CORRIGENDUM

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ABSTRACT

Corrections are made to the results, and interpretation thereof, presented in earlier work by Baker and Lawson. The main results regarding the improvement obtained using additional image parameters are unchanged. Secondary results regarding the applicability of subgroup parameterizations are corrected. Whereas it was found in the earlier work that very few subgroup parameterizations could be applied, it is now found that more subgroup parameterizations could be applied in situations in which crystal habits are sufficiently identifiable.

1. Introduction

The ice particle image and mass data of Mitchell et al. (1990, hereinafter M90) and Baker and Lawson (2006, hereinafter BL06) were revisited in a paper by B. Baker and R. P. Lawson (2016, unpublished manuscript, hereinafter BL16) to extend results to large dendritic crystals, which were underrepresented in those previous studies. During this work, errors were discovered in BL06. These errors and their correction are the topic of this corrigendum. The main results of BL06, which are inclusion of mass parameterizations based on the additional image parameters, area, width, and perimeter, are not affected here.

BL06 also presented a bootstrap reanalysis of the dataset used by M90, which parameterized masses M on the basis of image lengths L alone. Those results were presented in Table 2 of BL06. Our corrections here are of the values and interpretation of those results. We concluded that, for the majority of the subgroups, the results were not sufficiently robust and different from the ''all'' category result to support the use of the subgroup results.

This was erroneous for two reasons. First, BL06 did not present the differences and uncertainties of both of the parameters (α and β) in the parameterization $M = \alpha L^{\beta}$. Only the differences and uncertainties of the values for β were presented. As the results of BL16 exemplify, a significant difference in α is all that is required to warrant the use of a subgroup parameterization. Second, the β uncertainties reported in BL06 are not reproducible and are corrected herein. The new values are about two-thirds of the magnitude of those reported in BL06.

2. Corrections

Both errors are corrected in the results shown in Table 1. Table 1 is similar to Table 2 of BL06. The differences are corrected uncertainties in β, which are column 5 in Table 1, and the inclusion of the uncertainties in α, which are column 8. The β uncertainties are estimated through the standard deviation of the β values of 50 000 bootstrapped linear regressions, and the means of the β values for those 50 000 regressions determine the β values themselves. Here, β is the slope m in each linear-regression result $Y = mX + b$, where Y is the natural logarithm of the masses and X is the natural logarithm of the lengths L. Parameter α is

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TABLE 1. Bootstrap reanalysis of the data of M90. The first column indicates subgroup crystal type, the second column indicates the number of data in the group, the third column presents the power law parameterization of $M(L)$ in the form $M = \alpha L^{\beta}$ determined by least squares linear regression of the logarithms of M and L as described in the text. The fourth column ($\Delta\beta$) is the absolute value of the difference in β between the all-types-above case and each other subgroup. The fifth column is the β uncertainty, and the sixth column is the β ratio, as described in the text. Columns 7–9 are, respectively, the analogous differences, uncertainties, and ratio for α.

