Virtual and Real Topography for Flows across Mountain Ranges

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(Manuscript received 8 September 2014, in final form 30 January 2015)

ABSTRACT

A combination of real and virtual topography is shown to be crucial to describe the essentials of stratified flow over mountain ranges and leeside valleys. On 14 March 2006 [Intensive Observation Period 4 of the Terrain-Induced Rotor Experiment (T-REX)], a nearly neutral cloud-filled layer, capped by a strong density step, overflowed the Sierra Nevada and separated from the lee slope upon encountering a cooler valley air mass. The flow in this lowest layer was asymmetric across and hydraulically controlled at the crest with subcritical flow upstream and supercritical flow downstream. The density step at the top of this flowing layer formed a virtual topography, which descended 1.9 km and determined the horizontal scale and shape of the flow response aloft reaching into the stratosphere. A comparison shows that the 11 January 1972 Boulder, Colorado, windstorm case was similar: hydraulically controlled at the crest with the same strength and descent of the virtual topography. In the 18 February 1970 Boulder case, however, the layer beneath the stronger virtual topography was subcritical everywhere with a symmetric dip across the Continental Divide of only 0.5 km. In all three cases, the response and strength of the flow aloft depend on the virtual topography. The layer up to the next strong density step at or near the tropopause was hydraulically supercritical for the 18 February case, subcritical for the T-REX case, and critically controlled for the 11 January case, for which a weak density step and isolating layer aloft made possible the strong response aloft for which it is famous.

1. Introduction

A combination of real and virtual topography, as opposed to the real or actual topography alone, will be shown to describe the essentials of stratified flow over mountain ranges and leeside valleys or plains when a layer capped by a strong density step exists above the topography. This cap acts as virtual topography for the stratified flow aloft and will control its response. Vosper (2004) and Jiang (2014) explored the role of such a cap in theoretical and idealized numerical simulations.

The shape of the virtual topography, described by its altitude in the along-flow direction, was observed in the data discussed in this paper. Data are from the afternoon of 14 March 2006, Intensive Observation Period 4 of the Terrain-Induced Rotor Experiment (T-REX), which took place over the Sierra Nevada, Owens Valley, and the Inyos (Fig. 1a). Doyle et al. (2011) and Smith et al. (2008) used this case to study mountain waves in the stratosphere.

We will focus on how the layer close to the ground and its cap react to the topography and in turn provide the virtual topography for the flow response aloft. Mayr and Armi (2010) and Armi and Mayr (2011) showed that a cross-barrier density difference is a necessary condition for this downslope flow in contact with the topography to overflow. Potential temperature at the crest or pass needs to be lower than on the leeside valley. Otherwise the downslope flow will separate from the slope at an altitude at which it encounters the denser valley air mass with a lower potential temperature.

We will appeal to simple single-layer reduced-gravity hydraulic concepts to help to describe the flow of the nearly neutrally stratified lowest layer. We will also compare our results with the famous Boulder, Colorado, studies carried out by Lilly and his colleagues (Lilly 1971, 1978).
2. Observations

The T-REX observations were taken in the afternoon of 14 March 2006 over the Sierra Nevada and the Inyos on the western and eastern sides of Owens Valley, respectively. The Sierra Nevada crest shown in Fig. 1c is on average at 4 km MSL but is intersected by several gaps, including Kearsarge Pass just upstream of Independence (location O in Fig. 1a). We use in situ data from the NCAR Gulfstream V (G-V) and University of Wyoming King Air aircraft (Fig. 1a). In addition, reflectivity from the cloud radar (WCR) on board the King Air, and profiles from sondes dropped from the G-V, for which the tracks are shown in Fig. 1b, are used. Clouds in the visible satellite image extend from upstream (west) past the Sierra Nevada crest, evaporate as the air descends into Owens Valley, and reform upon ascent over the Inyos.

a. The neutrally stratified cloud layer

The WCR reflectivity (Fig. 2b) shown at 1:1 aspect ratio depicts the turbulent structures within the cloud layer. In the turbulent boundary layer above the Sierra Nevada topography, the radar shows typical eddy signatures spanning the thickness of the cloud layer with an angle of approximately 30° as seen in laboratory smoke visualizations of turbulent boundary layers (cf. Falco 1977). Downstream of the topography over the valley, the turbulent structures are more typical of a mixing layer (cf. Brown and Roshko 1974) and the angle becomes steeper—approximately 45°. Dropsonde 1 in Fig. 2a penetrated the cloud layer just upstream of the crest. The constant equivalent potential temperature confirms the finding from the radar that this layer is actively mixing and is of constant density.

