Estimating Spatially Varying Severity Thresholds of a Forest Fire Danger Rating System Using Max-Stable Extreme-Event Modeling*

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(Manuscript received 25 February 2014, in final form 19 November 2014)

ABSTRACT

Fire danger indices are used in many countries to estimate the potential fire danger and to issue warnings to local regions. The McArthur fire danger rating system is used in Australia. The McArthur forest fire danger index (FFDI) uses only meteorological elements. It combines information on wind speed, temperature, relative humidity, and recent rainfall to produce a weather index of fire potential. This index is converted into fire danger categories to serve as warnings to the local population and to estimate potential fire-suppression difficulty. FFDI values above the threshold of 75 are rated as extreme. The spatial behavior of large values of the FFDI is modeled to investigate whether a varying threshold across space may serve as a better guide for determining the onset of elevated fire danger. The authors modify and apply a statistical method that was recently developed for spatial extreme events, using a “max-stable” process to model FFDI data at approximately 17 000 data sites. The method that is described here produces a quantile map that can be employed as a spatially varying fire danger threshold. It is found that a spatially varying threshold may serve to more accurately represent high fire danger, and an adjustment is proposed that varies by local government area. Temporal change was also investigated, and evidence was found of a recent increase in extreme fire danger in southwestern Australia.

1. Introduction

Fire danger rating systems are used around the world to estimate the potential for wildfires to break out, spread, do damage, or be controlled (Chandler et al. 1983, p. 409). Wildfires can be started by natural causes such as lightning (Dowdy et al. 2009), by accidental

ignition such as vegetation contacting overhead power lines (Lin et al. 2011), and by arson (Beale and Jones 2011). Fire danger rating systems may be used to assist in suppression-preparedness planning, to inform land-planning and life-safety management policies, and to issue warnings to the general public. In Australia, where wildfires are more commonly called bushfires, the fire danger rating systems of McArthur (1966, 1967) are used. These systems combine weather and fuel information to calculate an index of fire danger that is used to define the fire danger rating. Of the two fire danger rating systems in use in Australia, we constrain our focus to the forest fire danger index (FFDI), but our method could also be applied to the grassland fire danger index (Sullivan et al. 2012).

*Supplemental information related to this paper is available at the Journals Online website: <http://dx.doi.org/10.1175/JAMC-D-14-0041.s1>.

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A forecast of high FFDI values may trigger clauses in legislation intended to reduce the potential for fire outbreaks, and may lead to advice for early evacuation of residential properties. For example, on a day of predicted extreme fire danger (currently a maximum FFDI of 75 or greater), the state of New South Wales (https://www.fire.nsw.gov.au/gallery/files/pdf/bushfires/fire_danger_ratings.pdf) advises the public that “staying and defending should only be considered if your home is well prepared, specifically designed and constructed for bush fire and you are currently capable of actively defending it.” We emphasize that the word “extreme” is currently the standard terminology used in Australia to refer to FFDI values above 75 and should be interpreted in this context for the remainder of the article.

There are many social and economic factors involved in fire prevention and suppression. In this article we focus only on the information provided by the FFDI. The FFDI is widely used across all of Australia, not only for fire management but also for community awareness through street signs and news reports. The FFDI is used to implement restrictions on, for example, campfires, incinerators, and solid-fuel barbecues, with the precise implementation varying by Australian state. The FFDI, as discussed in [appendix A](#), is based only on meteorological data. It is of particular importance in heavily forested and populated areas.

While the underpinnings of the McArthur fire danger rating systems are based on the consideration of the behavior and difficulty of suppression of a large number of individual fires burning under a range of conditions ([McArthur 1967](#)), the systems are intended for application at a regional level in a nationally uniform manner. Recent work by, for example, [Dowdy et al. \(2009\)](#) has suggested that the FFDI should be interpreted more locally. For example, Tasmania has lower temperatures and a generally lower FFDI than the rest of Australia, but it still has serious bushfires despite the fact that an index value of 75 is rarely exceeded ([Fox-Hughes 2008](#)).

[Dowdy et al. \(2009\)](#) assert that the practical significance of the FFDI can vary among different regions even though the warning system applies the same severity threshold across the whole of Australia. We use modern statistical methods to quantify this assertion and attempt to determine whether a spatially varying severity threshold across Australia would be of benefit. There is clearly a trade-off between simplicity and spatial variation of the threshold, and so we seek to determine whether a varying threshold can retain simplicity while providing a better assessment of practical significance. We also investigate temporal change by applying our method to each decade of FFDI data. We only discuss the Australian fire danger rating system here, but there are

many other implementations used in different countries. The fire weather index that was developed in Canada ([Van Wagner 1987](#)) is perhaps the most widespread. It is used by Canada, New Zealand, and several countries in Europe and Asia. The United States uses the National Fire-Danger Rating System of [Deeming et al. \(1978\)](#).

To model the largest values of the FFDI we employ modern spatial statistical techniques derived from extreme-value theory. We apply the Bayesian “max-stable” hierarchical model of [Reich and Shaby \(2012\)](#), with amendments intended to allow the approach to be applied to a large number of sites (see [section 3](#)). Our primary focus is in the construction of quantile maps from which we can identify suitable severity thresholds. [Reich and Shaby \(2012\)](#) use a latent-variable modeling approach ([Casson and Coles 1999](#)) that enables a good assessment of the variation of return levels across space ([Davison et al. 2012](#)), and they also incorporate spatial-dependence parameters to provide a more realistic spatial structure on extreme events. Some other recent applications of Bayesian models to spatial extremes include [Cooley et al. \(2007\)](#), [Gaetan and Grigoletto \(2007\)](#), [Sang and Gelfand \(2009\)](#), [Schliep et al. \(2010\)](#), and [Apputhurai and Stephenson \(2013\)](#). For alternative approaches that are based on composite likelihoods see, for example, [Smith and Stephenson \(2009\)](#), [Padoan et al. \(2010\)](#), and [Ribatet et al. \(2012\)](#).