Crystal type	N	Equation	$\Delta\beta$	β uncertainty	β ratio	α factors	α uncertainty	α ratio
Elementary needles	16	$0.00493L^{1.80}$	0.20	0.33	0.55	4.31	1.12	**3.75**
Rimed elementary needles	7	$0.00619L^{2.10}$	0.10	0.27	0.32	3.52	1.16	**2.96**
Long columns	64	$0.0124L^{1.85}$	0.15	0.25	0.54	1.69	1.17	**1.41**
Rimed long columns	27	$0.0233L^{1.83}$	0.17	0.21	0.68	1.11	1.15	*0.94*
Combinations of long columns	62	$0.0167L^{1.83}$	0.18	0.13	*1.03*	1.26	1.08	*1.13*
Rimed combinations of long columns	54	$0.0251L^{1.93}$	0.07	0.09	0.55	1.20	1.08	*1.08*
Short columns	12	$0.0642L^{2.61}$	0.61	0.28	**1.92**	3.05	1.48	**2.00**
Combinations of short columns	17	$0.0298L^{1.88}$	0.12	0.28	0.37	1.42	1.24	*1.11*
Hexagonal plates	30	$0.0274L^{2.47}$	0.47	0.30	**1.39**	1.30	1.29	*0.98*
Radiating assemblages of plates	63	$0.0187L^{2.09}$	0.09	0.11	0.61	1.12	1.06	*1.03*
Side planes	77	$0.0206L^{2.30}$	0.30	0.09	**2.44**	1.02	1.05	*0.94*
Heavily rimed dendritic crystals	20	$0.0717L^{2.23}$	0.23	0.25	*0.79*	3.41	1.23	**2.71**
Fragments of heavily rimed dendritic crystals	39	$0.0268L^{1.71}$	0.29	0.25	*1.00*	1.27	1.23	*1.01*
Aggregates of side planes	35	$0.0212L^{2.17}$	0.17	0.17	*0.82*	1.01	1.15	*0.86*
Aggregates of side planes, bullets, and columns	31	$0.0222L^{2.13}$	0.13	0.15	0.70	1.05	1.12	*0.92*
Aggregates of radiating assemblages of plates	30	$0.0229L^{1.81}$	0.19	0.14	*1.11*	1.09	1.13	*0.94*
Aggregates of fragments of heavily rimed dendritic crystals	46	$0.0341L^{1.96}$	0.05	0.14	0.26	1.63	1.08	**1.46**
All types above	630	$0.0210L^{2.00}$	—	0.04	—	—	1.03	—

similarly estimated as $\alpha = e^{\langle b \rangle}$, where $\langle b \rangle$ represents the average y intercept b of the 50 000 bootstrapped regressions and the α uncertainty is estimated as $e^{[\mathrm{stdev}(b)]}$, where stdev(b) is the standard deviation of the 50 000 bootstrapped linear-regression y intercepts b.

In addition, columns are added to facilitate visualizing for which subgroups the coefficients α and β are significantly different from the "all types above" case. These are columns 6 and 9 for α and β, respectively. For column 6 (labeled β ratio), we divide the differences in the β values between the all-above-types case and the subgroup cases, which is column 4 and is labeled $\Delta\beta$, by the combined β uncertainty estimate for each case, which is column 5 plus 0.0365; 0.0365 is the β uncertainty estimate for the all-types-above case. If the β ratio is significantly greater than 1, then the difference in β values is large relative to their combined uncertainties and we have confidence that the differences in β are real. These cases are in boldface font. If, however, the β ratio is significantly less than 1, then the difference in β values is small relative to their combined uncertainties and we cannot have confidence that the differences in β are real. These cases are in lightface font. For values of β ratio near 1, one has a borderline situation, which is indicated in italic font.

Column 9 (labeled α ratio) is similar to column 6 but for α. Here, in place of $\Delta\beta$, the numerator of the α ratio is taken as the value of α for each subgroup divided by the value of α for the all-types-above case, or its inverse, whichever is larger, that is, greater than 1. This is column 7 and is labeled "α factors." It is calculated as this ratio because α is calculated as $\alpha = e^{\langle b \rangle}$ and because the laws of exponentials and logarithms indicate that subtracting y intercepts ($\langle b \rangle$ values) is equivalent to dividing their α values. In a similar way, the denominator is taken as the product of α uncertainties in place of the sum of β uncertainties. That is, the denominator of the α ratio is taken as the product of subgroup α uncertainties (column 8) and 1.0273, which is the α uncertainty estimate for the all-types-above case.

Just as for the β ratio, if the α ratio is significantly greater than 1, then the difference in α values is large relative to their combined uncertainties and we have confidence that the differences in α are real. These cases are shown in boldface font. If, however, the α ratio is significantly less than 1, then the difference in α values is small relative to their combined uncertainties and we cannot have confidence that the differences in α are real. For values of α ratio values near 1, one again has a borderline situation, which is shown in italic font.