In the unsaturated parts of the sounding, potential temperature has to be used to assess static stability. Increased color saturation of the appropriate parts of the equivalent potential temperature and potential temperature profiles in Fig. 2a are used to assist the reader in choosing the appropriate type of potential temperature. A strong cap formed by a potential temperature increase of 9 K sits on top of the cloud layer and isolates the stably stratified, cross-barrier flow aloft from the underlying terrain. As such it forms a different, virtual topography. Over Owens Valley, the virtual topography as captured by dropsonde 2 has descended by 0.9 km at this location and the cap has become stronger. Air originating from above the crest (black curve in Fig. 2a) has a higher potential temperature than the air in the valley below 3.8 km (blue curve) and will thus separate from the leeside slope above the valley floor. Air passing through
the gap, on the other hand, is dense enough to descend to the valley floor. The lowest portion of dropsonde 1 captures the lower potential temperature of the gap flow relative to the valley air mass. It also has a higher mixing ratio.

b. Descent of the overflowing cloud layer and internal hydraulic jump

Figures 3a–c and 4b show the spatial structure of the flow using aircraft in situ measurements and the WCR reflectivity, with a 3.7:1 aspect ratio—unlike Fig. 2a, which is at 1:1. Figure 4b also includes dropsonde data from the G-V. The cloud layer overflows the Sierra Nevada and descends into Owens Valley where it separates from the lee slope at 3.2 km MSL (Fig. 2b). At the separation altitude, its potential temperature matches that of the air mass in the valley. The overflow rebounds in an internal hydraulic jump past the separation. The jump face is marked in Fig. 3c and can also be seen in the isentropes shown in Fig. 4b. The decrease in altitude above the crest of the cloud layer to two-thirds of its upstream altitude measured from the crest indicates a hydraulically controlled flow: The top of the cloud layer in the WCR image (Fig. 2b) descends continuously from 6.0 km MSL upstream to 5.5 km MSL above the crest and 4.9 km MSL at the separation downstream. The height of the top of the cloud-filled layer measured relative to the crest at 3.85 km MSL is 2.15 km upstream and 1.65 km at the crest, giving the two-thirds ratio.
The flow in the valley downstream of the jump is turbulent. Vertical velocity (Fig. 3a) is upward downstream of the jump face at the edge of the reflectivity signal. Behind the jump an up-slope flow direction (Fig. 3c) and consequently a negative cross-barrier component (Fig. 3b) are observed. Only the gap flow going through Kearsarge Pass descends to the valley floor. The lowest flight legs sampled the narrow gap-flow streak, which can be identified by its higher downslope speed component.

c. Response in the stratosphere

The shape of the cap at the top of the overflow, which forms the virtual topography, differs from that of the underlying real topography. It launches a smooth response in the flow aloft with a coherent updraft region that is approximately 20 km wide and propagates into the stratosphere. Note in Fig. 3a that the vertical velocities along the G-V track at 11 km are weaker than at 13 km. Smith et al. (2008) also included these G-V data in their analysis of mountain waves in the stratosphere.

3. Discussion and comparison with Boulder cases

Virtual topography is not uncommon and is also seen for the flows across the Rocky Mountains. We compare our case with the famous 11 January 1972 Boulder windstorm (Lilly 1978) and the less-known 18 February 1970 case (Lilly 1971). We primarily use the potential temperatures contoured in Fig. 4 and the sketches shown in Fig. 5. To make the comparison, we have aligned the positions of maximum descent of the virtual topography and stretched the sections across the Rockies to the same horizontal and vertical scales as our Sierra Nevada case. Figure 4a is adapted from Lilly and Zipser (1972), reprinted in Lilly (1978) as his Fig. 7. We followed the suggestion of Lilly and Zipser (1972, 60–61) and shifted the part above the dashed line 25 km east to account for temporal changes. Isentrope stubs from the 0000 UTC 12 January 1972 soundings of Grand Junction (upstream) and Denver (downstream), Colorado, are added to show stratification to 14 km MSL. Figure 3c is adapted from Lilly (1971, his Fig. 3), adding isentropes between 300 and 340 K to indicate that the contouring interval there is 10 K instead of 2 K.