As far as we are aware, this is the first published application of spatial extreme-event models to fire danger indices, although [Mendes et al. \(2010\)](#) apply extreme-event models to wildfire sizes. Publications using more general statistical modeling and point-process methods to fire locations and sizes include [Preisler et al. \(2004\)](#), [Turner \(2009\)](#), and [Moreira et al. \(2010\)](#). Applications to fire danger indices are less common, perhaps because of the difficulty of obtaining suitable historical data. [Lucas \(2010\)](#) constructs historical fire danger indices at 77 observation stations in Australia, and [Dowdy et al. \(2009\)](#) is the only publication we know of that presents maps of gridded fire danger indices for Australia. [Sanabria et al. \(2013\)](#) fit univariate models to 77 sites using the data of [Lucas \(2010\)](#) and subsequently perform spatial interpolation on these univariate-model outputs.

We modify the methodological framework of [Reich and Shaby \(2012\)](#) to allow for both flexibility and computational feasibility when applied to many thousands of sites. In particular, we propose a local spatial-dependence function so that observations separated by a predefined distance are independent. This feature allows fast updates of site-specific parameters. We also select knot locations with varying densities within different climates. These modifications lead to dramatic computational gains. Our implementation will be incorporated into

version 2 of the “extRemes” statistical software package (Gilleland and Katz 2011; Gilleland et al. 2013), which will allow any applied scientist to reproduce the type of analysis given here.

This article is structured as follows. In section 2 we discuss the derivation of the FFDI data, calculated on a $0.2^\circ \times 0.2^\circ$ resolution grid over approximately 17 000 sites. The derivation of historically accurate FFDI data is challenging, and so it is discussed in some detail. In section 3 we describe the model that we apply to our data, and in section 4 we present the model fit and results. Section 5 provides a brief discussion.

2. FFDI data

The FFDI is calculated from four variables: wind speed, air temperature, relative humidity, and a drought factor. The drought factor is a numerical index from 0 to 10 (Griffiths 1999) that is based on recent rainfall events and longer-term soil-moisture deficit. The calculation of the FFDI is performed using an equation of Noble et al. (1980) that was derived by fitting a model to data that were read from the original cardboard Mark 5 Forest Fire Danger Meter (McArthur 1967). The equation is given by

$$\text{FFDI} = 2D^{0.987} \exp(-0.45 - 0.0345H + 0.0338T + 0.0234V), \quad (1)$$

where $D \in [0, 10]$ is the drought factor, H is relative humidity (%), T is air temperature ($^\circ\text{C}$), and V is (typically 10 min) average wind speed at a height of 10 m in the open (km h^{-1}). In our implementation, T is the daily maximum temperature and H is the relative humidity at the daily maximum temperature. The index is purely meteorological and does not take into account specifics related to fuel or topography. Appendix A discusses this equation and other mathematical details regarding the calculation of the FFDI.

The implementation of the FFDI is not consistent across Australia because different methods are employed for determining the soil-moisture deficit used in the drought-factor calculation. The most widely used measure is the Keetch–Byram drought index (KBDI) of Keetch and Byram (1968), which is employed in Queensland, New South Wales, South Australia, and Victoria. To ensure consistency we exclusively use the KBDI. For alternative approaches see, for example, Finkele et al. (2006), Liu et al. (2003), and Li et al. (2003).

To calculate the FFDI we use “SILO” (not an acronym) gridded climate data on rainfall, temperature, and relative humidity (Jeffrey et al. 2001). SILO is an archive of Australian climate and rainfall data that are derived from observational data. The SILO data provide daily

aggregate rainfall, daily temperature maxima, and relative humidity at the daily maximum temperature that is directly calculated from atmospheric water vapor pressure. The values are spatially interpolated from recorded observations at observation stations maintained by the Australian Bureau of Meteorology. Interpolation was (from 1958 onward) performed using ordinary kriging for daily rainfall and a thin-plate-smoothing spline for daily climate variables (Jeffrey et al. 2001). The accuracy of the data is dependent on both the spatial interpolation and the coverage of the observation stations for each meteorological element.

Further information about the SILO data was available online at the time of writing (<http://www.longpaddock.qld.gov.au/silo/>). For the rainfall data, see also Tozer et al. (2011) and Zajackowski et al. (2013). The SILO gridded data products are available for purchase on their website. In our modeling we treat the SILO gridded data products as raw data, and we therefore ignore any uncertainty that derives from their interpolation method.

The FFDI also requires information about average wind speed. This requirement for historical wind speed data presents a difficulty because available historical wind speed data cannot be usefully interpolated. Even at data-site locations it is common for wind speed data to have artificial discontinuities and trends that result from changes in instrumentation and observation practices (e.g., Clarke et al. 2013). Lucas (2010) developed a method to correct for these factors in constructing historical fire danger indices at 77 observation stations, but the method is of limited use for spatial interpolation.

Dowdy et al. (2009) present maps of gridded FFDI data for Australia. They use numerical weather prediction models for wind speed, temperature, and relative humidity. They state that their numerical wind speed predictions are known to underestimate site-based observations, and they therefore make a bias correction by incorporating gust-speed prediction. This approach presents a problem when the interest is in large values since the incorporation of spatial information on high mean wind speeds will be largely inaccurate. Incorporating gust speed is also inconsistent with the design of the FFDI, which uses mean wind speed averaged over periods of 10–20 min.

We therefore decided to use wind speeds that represent only the general climate. We employ average monthly wind speed grids produced from the “MesoLAPS_PT125” numerical weather prediction model, which is a mesoscale version of the Limited Area Prediction System (LAPS) of the Australian Bureau of Meteorology. This decision means that our fire danger indices will not take into account short-term effects such as

FIG. 1. Fire danger index maxima data on a grid resolution of $0.2^\circ \times 0.2^\circ$ for the period from 1 Sep 2011 to 30 Apr 2012, using average monthly wind data. In scenarios of high daily wind speed the actual index may be much greater than the values given here. Temperature, relative humidity, and drought factors are interpolated from observation site networks.

wind shifts and also will not take into account scenarios with high mean daily wind speeds. Lucas (2010) investigates wind speed readings at several sites: in extreme cases, the daily wind speed can be 30 km h^{-1} faster than monthly averages. For example, if the average monthly wind speed at a grid location is 25 km h^{-1} , and the daily mean wind speed is 55 km h^{-1} , the FFDI would be roughly double the value that we calculate. This effect is not a significant problem for modeling because the FFDI is a purely relative measure; we are effectively fixing the wind speed to a large value on days of extreme fire danger. It does require adjustment of the model inferences, as discussed in section 4.

