Gamma Drop Size Distribution Assumptions in Bulk Model Parameterizations and Radar Polarimetry and Their Impact on Polarimetric Radar Moments

KATHARINA SCHINAGL, PETRA FRIEDERICH, SILKE TRÖMEL, AND CLEMENS SIMMER

Meteorological Institute, University of Bonn, Bonn, Germany

(Manuscript received 9 July 2018, in final form 8 November 2018)

ABSTRACT

A suitable formulation of the rain drop size distribution (DSD) is a prerequisite for a successful assimilation of radar polarimetric information on rain into a numerical weather prediction model. Popular DSD parameterizations in two-moment bulk microphysics schemes use relations between the so-called mean-mass diameter D'_m and the DSD shape parameter μ , in order to prevent overly strong size sorting in the models. In radar polarimetry constrained-gamma DSDs with empirical relations between the shape and scale parameter are commonly used. This study compares the different DSD formulations and highlights the differences. Synthetic polarimetric radar observations for X band (9.39 GHz) and S band (3 GHz) were calculated from the different DSDs using the T-matrix method. Depending on the constraint that is assumed for the DSDs, the polarimetric moments exhibit quite different dependencies on the mean diameter, which are particularly striking for differential reflectivity Z_{DR} . To successfully assimilate observed polarimetric moments into atmospheric models, formulations—possibly more flexible than those investigated in this study—have to be found that sufficiently represent microphysical processes and at the same time are consistent with empirical relations derived from disdrometer and radar polarimetric measurements.

1. Introduction

Assumptions on the structure of the rain drop size distribution (DSD) are essential both for quantitative precipitation estimation from radar polarimetric observations and for the parameterization of precipitation microphysical processes in numerical weather prediction (NWP) models. Most polarimetric variables, traditionally also called moments, are sensitive to the DSD (e.g., Kumjian 2013), as are the computations of bulk microphysics schemes. In this paper, assumptions on the shape of the DSD such as the constrained-gamma DSD (e.g., Zhang et al. 2001) or the two-moment bulk microphysical parameterizations (e.g., Milbrandt and Yau 2005) are studied. We particularly focus on the impact of constraints on the DSD formulation on synthetic polarimetric radar observables. A realistic simulation of radar polarimetric moments from the output of NWP models is mandatory when polarimetric data are to be assimilated.

A wealth of mathematical formalizations of the DSD exist, the exponential Marshall–Palmer model (Marshall and Palmer 1948) being one of the first. Nowadays, the so-called gamma DSD (m^{-4}),

$$N(D) = N_0 D^\mu \exp(-\Lambda D) \quad (1)$$

[see Ulbrich (1983) and many others], is typically used to capture the variability of natural DSDs. Here, D is the equivalent spherical drop diameter (m), Λ is the slope parameter (m^{-1}), N_0 is the number concentration parameter [$m^{-(\mu+4)}$], and μ is a dimensionless shape parameter.

Early microphysics parameterizations in NWP models were single-moment schemes (e.g., Kessler 1969), which required assumptions on all DSD parameters that are only based on the rain rate or the total mass content of rain. More recently, two-moment schemes are used to forecast both number concentration and mass content of hydrometeors. Although single-moment schemes are still widely used in operational NWP, in mesoscale weather research two-moment schemes became a standard because they provide more flexibility and physical realism in describing cloud microphysics. A particular challenge for one- or two-moment bulk schemes is the realistic representation of size sorting. Single-moment

Denotes content that is immediately available upon publication as open access.