a. Response of the virtual topography to the real topography

In all three cases the layer next to the topography is nearly neutrally stratified upstream and is capped by a strong density (potential temperature) step that forms.
the virtual topography. The 11 January case and our 14 March case have similar descents of the virtual topography of 1.9 km and also have comparable potential temperature steps of ~10 K as shown in Table 1. In the 14 March case, the flow outside the gap-flow region separated already over the lee slope, whereas the downslope storm reached the plains in the 11 January case. Another similarity is that they are both hydraulically controlled at the crest. We define the internal Froude number of layer $i$ as

$$F_i = U_i/(g' y_i)^{1/2},$$

where $U_i$ is the average speed of the layer, $g' = g \Delta \theta / \theta$ is reduced gravity computed using the potential temperature step $\Delta \theta$ at the top of the layer, and $y_i$ is the thickness of the layer. Froude numbers in the layer in contact with the topography at the crest are approximately critical,$^1$ and the flow is asymmetric with thicker flow upstream and thinner flow on the leeside slope. As a consequence, the virtual topography formed by the strong cap above the descending flow also takes this shape and is not parallel to the real topography. The virtual topography is smoother and less steep than the real topography and descends continuously from upstream of the crest as it accelerates across the crest.

The 18 February 1970 case (Fig. 4c) differs completely. Virtual topography descends only by 0.5 km—about one-quarter of the distance of the other two cases (Table 1). The density step, on the other hand, is approximately 4 times as large—46 K—and the cap forming the virtual topography is thicker. In addition, the depth of the layer between real and virtual topography is also much thicker: 3.5 km. As a consequence, the Froude number at the crest is subcritical (Table 1) despite similar cross-barrier speeds as in the other cases. The flow in the virtual topography layer is subcritical everywhere else and the virtual topography is symmetric about the crest and only dips slightly there because of the slightly higher speed across the crest.

The concept of a constant potential temperature layer capped by a step in potential temperature was recently studied theoretically and numerically by Jiang (2014). His Fig. 14 for a cap of 10 K shows flows of the layer in contact with the topography that are controlled at the crest (his Figs. 14a,c,d) and that are subcritical everywhere (his Fig. 14b), respectively. Although not commented on by Jiang, his simulations also show that the response aloft is to the virtual topography formed by the cap at the top of the constant potential temperature layer. These simulations for controlled flows and the simulation of Vosper (2004, his Fig. 8) are most similar to our 14 March 2006 case shown in Fig. 4b, although the position of his hydraulic jump downstream is not established by separation on the lee slope since the cold valley air mass influences the altitude and location of the separation on the lee slope, as discussed by Mayr and Armi (2010). This is very different from the flat terrain in the lee of the Rockies and the idealized topography of Vosper and Jiang. It affects the downstream shape of the virtual topography and can also be seen in the response aloft to this virtual topography in Figs. 4a and 4b.

Jiang’s subcritical case is most like that of 18 February 1970 shown in Fig. 4c with only a slight dip in the virtual topography and a weak response aloft to this dip downstream of the real topography. The virtual topography is not as clear in this case, and the additional contours were added in Fig. 4c to show that above 340 K the stratification weakens significantly. In our simple two-layered approach this may be somewhat of an oversimplification since the thickness of the density step capping the virtual topography layer is substantial relative to the thickness of the layer itself. As will be seen below, however, the effect on the flow aloft is weak since the virtual topography dips only slightly.

**b. The response of the flow aloft to the virtual topography**

As the virtual topography responds to the real topography beneath, the flow aloft in the troposphere responds to changes in the height of the virtual topography. When the well-mixed layer is controlled at the crest, the shape of the virtual topography is asymmetric across the crest. It continuously descends although the real topography rises and falls to form a crest. Hence the response in the troposphere can be displaced downstream with respect to the real topography.