After calculating the FFDI for every site on every day, we take site maxima over an extended fire season, which we define as the period from 1 September to 30 April. This period captures the fire season for the majority of the areas that are prone to high-intensity bushfires (Luke and McArthur 1978). We use meteorological data in August to initialize the KBDI (see appendix A). Our data span from the 1958/59 fire season to the 2011/12 fire season. We subsequently refer to these maxima as annual maxima, even though they are not taken over a calendar year. The SILO data have some observations available for years prior to 1958, but the data quality is poorer and the interpolation method is different from that used for the post-1958 data, and so we do not use this older information here.

Figure 1 shows the FFDI annual maxima from the 2011/12 season. The resolution of the gridded data is $0.2^\circ \times 0.2^\circ$, and so the distance between site neighbors is approximately 20 km. The FFDI is intended to determine relative fire danger for forested regions, and so there are large parts of the continent for which this index

is less relevant. In particular, the calculated FFDI is often high in arid desert regions where there is little interest in fire danger because of the lack of fuel and the lack of assets at risk. We take advantage of this fact in section 3c, where we select knot locations to focus our computation effort only on areas of interest.

3. Max-stable extreme-event model

a. General concepts

The general concepts described here are based on the construction of Reich and Shaby (2012). We briefly outline a general framework before discussing our specific implementation to the fire danger index data. Computational details and practical issues for reproducibility are deferred to appendix B. A simulation study for model validation is given in section 4 of Reich and Shaby (2012). A theoretical justification for using max-stable processes to model spatial maxima is given by Schlather (2002). This justification is an extension of standard asymptotic arguments for the componentwise maxima of random vectors (e.g., Tawn 1990).

For ease of exposition, we initially consider only a single year and focus only on the sites $\mathbf{s}_i$, $i = 1, \dots, N$, for which we have data (here, $\mathbf{s}$ is a two-element vector: latitude and longitude). Let $Y(\mathbf{s}_i)$ be the annual maximum fire danger index at the data site $\mathbf{s}_i$. For any max-stable process the marginal distribution of $Y(\mathbf{s}_i)$ is generalized extreme value (e.g., Coles 2001) with location, scale, and shape parameters given by $\mu(\mathbf{s}_i)$, $\sigma(\mathbf{s}_i) > 0$, and $\xi(\mathbf{s}_i)$, respectively. We denote this by $Y(\mathbf{s}_i) \sim \text{GEV}[\mu(\mathbf{s}_i), \sigma(\mathbf{s}_i), \xi(\mathbf{s}_i)]$, where the large tilde means “is distributed as.”

To specify spatial dependence, we transform to residuals with a common marginal distribution. We define

$$X(\mathbf{s}_i) = \left\{ 1 + \frac{\xi(\mathbf{s}_i)}{\sigma(\mathbf{s}_i)} [Y(\mathbf{s}_i) - \mu(\mathbf{s}_i)] \right\}^{1/\xi(\mathbf{s}_i)}, \quad (2)$$

so that the marginal distribution of $X(\mathbf{s}_i)$ is given by the standard Fréchet distribution $X(\mathbf{s}_i) \sim \text{GEV}(1, 1, 1)$, where $\Pr[X(\mathbf{s}_i) < x] = \exp(-1/x)$. The joint distribution of the $X(\mathbf{s}_i)$ is then modeled as

$$\begin{aligned} \Pr[X(\mathbf{s}_i) < x_i, i = 1, \dots, N] \\ = \exp \left(- \sum_{k=1}^K \left\{ \sum_{i=1}^N \left[\frac{w_k(\mathbf{s}_i)}{x_i} \right]^{1/\alpha} \right\}^\alpha \right), \end{aligned} \quad (3)$$

where $\alpha \in (0, 1]$ is a spatial-dependence parameter and $w_k(\cdot)$ are kernel basis functions with $w_k(\mathbf{s}_i) \geq 0$ and $\sum_{k=1}^K w_k(\mathbf{s}_i) = 1$ for all sites $\mathbf{s}_i$. The value K is the number of spatial knots. A knot is simply a spatial

location (see [section 3c](#)) and should not be confused with the unit of speed. The specification of $w_k()$ and K is discussed in [section 3c](#). Equation (3) is a multivariate GEV distribution. The general expression for this distribution, as given in, for example, [Tawn \(1990\)](#), has no finite parameterization. Equation (3) uses an asymmetric logistic dependence structure ([Tawn 1990](#)), which is a parameterized subclass of the more general form [see also [Coles and Tawn \(1991\)](#), [Kotz et al. \(2000\)](#), and [Stephenson \(2003\)](#)]. This equation defines the distribution of our max-stable process at $\mathbf{s}_1, \dots, \mathbf{s}_N$. In our application, the data sites $\mathbf{s}_1, \dots, \mathbf{s}_N$ are gridded, but Eq. (3) can apply more generally to any N locations.

The dependence in the model defined by Eqs. (2) and (3) is derived from two sources. First, spatial dependence is derived through the parameter α and the kernel basis functions $w_k()$. Second, dependence can be induced through stochastic-model specifications for the parameters $\mu(\mathbf{s}_i)$, $\sigma(\mathbf{s}_i)$, and $\xi(\mathbf{s}_i)$, as specified in [section 3b](#). When $\alpha = 1$, then the $X(\mathbf{s}_i)$ are independent, and the $Y(\mathbf{s}_i)$ are conditionally independent given $[\mu(\mathbf{s}_i), \sigma(\mathbf{s}_i), \xi(\mathbf{s}_i)]$. In this case, the only dependence that remains is that induced by integration over $[\mu(\mathbf{s}_i), \sigma(\mathbf{s}_i), \xi(\mathbf{s}_i)]$. This is commonly known as the latent-variable model ([Davison et al. 2012](#)). The model defined by Eqs. (2) and (3) is thus far unique in that it can include both forms of dependence and it permits an exact Bayesian analysis using the techniques detailed in [appendix B](#). It also permits straightforward spatial prediction at unobserved sites.

