single set of functions stood out as producing better agreement between scintillometer and eddy-covariance fluxes than the other functions did. The correlation coefficient, the mean bias error, and the root-mean-square error that I used as quality metrics for scatterplots of $u_\ast$ and $H_s$ were generally similar for the four sets of functions. The Wyngaard functions did converge to solutions more often than the other three functions, but the resulting scintillometer heat fluxes, especially, were often anomalous.

Alternatively, I used the scintillometer $C_n^2$ and $\epsilon$ data, in combination with the eddy-covariance measurements of $u_\ast$, $\theta_w$, and $L$, to calculate the similarity functions $g(z/L)$ and $\phi_e(z/L)$. This, I believe, is the first time that scintillometer data have been used to determine $\phi_e(z/L)$. My analysis also extends previous scintillometer estimates of $g(z/L)$ into very stable stratification. These calculations unfortunately also failed to provide any clearer guidance for choosing among the four candidate sets of similarity functions than did the flux comparisons.

Only for a small portion of the data in unstable stratification did the $g(\zeta)$ values show any tendency to collapse to a common curve, as predicted by similarity theory. Otherwise, the $g(\zeta)$ and $\phi_e(\zeta)$ data showed no collapse to “universal” similarity functions. As for the question of choosing among the similarity functions, the only conclusions I could make were that the budget function for $g(\zeta)$ had more appropriate values when $\zeta > 1$ than did the other three functions but that the budget function for $\phi_e(\zeta)$ is too small when $\zeta > 1$.

An important result from these calculations is that the scintillometer data give $\phi_e$ estimates at neutral stratification that are near 0.3 instead of the common result $\phi_e(\zeta = 0) = 1$. The reason is that the Scintec scintillometer underestimated $\epsilon$ during both the SHEBA and
Rapid Forcing experiments. This underestimate of $e$ is the fundamental reason why the scintillometer-derived $u_*$ was negatively biased when compared with the eddy-covariance $u_*$ when $u_* > 0.1$ m s$^{-1}$.

The lack of any consistent behavior in the scintillometer-derived $g(\xi)$ and $\phi_e(\xi)$ values and this serious underestimate of $\phi_e$ at neutral stratification make it impossible for me to conclude that my Scintec scintillometer data follow Monin–Obukhov similarity theory, at least as it is commonly practiced. Although other experiments with other types of scintillometers have indirectly confirmed that Monin–Obukhov similarity works for scintillometer data by producing favorable comparisons between scintillometer and eddy-covariance fluxes (most often, the sensible heat flux), my results urgently recommend direct validation that scintillometers satisfy similarity theory. That is, we need to test further whether $g(\xi)$ and $\phi_e(\xi)$ functions calculated from scintillometer data collapse to universal functions, as I have tried in Figs. 5 and 6. Only when we verify this universal behavior for both stable and unstable stratification can we confidently use scintillometers for estimating path-averaged fluxes.

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REFERENCES