In Table 1, the internal Froude numbers of the troposphere layers are computed for all three cases shown in Fig. 4. The computations were all made for data at the start of the descent of the troposphere layer. Essential aspects of the virtual topography layer and the response aloft to each of the three cases in Fig. 4 are summarized in Fig. 5. The Froude numbers were approximated on the basis of the strength of the potential temperature step above the troposphere layer, ignoring the much weaker stratification within the layer itself and the possible interaction with the layer underneath. We use the Froude numbers to determine whether the flows are

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$^1$ Lilly (1978) did not have speed measurements in this layer at the crest, and therefore his isotachs there are somewhat uncertain.
controlled and hence close to critical in the descending portion, supercritical everywhere, or subcritical everywhere. Essential aspects of the virtual topography layer and the response aloft to each of the three cases in Fig. 4 are summarized in Fig. 5.

For the Sierra Nevada case, the tropopause rises (Figs. 4b and 5b) since the flow between it and the virtual topography decelerates downstream of the crest as the virtual topography continues to descend (Figs. 2a and 3b). The tropopause response is therefore out of phase.
with the virtual topography—a subcritical response. The Froude number of only 0.5 is due to the very large potential temperature step aloft.

For the famous 11 January 1972 case (Figs. 4a and 5a), the virtual topography is similar. Unlike the Sierra Nevada case, however, the descending troposphere layer is critical as it flows across the Continental Divide and accelerates in response to the descending virtual topography. The essential difference is that the cap of the troposphere layer is much weaker, with a potential

FIG. 5. Essential aspects of the vertical cross sections shown in Fig. 4. (a) Virtual topography layer is a controlled overflow with the flow above the virtual topography making a transition from subcritical to supercritical flow. (b) Virtual topography layer is a controlled overflow with the flow above the virtual topography remaining subcritical. (c) Virtual topography layer is subcritical flow over the real topography with a symmetric dip at the crest. The flow aloft is supercritical with no response to the topography aloft. All velocities are shown with arrows, the lengths of which are scaled as shown below.
Table 1. A comparison of the virtual topography layer and the flow above the virtual topography for three cases. Here, \( y_{c1} \) is the thickness of the mixed layer measured from the crest to the middle of the density/potential temperature cap \( \Delta \theta \) forming the virtual topography, \( U_{c1} \) is the average cross-barrier speed of the mixed layer at the crest and \( F_{c1} \) is the internal Froude number there, \( y_2 \) is the thickness above the virtual topography from its top to the middle of the next strong potential temperature step of strength \( \Delta \theta_2 \), and \( U_2 \) is the average cross-barrier speed in this layer at the beginning of the descent of the virtual topography with \( F_2 \) being the Froude number there. Also, VTD indicates virtual topography descent. The 11 January 1972 case is from Lilly (1978); 18 February 1970 is from Lilly (1971).

<table>
<thead>
<tr>
<th>Case</th>
<th>VTD (km)</th>
<th>Cap ( \Delta \theta_1 ) (K)</th>
<th>( y_{c1} ) (km)</th>
<th>( U_{c1} ) (m s(^{-1}))</th>
<th>( F_{c1} )</th>
<th>( y_2 ) (km)</th>
<th>Cap ( \Delta \theta_2 ) (K)</th>
<th>( U_2 ) (m s(^{-1}))</th>
<th>( F_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 Jan 1972</td>
<td>1.9</td>
<td>11</td>
<td>1.4</td>
<td>(-30)</td>
<td>(-1.3)</td>
<td>6</td>
<td>8</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>14 Mar 2006</td>
<td>1.9</td>
<td>9</td>
<td>2.0</td>
<td>25</td>
<td>1.0</td>
<td>5</td>
<td>60</td>
<td>45</td>
<td>0.5</td>
</tr>
<tr>
<td>18 Feb 1970</td>
<td>0.5</td>
<td>46</td>
<td>3.5</td>
<td>25</td>
<td>0.3</td>
<td>2.5</td>
<td>8</td>
<td>80</td>
<td>3</td>
</tr>
</tbody>
</table>

A temperature step of only 8 K. This weak cap is bounded above by a stagnant isolating layer. Such isolating layers are well known. They are an essential component of the idealized downslope solution of Smith (1985) and the solution of Winters and Armi (2014), which also accounts for upstream blocking. They have also been observed at other locations such as the European and Dinaric Alps (e.g., Armi and Mayr 2007; Smith 1987). The formation of the isolating layer in a dynamically similar oceanographic context has been studied by Farmer and Armi (1999) and Armi and Farmer (2002).