The max-stable process model given above can be fitted to our data by using the following construction, as based on [Stephenson \(2009\)](#). Let $\mathbf{A} = (A_1, \dots, A_K)$ be K independent random variables that are distributed

according to the positive stable distribution with index equal to the spatial-dependence parameter α (see [appendix B](#)). Also, define

$$\theta(\mathbf{s}_i) = \left[\sum_{k=1}^K A_k w_k(\mathbf{s}_i)^{1/\alpha} \right]^\alpha. \quad (4)$$

Then it follows that $Y(\mathbf{s}_i)$ are conditionally independent ([Reich and Shaby 2012](#)), with

$$Y(\mathbf{s}_i) | \mathbf{A} \stackrel{\text{ind}}{\sim} \text{GEV}[\mu^*(\mathbf{s}_i), \sigma^*(\mathbf{s}_i), \xi^*(\mathbf{s}_i)], \quad (5)$$

where $\mu^*(\mathbf{s}_i)$, $\sigma^*(\mathbf{s}_i)$, and $\xi^*(\mathbf{s}_i)$ are defined by

$$\mu^*(\mathbf{s}_i) = \mu(\mathbf{s}_i) + \frac{\sigma(\mathbf{s}_i)}{\xi(\mathbf{s}_i)} [\theta(\mathbf{s}_i)^{\xi(\mathbf{s}_i)} - 1], \quad (6)$$

$$\sigma^*(\mathbf{s}_i) = \alpha \sigma(\mathbf{s}_i) \theta(\mathbf{s}_i)^{\xi(\mathbf{s}_i)}, \quad \text{and} \quad (7)$$

$$\xi^*(\mathbf{s}_i) = \alpha \xi(\mathbf{s}_i). \quad (8)$$

This formulation permits Bayesian inference using standard Markov chain Monte Carlo techniques ([Hastings 1970](#)) to simulate from the posterior distribution of the parameters and to perform spatial prediction at observed or unobserved sites. Suppose that the kernel basis functions $w_k()$ are specified using a single bandwidth parameter $\tau > 0$. Let $\boldsymbol{\mu}$ denote the vector $[\mu(\mathbf{s}_1), \dots, \mu(\mathbf{s}_N)]$, and similarly let $\boldsymbol{\sigma} = [\sigma(\mathbf{s}_1), \dots, \sigma(\mathbf{s}_N)]$ and $\boldsymbol{\xi} = [\xi(\mathbf{s}_1), \dots, \xi(\mathbf{s}_N)]$. Let $\mathbf{Y}$ denote the vector $[Y(\mathbf{s}_1), \dots, Y(\mathbf{s}_N)]$. Also suppose that $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\xi}$ are specified using the parameter vectors $\boldsymbol{\phi}_\mu$, $\boldsymbol{\phi}_\sigma$, and $\boldsymbol{\phi}_\xi$, respectively, and let $\boldsymbol{\phi} = (\boldsymbol{\phi}_\mu, \boldsymbol{\phi}_\sigma, \boldsymbol{\phi}_\xi)$. The posterior density is then proportional to

$$L(\mathbf{Y} | \boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\xi}, \mathbf{A}, \tau, \alpha) \pi(\boldsymbol{\mu} | \boldsymbol{\phi}_\mu) \pi(\boldsymbol{\sigma} | \boldsymbol{\phi}_\sigma) \pi(\boldsymbol{\xi} | \boldsymbol{\phi}_\xi) \pi(\mathbf{A} | \alpha) \pi(\tau, \alpha, \boldsymbol{\phi}), \quad (9)$$

where $L()$ is the likelihood function as defined in [appendix B](#), $\pi(\tau, \alpha, \boldsymbol{\phi})$ is the prior density function, which is also specified in [appendix B](#), and, for example, $\pi(\boldsymbol{\mu} | \boldsymbol{\phi}_\mu)$ is the density derived from the model for $\boldsymbol{\mu}$ given in [section 3b](#). The density for the positive stable random variables $\mathbf{A}$, given by $\pi(\mathbf{A} | \alpha)$, cannot be computed in closed form. We therefore introduce a further set of auxiliary variables $\mathbf{B}$ such that $\pi(\mathbf{A} | \alpha)$ is equal to $\int \pi(\mathbf{A}, \mathbf{B} | \alpha) d\mathbf{B}$, and where the joint density $\pi(\mathbf{A}, \mathbf{B} | \alpha)$ can be easily computed. We can then replace $\pi(\mathbf{A} | \alpha)$ with $\pi(\mathbf{A}, \mathbf{B} | \alpha)$ within Eq. (9). The extension of the model to more than one year of data follows by considering one set of auxiliary variables $(\mathbf{A}, \mathbf{B})$ for each year. See [appendix B](#) for details.

b. Latent-variable specification

We assign Gaussian spatial processes to the location, scale, and shape parameters of the generalized extreme-value distribution so that $\boldsymbol{\mu}$, $\log(\boldsymbol{\sigma})$, and $\boldsymbol{\xi}$ are distributed as multivariate normal (MVN), where the logarithm is applied componentwise. To be specific, we take

$$\boldsymbol{\mu} \sim \text{MVN}(\mathbf{X}_\mu \boldsymbol{\beta}_\mu, \delta_\mu \mathbf{Q}^{-1}),$$

$$\log(\boldsymbol{\sigma}) \sim \text{MVN}(\mathbf{X}_\sigma \boldsymbol{\beta}_\sigma, \delta_\sigma \mathbf{Q}^{-1}), \quad \text{and}$$

$$\boldsymbol{\xi} \sim \text{MVN}(\mathbf{X}_\xi \boldsymbol{\beta}_\xi, \delta_\xi \mathbf{Q}^{-1}),$$

where, for example, $\boldsymbol{\beta}_\mu = (\beta_{\mu,0}, \beta_{\mu,1}, \dots)$ is a vector of parameters and δ_μ is a single scaling parameter. The design matrices $\mathbf{X}_\sigma$ and $\mathbf{X}_\xi$ contain the intercept, the latitude, the longitude, and the interaction of latitude and longitude. The design matrix $\mathbf{X}_\mu$ additionally contains indicator variables that identify each Australian state. Fire policies in Australia are state based and therefore it is natural to include this information even though it may lead to discontinuous inferences across state boundaries. The parameters $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\xi}$ are constant across time. It would be possible to include some form of temporal specification, but we instead choose to investigate behavior over time by fitting the model to different periods.