TABLE 2. As in Table 1, but mass M is parameterized as a function of area A instead of length L.

Crystal type	N	Equation	$\Delta\beta$	β uncertainty	β ratio	α factors	α uncertainty	α ratio
Elementary needles	11	$0.0869A^{1.28}$	0.09	0.20	0.41	1.29	1.54	0.80
Rimed elementary needles	6	$0.128A^{1.46}$	0.27	0.85	0.31	1.15	5.39	0.20
Long columns	47	$0.164A^{1.31}$	0.13	0.19	0.59	1.47	1.71	*0.82*
Rimed long columns	26	$0.177A^{1.25}$	0.07	0.10	0.54	1.58	1.19	**1.26**
Combinations of long columns	46	$0.125A^{1.15}$	0.04	0.11	0.28	1.12	1.27	*0.83*
Rimed combinations of long columns	39	$0.117A^{1.09}$	0.09	0.08	*0.86*	1.04	1.18	*0.84*
Short columns	10	$0.418A^{1.63}$	0.44	0.28	**1.45**	3.74	2.23	**1.59**
Combinations of short columns	14	$0.217A^{1.44}$	0.26	0.24	*0.98*	1.94	1.73	*1.06*
Hexagonal plates	17	$0.0502A^{1.18}$	0.01	0.32	0.02	2.22	2.35	*0.90*
Radiating assemblages of plates	36	$0.0567A^{1.03}$	0.15	0.08	**1.41**	1.97	1.17	**1.60**
Side planes	59	$0.109A^{1.24}$	0.06	0.15	0.34	1.02	1.23	0.79
Heavily rimed dendritic crystals	15	$0.186A^{1.22}$	0.04	0.12	0.27	1.66	1.30	*1.21*
Fragments of heavily rimed dendritic crystals	36	$0.0992A^{1.11}$	0.08	0.14	0.46	1.13	1.43	0.75
Aggregates of side planes	17	$0.109A^{1.08}$	0.10	0.07	*1.06*	1.02	1.08	*0.90*
Aggregates of side planes, bullets, and columns	18	$0.112A^{1.07}$	0.11	0.13	0.72	1.00	1.08	*0.88*
Aggregates of radiating assemblages of plates	13	$0.0931A^{1.28}$	0.10	0.13	0.60	1.20	1.12	*1.02*
Aggregates of fragments of heavily rimed dendritic crystals	34	$0.125A^{1.06}$	0.12	0.10	*0.95*	1.12	1.09	*0.98*
All types above	444	$0.112A^{1.18}$	—	0.03	—	—	1.05	—

3. Discussion

From Table 1, we see that, similar to BL06's result, few subgroups (in fact, only three) are well distinguished by their values of β: the short-column, hexagonal-plate, and side-plane subgroups. Six subgroups are distinguished by their values of α: the elementary needles, rimed elementary needles, long columns, short columns, heavily rimed dendritic crystals, and aggregates of fragments of heavily rimed dendritic crystals.

Although the difficulties of automatic habit classification indicate that these subgroup results may not be widely applicable, BL16 demonstrate that in some situations the use of a subgroup result can be warranted and beneficial. Because of that, together with the main result of BL06 that area is a better predictor of mass than length is, we present Table 2. Table 2 is like Table 1 except the predictor is area instead of length.

Note that there are overall fewer data points in Table 2 than in Table 1. This is because only a subset of M90's original data was of sufficient quality that the automatic image processing of BL06, which determined area and perimeter, for example, was deemed acceptable. BL06 then included addition images for their analysis, bringing their total to 864, but those data were not classified by habit and thus could not be included in Table 2. Thus, Table 2 is included because area is a better mass predictor than length, but the caveat that the dataset is reduced must be acknowledged.

In fact, all of the results of M90, BL06, and this corrigendum are subject to the same caveat that the results are not expected to be applicable, with high accuracy, to general ice-crystal datasets. However, until such time as more broadly representative datasets are obtained and analyzed, these types of results, with/despite their caveats, are the best ones that are available for use.

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