For the 18 February 1970 case (Figs. 4c and 5c), the virtual topography only dips 0.5 km so that the flow response aloft is weak and decays rapidly. The layer aloft is only 2.5 km deep, which is the key ingredient for making it supercritical. As a consequence, and in contrast to the other two cases, the cross-barrier speed in this layer remains approximately constant (Lilly 1971, his Fig. 4). This speed was much higher than for the famous 11 January 1972 case (cf. Lilly 1978, his Fig. 9). At 10 km MSL the westerly wind component was 80 m s\(^{-1}\) on 18 February 1970 and 50 m s\(^{-1}\) on 11 January 1972. Despite the higher wind component aloft, no downslope wind was observed in the 18 February 1970 case as opposed to the severe downslope-wind of the 11 January 1972 case that occurred with weaker winds. The difference is due to the subcritical flow in contact with the topography and its small change in virtual topography as the flow crosses the crest. Higher cross-barrier winds do not always equate to strong downslope flows. Cases such as the 18 February 1970 one have remained unexplored in the literature.

4. Conclusions

We have introduced the concept of virtual topography to help explain the response of stratified flow over real topography when a layer of nearly constant density with a density step above it is in contact with the real topography. The essential aspects of the concept are shown in Fig. 5. We observed the layer in contact with the real topography to be hydraulically controlled for the case of 14 March 2006 (Fig. 5b), and the famous Boulder case of 11 January 1972 (Fig. 5a) is similar. In both of these cases the layer in contact with the real topography makes a transition from subcritical to critical flow at the crest and becomes supercritical downstream. It becomes subcritical again farther downstream after transitioning through a hydraulic jump. For the less-well-known case of 11 February 1970 (Fig. 5c) the layer in contact with the real topography was subcritical everywhere with only a slight dip above the crest that was due to an acceleration and deceleration as it crossed the crest, typical of subcritical flows.

The response aloft to the virtual topography at the top of the step capping the layer in contact with the real topography differed among these three cases. This response can be analyzed with the internal Froude numbers of the troposphere layers in all three cases in Fig. 5 and the virtual topography formed by the layer in contact with the real topography. For the Sierra Nevada case (Fig. 5b) the troposphere layer is subcritical and hence decelerates and slows with a decrease in the height of the virtual topography. This is a subcritical response with an associated rise in the elevation of the tropopause. Although the famous 11 January 1972 case (Fig. 5a) has virtual topography that is similar to that of the Sierra Nevada case, the troposphere layer makes a transition from subcritical to supercritical flow. The difference is in the existence of a weak cap bounded by a stagnant isolating layer. The 18 February 1970 case (Fig. 5c) had no downslope windstorm at Boulder and no response in the supercritical flow in the troposphere layer aloft.

Acknowledgments. This study would not have been possible without the careful planning and flying of the University of Wyoming King Air operations group. As mission scientist, LA is particularly thankful for their willingness and ability to carry out these turbulent flights close to the eastern slope of the Sierra Nevada.
particularly thank the following individuals from the University of Wyoming King Air (N2UW): Al Rodi, facility manager; Tom Drew, pilot; Larry Oolman, data and project manager; and from the Gulfstream V: Ron Smith, mission scientist. Georg Mayr’s participation in T-REX was funded by Austrian Science Foundation Grant P18940-N10. Larry Armi’s participation in T-REX was partly funded by The University of California and by Lucky Larry’s Auto Repair; preparation of this publication was partially funded under NSF Grant OCE-1061027. We thank the editor J. Charney and, in particular, one of the reviewers for their careful comments on the manuscript.

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