The matrix $\mathbf{Q}$ as given above is an $N \times N$ neighborhood matrix, with the i th diagonal element being equal to the number of neighbors of the corresponding site and the off-diagonal elements being equal to -1 if the sites are neighbors and 0 otherwise. Our sites are gridded, and therefore we define site neighbors using the four cardinal directions. We have additionally forced neighborhood relationships between islands, ensuring that $\mathbf{Q}$ has nullity equal to 1. This specification gives an intrinsic conditional autoregressive model for each spatial process (Banerjee et al. 2004). The rows and columns of $\mathbf{Q}$ each sum to zero, and therefore the density of each process is invariant to the addition of a constant to all components (Besag et al. 1991). This invariance implies that, for example, the density for $\boldsymbol{\mu}$ does not depend on the intercept parameter $\beta_{\mu,0}$.

The above framework defines the density $\pi(\boldsymbol{\mu} | \boldsymbol{\phi}_\mu)$ in Eq. (9) as (degenerate) multivariate normal, with $\boldsymbol{\phi}_\mu = (\boldsymbol{\beta}_\mu, \delta_\mu)$. Similar definitions apply for $\pi(\boldsymbol{\sigma} | \boldsymbol{\phi}_\sigma)$ and $\pi(\boldsymbol{\xi} | \boldsymbol{\phi}_\xi)$. It is also possible to incorporate parameter specifications for $\mathbf{Q}$, subject to the conditions that it must be nonnegative definite and that the computation must be feasible (e.g., Rue and Held 2005). In initial experiments we employed the formulation $\mathbf{Q}^* = \lambda \mathbf{Q} + (1 - \lambda) \mathbf{I}$ for $\lambda \in [0, 1]$, where $\mathbf{I}$ is the identity matrix (e.g., MacNab et al. 2004), but for our data the marginal posterior for λ was concentrated around 1 in all three processes, and we therefore removed it from the final model.

c. Kernel and knot selection

Let $\tau > 0$ be a kernel bandwidth, as in Eq. (9). A natural definition for the kernel basis functions in Eq. (3) is to take

$$w_k(\mathbf{s}_i) = \frac{\mathcal{K}(\|\mathbf{s}_i - \mathbf{v}_k\|/\tau)}{\sum_{j=1}^K \mathcal{K}(\|\mathbf{s}_i - \mathbf{v}_j\|/\tau)} \quad (10)$$

for $k = 1, \dots, K$, where $\mathcal{K}()$ is a kernel and $\mathbf{v}_1, \dots, \mathbf{v}_K$ are a fixed set of K locations (each a two-element vector: latitude and longitude) that represent spatial knots. We

FIG. 2. A map of the six major Australian climate zones that were identified by Stern et al. (2000), reproduced using data provided by the Australian Bureau of Meteorology. We combine the equatorial, tropical, subtropical, and temperate regions (top-left legend) into a single coastal region, giving three regions of differing knot density.

employ the triweight kernel $\mathcal{K}(u)$, which is proportional to $(1 - u^2)^3$ for $|u| < 1$ and zero otherwise. Initial experiments have shown that it gives results that are similar to those of a Gaussian kernel but has the computational advantage of a closed support that is informed by the data rather than by an arbitrary cutoff.

The specification of the number of spatial knots K presents a trade-off between computational burden and the accuracy of the fit. The computational burden within the fitting algorithm lies primarily in the updating of the $\mathbf{A}$ variables, and if there are fewer knots then there are fewer variables to update. Although knots are often selected as a regularly spaced grid of points, in our application we are more interested in some parts of the space than others. For example, there is less interest in the FFDI in arid desert regions where there is little vegetation. We therefore choose our knot locations by sampling data site locations at different rates.

To specify our knot locations $\mathbf{v}_1, \dots, \mathbf{v}_K$, we divide Australia into three regions by using six major climate zones (see Fig. 2) that were identified by the Australian Bureau of Meteorology (Stern et al. 2000) and are based on the Köppen climate classification (e.g., Peel et al. 2007). The three regions are desert, grassland, and coastal. The coastal region encompasses the equatorial, tropical, subtropical, and temperate climates. We sample $K = N/10$ knots in total: the relative proportions of sampled knots in the desert, grassland, and coastal regions are 0.1, 0.3, and 0.6, respectively.

4. Model inference and results

a. National results

Figure 3 is the primary result of this paper: it shows a quantile map that is derived from the model of section 3.

FIG. 3. A map of 10-yr return levels for the FFDI, as derived from the model of section 3. The return levels could be used as the basis for a spatially varying fire danger severity threshold.

FIG. 4. Half-lengths of 90% Bayesian credible intervals for 10-yr return levels for the FFDI, as derived from the model of section 3. The half-lengths represent the level of uncertainty for the return-level estimates.

It was calculated using 14 000 iterations that were simulated from the posterior distribution of the parameters via the Markov chain Monte Carlo method. We removed data sites from some smaller islands, leaving the following landmasses in the model: mainland Australia, Tasmania, Flinders Island, Kangaroo Island, Melville Island, and Groote Eylandt. This gave $N = 17\,363$ sites, using a grid resolution of $0.2^\circ \times 0.2^\circ$. As discussed in section 2, forced neighborhood relationships were used to link the islands. Figure 3 gives the FFDI values that would be exceeded in any given fire season with 10% probability. These values are posterior mean quantile estimates, derived from the model using the GEV parameters at each site and at each iteration of the Markov chain. In extreme-value terminology, the quantile estimates are often referred to as 10-yr return levels. As discussed in section 2, we adjust the quantile estimates to account for scenarios of high daily wind speed. To perform this adjustment we use the multiplier $\exp(0.0234V_+)$, where $V_+ = 30$, derived using Eq. (1).

Figure 4 displays half-lengths of 90% Bayesian credible intervals for the estimates of 10-yr return level that are depicted in Fig. 3. Posterior mean estimates for model parameters are tabulated in the online supplemental material. The only significant state indicator variable is for the state of Tasmania, where FFDI values tend to be lower. The latitude and longitude parameters, and their interaction, are significant for the location parameter but either not significant or only marginally significant for the scale and shape parameters.

Figure 3 shows that FFDI values tend not to reach extremely high levels in Tasmania, along much of the eastern coastline, and in the mountainous terrain of the Great Dividing Range. Based on the conclusions of Dowdy et al. (2009), Fig. 3 can be used as the basis for a spatially varying fire danger severity threshold across Australia. The

country can be partitioned into distinct areas, and the quantiles can be aggregated within each area to determine the threshold. In section 4b we choose to use local government areas for this purpose: the 2011 Australian Standard Geographical Classification defines 564 local government areas, including unincorporated areas.

Figure 4 shows that variability also exists in the uncertainty of the quantile estimates. This uncertainty tends to be larger in coastal areas, particularly for the Victorian and New South Wales coastlines and for the area around Perth. The uncertainty is much lower in desert regions. The variability in the uncertainty of the FFDI appears to be due to variability in the uncertainty of low relative humidity and of high temperatures (see section 4d), which both show more uncertainty in coastal regions. Note that we ignore any uncertainty deriving from the interpolation of the SILO gridded data products, and therefore the variability in the uncertainty is not due to the spatial coverage of weather stations.

Taking a flat severity threshold across Australia results in areas such as Tasmania rarely being in extreme fire danger even when there is a clear fire risk (Fox-Hughes 2008). Our method, using the spatially varying threshold depicted in Fig. 3, results in a 10% probability of each location being in extreme fire danger at least once per year. To link this explicitly to fire risk, it needs to be interpreted relative to the historical fire events at that location. In particular, if there is no vegetation, then there is no risk irrespective of the threshold value.

b. Local results

The states of Victoria and New South Wales are the most populous states of Australia, and they appear to show the most spatial variation in the threshold. Both states are located in the southeast of the country. Figure 5 shows the severity threshold in each local government

FIG. 5. A suggested spatially varying fire danger severity threshold for local government areas of Victoria, as based on regional means that were derived from the Australian map in Fig. 3. Unshaded areas did not contain any gridded data site.

area for Victoria, calculated using area means derived from Fig. 3. Around the city of Melbourne the suggested severity threshold is approximately equal to 75, which is the value currently used as a threshold for extreme fire danger across all of Australia. To the east and northeast the threshold decreases, suggesting that an FFDI value of above only 65 may be considered as extreme in these regions. Conversely, the threshold increases to the northwest, where the fuel load is typically lower.

Figure 6 presents the same information as Fig. 5 for the state of New South Wales. The general pattern here is for the threshold to increase as we get farther from the coastline, although there are some deviations. For example, regions to the west of Sydney have a lower threshold than do regions directly north or south. The north coast also appears to have a lower threshold than the rest of the state. The severity threshold values for local government areas depicted in Figs. 5 and 6 are available in the online supplemental material in tabulated form. It is clear that there is practically significant spatial variation in the largest values of the FFDI.

Figures 5 and 6 present 10-yr return levels, but our model can be used to present return levels of any period, allowing higher thresholds such as 20- and 50-yr return levels to be investigated. It would also be possible to use regions that are based on fire management strategies or fire weather behavior rather than local government areas, although the boundaries of such regions would be more difficult to communicate to the general public.

c. Decadal comparisons

We additionally investigated possible temporal changes by fitting the model of section 3 to data from the five decades from the 1960s up to the 2000s. Figure 7 shows the Australian quantile maps for the decadal differences within the period 1960–2010. The main feature appears to be the recent increase in the southeast of Australia, suggesting an

FIG. 6. As in Fig. 5, but for New South Wales and Australian Capital Territory.

increase in the frequency of extreme FFDI values for this region. There are also increasing values from the 1970s to the 1980s, but these increases are mainly constrained to desert regions with little vegetation. The raw estimates for decadal 10-yr return levels and their corresponding estimates of variability are mapped in the online supplemental material. Figure 8 shows time series plots of the return levels at a small number of sites. The half-lengths of 90% Bayesian credible intervals for these return levels range from 2 to 6 units, depending on the site. There again appears to be some evidence of an increasing fire risk in recent times, although this only applies to particular sites (e.g., Melbourne), with other sites remaining largely flat.

d. Temperature and relative humidity

As discussed in section 2, we employ average monthly wind speed grids produced from the MesoLAPS_PT125 numerical weather prediction model. This means that our fire danger indices will not take scenarios of high mean daily wind speed into account. In addition, the dryness index that is used to calculate the FFDI is often close to the maximum value of 10, at which most of the largest FFDI values occur. The principal drivers for the spatial FFDI variability are therefore the temperature and the relative humidity. We have analyzed both of these drivers using the same techniques as for the FFDI data, using annual temperature maxima and relative humidity minima. The resulting quantile maps and estimates of uncertainty that were derived from these models are given in the online supplemental material.

The figures in the online supplemental material show that annual temperature maxima tend to be smaller in Tasmania and in those regions of high altitude that extend from Melbourne to Brisbane. Annual relative humidity minima are clearly lowest in desert regions and in most of Western Australia, but they do not reach such low values in Tasmania or along the eastern coastline. It is largely the combination of these two features that results in the low FFDI threshold values

FIG. 7. Maps of decadal differences of 10-yr return level estimates for the FFDI, as derived from the model of section 3, for the (top left) 1970s vs 1960s, (top right) 1980s vs 1970s, (bottom left) 1990s vs 1980s, and (bottom right) 2000s vs 1990s. Red shading denotes an increase in the return level. Blue shading denotes a decrease in the return level.

that can be seen in Tasmania and in the southeast of the country within Fig. 3.

The estimates of uncertainty for the temperature maxima are low in desert regions, and they tend to increase a little as one moves toward the coastline. For relative humidity, the spatial variability of the uncertainty appears to be more pronounced, with a larger amount of uncertainty present in Tasmania and in the southeast.

5. Discussion

This article has presented a derivation of a gridded interpolated forest fire danger index dataset across Australia and has analyzed the data using modern statistical methods. We find that there is practically significant spatial variation in the largest values of the FFDI. Dowdy et al. (2009) compare actual fire events with the FFDI and suggest that the FFDI as a measure of fire danger should be interpreted relative to the local region. This result suggests the utility of using a model such as the one presented here as a basis for fire danger severity thresholds that vary in space. We have depicted specific examples of this for the states of Victoria and New South Wales where we specify the threshold value in each local government area.

We find that the fire danger severity threshold might be lowered in areas such as Tasmania, parts of Victoria and New South Wales, and along the eastern coastline of Australia. The lowering of the threshold would lead to

earlier warnings of fire danger in these regions. In particular, the areas to the east and northeast of Victoria and areas within about 200 km of the coastline of New South Wales would each be subject to earlier fire warnings. We conclude that the use of a spatially varying severity threshold such as that suggested here would better serve the community and would better represent the practical significance of fire danger.

The method that we use here is novel in many aspects and is applicable beyond fire science applications. It can be used in any spatial setting in which there is

FIG. 8. A plot of 10-yr return level estimates for the FFDI at nine selected sites, as derived from the model of section 3, applied to data from several decades. The sites are approximately located at the cities given in the legend.

interest in the largest or smallest values of a process. It properly accounts for the spatial structure but can still be applied in cases in which there are a very large number of data sites. In our application, we modeled gridded data, but it can be used more generally for arbitrary locations, requiring only that the neighborhood matrix be specified. The inferential method does require the manual specification of certain algorithmic variables, such as parameter starting values and the standard deviations of proposal distributions, but our software is easy to use for those with experience in Markov chain Monte Carlo techniques.

APPENDIX A

Fire Danger Index Calculation

The McArthur FFDI is given by Eq. (1). The complexity in calculating the FFDI lies in the calculation of

$$ET_t = \frac{(203.2 - KBDI_t)[0.968 \exp(0.0975T_t + 1.5552) - 8.3]}{1000[1 + 10.88 \exp(-0.6341\bar{R})]}, \quad (A2)$$

where T_t is the daily maximum temperature on day t and $\bar{R}$ is the average daily rainfall across the extended fire season. The net rainfall NR uses a canopy-drainage model in which the interception amount is approximated as the first 5 mm within consecutive wet days, where wet days are defined using a threshold of 0.2 mm. Let R_t be the rainfall and let $I_t = \sum_{i=1}^{N_R-1} R_{t-i}$ be the current interception amount, where N_R is the smallest positive integer such that $R_{t-N_R} < 0.2$. The net rainfall is then given by

$$NR_t = \begin{cases} R_t + I_t - 5 & \text{if } R_t \geq 0.2 \text{ and } I_t < 5 \\ R_t & \text{if } R_t \geq 0.2 \text{ and } I_t \geq 5 \\ 0 & \text{if } R_t < 0.2 \end{cases} \quad (A3)$$

The calculation of the KBDI from Eq. (A1) requires a starting value and an initialization period. For each extended fire season we use a starting value of zero millimeters, and we use meteorological data from August to perform the initialization. We then calculate the daily fire index from 1 September to 30 April and take the maxima over this period.

To calculate the drought factor $D \in [0, 10]$ from the KBDI values, we use the formula of Griffiths (1999). It is given by $D = \max[\min(D^*, 10), 0]$, where

D . As discussed in section 2, we calculate D via the $KBDI \in [0, 200]$, which measures the amount of water in millimeters needed to bring the soil to field capacity. The daily change in the KBDI is calculated using the difference between rainfall and an estimate of evapotranspiration that is based on daily maximum temperature. To calculate the drought factor from the KBDI, we use the formula of Griffiths (1999) and also employ the adjustment given by Finkele et al. (2006). Alternative proposals have been given by Liu et al. (2003) and Li et al. (2003).

The KBDI on day t is defined by

$$KBDI_t = KBDI_{t-1} + ET_{t-1} - NR_{t-1}, \quad (A1)$$

where ET is the evapotranspiration in millimeters and NR is the net rainfall in millimeters, which is the rainfall decreased by an amount to allow for interception and runoff. The evapotranspiration is defined by

$$D^* = 10.5 \left\{ 1 - \exp \left[- \left(\frac{KBDI + 30}{40} \right) \right] \right\} \frac{\lambda + 42}{\lambda^2 + 3\lambda + 42} \quad \text{and} \quad (A4)$$

$$\lambda = \max\{\psi_*, \psi_t, \psi_{t-1}, \dots, \psi_{t-19}\}, \quad \text{with} \quad (A5)$$

$$\psi_{t-i} = \begin{cases} (R_{t-i} - 2)/i^{1.3} & \text{if } i \geq 1 \text{ and } R_t \geq 2 \\ (R_{t-i} - 2)/0.8^{1.3} & \text{if } i = 0 \text{ and } R_t \geq 2 \\ 0 & \text{if } R_t < 2 \end{cases} \quad (A6)$$

for integers $i = 0, \dots, 19$. Note that 2 mm is used as a threshold here, whereas 0.2 mm is used for the canopy-drainage model. Also, ψ_* is an adjustment of Finkele et al. (2006), given by

$$\psi_* = \begin{cases} 0.1135 \times KBDI & \text{if } KBDI < 20 \\ 2.607 - 0.01689 \times KBDI & \text{if } KBDI \geq 20 \end{cases} \quad (A7)$$

APPENDIX B

Model Estimation

Model estimation is achieved through standard Markov chain Monte Carlo simulations (Hastings 1970) that are applied to the posterior distribution of the parameters. For a single year the posterior density is proportional to

Eq. (9), with $\pi(\mathbf{A}|\alpha)$ replaced by $\pi(\mathbf{A}, \mathbf{B}|\alpha)$, as discussed below that equation. For the extension to more than one year, suppose we have T years of data. We redefine $\mathbf{A} = [\mathbf{A}_{(1)}, \dots, \mathbf{A}_{(T)}]$, where $\mathbf{A}_{(t)} = (A_{t,1}, \dots, A_{t,K})$ for $t = 1, \dots, T$ and $A_{t,K} \geq 0$ is the positive stable

random variable corresponding to year $t \in \{1, \dots, T\}$ and spatial knot $k \in \{1, \dots, K\}$. We redefine $\mathbf{B}$ similarly, with $B_{t,k} \in [0, 1]$, and let $Y_t(\mathbf{s}_i)$ be the observation at time t and site $\mathbf{s}_i$. Then the posterior density is proportional to

$$L(\mathbf{Y}|\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\xi}, \mathbf{A}, \tau, \alpha)\pi(\boldsymbol{\mu}|\boldsymbol{\phi}_\mu)\pi(\boldsymbol{\sigma}|\boldsymbol{\phi}_\sigma)\pi(\boldsymbol{\xi}|\boldsymbol{\phi}_\xi)\pi(\mathbf{A}, \mathbf{B}|\alpha)\pi(\tau, \alpha, \boldsymbol{\phi}), \quad (\text{B1})$$

where the likelihood function is given by

$$L(\mathbf{Y}|\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\xi}, \mathbf{A}, \tau, \alpha) = \prod_{t=1}^T \prod_{i=1}^N \left(\frac{1}{\sigma^*(\mathbf{s}_i)} g[Y_t(\mathbf{s}_i)]^{\xi^*(\mathbf{s}_i)+1} \exp\{-g[Y_t(\mathbf{s}_i)]\} \right),$$

with

(B2)

$$g[Y_t(\mathbf{s}_i)] = \left\{ 1 + \frac{\xi^*(\mathbf{s}_i)}{\sigma^*(\mathbf{s}_i)} [Y_t(\mathbf{s}_i) - \mu^*(\mathbf{s}_i)] \right\}_+^{-1/\xi^*(\mathbf{s}_i)} \quad (\text{B3})$$

for $\xi^*(\mathbf{s}_i) \neq 0$, where $\mu^*(\mathbf{s}_i)$, $\sigma^*(\mathbf{s}_i)$, and $\xi^*(\mathbf{s}_i)$ are defined in section 3a and $\lambda_+ = \max(\lambda, 0)$. If $\xi^*(\mathbf{s}_i) = 0$, then Eq. (B2) is defined in the limit $\xi^*(\mathbf{s}_i) \rightarrow 0$.

The densities $\pi(\boldsymbol{\mu}|\boldsymbol{\phi}_\mu)$, $\pi(\boldsymbol{\sigma}|\boldsymbol{\phi}_\sigma)$, and $\pi(\boldsymbol{\xi}|\boldsymbol{\phi}_\xi)$ are multivariate normal, as defined in section 3b, where, for example, $\boldsymbol{\phi}_\mu = (\boldsymbol{\beta}_\mu, \delta_\mu)$. The density of the auxiliary variables ($\mathbf{A}$ and $\mathbf{B}$) is given by

$$\pi(\mathbf{A}, \mathbf{B}|\alpha) = \prod_{t=1}^T \prod_{k=1}^K \frac{\alpha A_{t,k}^{-1/(1-\alpha)}}{1-\alpha} h(B_{t,k}) \exp[-h(B_{t,k}) A_{t,k}^{-\alpha/(1-\alpha)}],$$

with

(B4)

$$h(B_{t,k}) = \left[\frac{\sin(\alpha\pi B_{t,k})}{\sin(\pi B_{t,k})} \right]^{1/(1-\alpha)} \frac{\sin[(1-\alpha)\pi B_{t,k}]}{\sin(\alpha\pi B_{t,k})}. \quad (\text{B5})$$

Last, $\pi(\tau, \alpha, \boldsymbol{\phi})$ is defined using the vague independent prior distributions. For the spatial-dependence parameters we take $\alpha \sim \text{Unif}(0, 1)$ and $\log(\tau) \sim N(0, 1)$. For the latent model parameters we take $\delta_\mu \sim \text{InvGam}(0.1, 0.1)$ and $\boldsymbol{\beta}_\mu \sim \text{MVN}(\mathbf{0}, 100\mathbf{I})$, where $\mathbf{I}$ is the identity matrix. The prior distributions for $\boldsymbol{\phi}_\sigma = (\boldsymbol{\beta}_\sigma, \delta_\sigma)$ and $\boldsymbol{\phi}_\xi = (\boldsymbol{\beta}_\xi, \delta_\xi)$ are defined similarly.

The Markov chain Monte Carlo simulations are performed using standard Metropolis–Hastings proposals (Hastings 1970). We individually update the spatial-dependence parameters (τ and α), the generalized

extreme-value parameters ($\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\xi}$), and the auxiliary variables ($\mathbf{A}$ and $\mathbf{B}$) using either normal, lognormal, or logit-normal proposal distributions. The parameters $\boldsymbol{\beta}_\mu$ and δ_μ have closed-form conditional posterior distributions, and so our proposals for these parameters simulate from the posterior directly. In particular, the conditional posterior distributions are given by

$$\boldsymbol{\beta}_\mu | \boldsymbol{\mu}, \delta_\mu \sim \text{MVN}(\mathbf{V}_\mu \mathbf{X}_\mu^T \mathbf{Q} \boldsymbol{\mu} / \delta_\mu, \mathbf{V}_\mu) \quad \text{and}$$

$$\delta_\mu | \boldsymbol{\mu}, \boldsymbol{\beta}_\mu \sim \text{InvGam}(N/2 + 0.1, S_\mu/2 + 0.1),$$

where $\mathbf{V}_\mu = (\mathbf{X}_\mu^T \mathbf{Q} \mathbf{X}_\mu / \delta_\mu + \mathbf{I}/100)^{-1}$ and $S_\mu = (\boldsymbol{\mu} - \mathbf{X}_\mu \boldsymbol{\beta}_\mu)^T \mathbf{Q} (\boldsymbol{\mu} - \mathbf{X}_\mu \boldsymbol{\beta}_\mu)$. In our case the entry in the first row and first column of $\mathbf{X}_\mu^T \mathbf{Q} \mathbf{X}_\mu$ is equal to zero. The marginal posterior variance of the corresponding intercept parameter $\beta_{\mu,0}$ is unchanged by the data because the Gaussian spatial process for $\boldsymbol{\mu}$ does not depend on $\beta_{\mu,0}$. Similar results apply to $(\boldsymbol{\beta}_\sigma, \delta_\sigma)$ and $(\boldsymbol{\beta}_\xi, \delta_\xi)$.

For the results of section 4 we simulated chains of 14 000 iterations following a burn-in period of 6000 iterations. In our application we found that the individual proposals for the positive stable variables $\mathbf{A}$ take much of the computing time required and that relatively long chains are needed because of the relatively slow mixing of α and τ . We also found that the standard deviations of the jump proposal distributions for $\mathbf{A}$ require some care to get reasonable acceptance rates across all times and all knots. For information on output diagnostics for Markov chains, see Brooks and Roberts (1998) and Cowles and Carlin (1996). Our main chain took approximately 50 h to run, and we believe that algorithmic alterations would be required to extend implementations to more-complex models or to beyond 17 000 sites. Our experiments with block-updating alterations were generally unsuccessful.

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