Corresponding author: Petra Friederichs, pfried@uni-bonn.de

schemes are unable to model size sorting. In two-moment schemes with a fixed DSD shape parameter μ , however, an excessive amount of size sorting has been found (Wacker and Seifert 2001), which is critically influenced by the assumptions about μ (Milbrandt and Yau 2005; Milbrandt and McTaggart-Cowan 2010). To represent natural DSD variability and at the same time curtail excessive size sorting, different approaches emerged. Milbrandt and Yau (2005) postulated μ - D'_m relations, with D'_m being the so-called mean-mass diameter. In a three-moment model μ can be directly computed, which potentially alleviates the sedimentation-induced problems (e.g., Milbrandt and McTaggart-Cowan 2010; Kumjian and Ryzhkov 2012). A thorough review of the advantages and disadvantages of different configurations of bulk microphysics schemes can be found in Khain et al. (2015).

Polarimetric radar variables such as reflectivity at horizontal polarization Z_H , differential reflectivity Z_{DR} , or specific differential phase shift K_{DP} contain information on the shape and size of rain droplets. The gamma DSD can be estimated from polarimetric measurements, when its three parameters N_0 , Λ , and μ are confined to two, such as by empirical relations between μ and Λ , resulting in so-called constrained-gamma models (e.g., Zhang et al. 2001, hereinafter Zhang01). This article investigates how the assumptions inherent to the constrained-gamma approach compare to typical DSDs based on μ - D'_m relations, and assesses the consequences for the estimation of radar polarimetric moments.

Bringing together polarimetric radar research and numerical weather prediction models is an active area of research. Zhang et al. (2006) applied a simplified one-parameter version of their constrained-gamma relation to characterize rain microphysics and physical processes within a single-moment numerical model. Ryzhkov et al. (2011) show that realistic polarimetric signatures can be reproduced with a spectral microphysics model, while results with one- or two-moment bulk models were less convincing. Kumjian and Ryzhkov (2012) studied the effects of size sorting on polarimetric variables simulated by bulk schemes; in accordance with the findings of Wacker and Seifert (2001) and others, they conclude that in two-moment schemes with fixed μ , size sorting is overrepresented and results in a dramatic overestimation of Z_{DR} . Jung et al. (2010) applied a polarimetric forward operator to DSDs generated by a two-moment scheme with fixed μ and showed improvements in the quality of the polarimetric moments compared to those derived using a one-moment scheme. Lupidi et al. (2011) applied a polarimetric forward operator to the two-moment variant of the Weather Research and Forecasting (WRF) Model with different fixed values for μ . To our knowledge,

Kumjian (2012) first noted a difference between μ - D'_m relations from disdrometer observations and those from microphysical modeling. Schinagl et al. (2018) studied the effects of μ - D'_m relations on polarimetric radar moments. Here we extend the latter work by including the constrained-gamma relations.

In this study we focus on two-moment scheme parameterizations with a diagnostic μ parameter. Milbrandt and Yau (2005) studied the impact of particle sedimentation on DSDs and formulated a relation between μ and D'_m . Seifert (2008) proposed a different μ - D'_m relation taking into account drop sedimentation as well as evaporation, collision-coalescence, and collisional breakup. Milbrandt and McTaggart-Cowan (2010) modified the μ - D'_m relation of Milbrandt and Yau (2005). The parameterization of Seifert (2008) is implemented in the Consortium for Small-Scale Modeling (COSMO) numerical weather prediction model (Baldauf et al. 2011) and its follow-up, the Icosahedral Non-hydrostatic (ICON) model (Zängl et al. 2015). Since this study originated from an attempt to use the output of these models in the framework of DSD retrievals, the Seifert (2008) parameterization especially attracted our attention. The parameterization of Morrison et al. (2009), as implemented in the WRF Model, uses $\mu = 0$ for rain and cloud ice and is not considered here. We investigate parameterizations for rain-only DSDs. Parameterizations for mixed-phase precipitation also exist (e.g., Morrison and Milbrandt 2015) but are beyond the scope of the paper.

We compare the polarimetric moments resulting from μ - D'_m -based DSD parameterizations with known empirical relations derived from disdrometer measurements. In section 2a we introduce the μ - D'_m relations, and in section 2b the two constrained-gamma DSDs followed by a discussion of the different DSD parameterizations in section 2c. Section 2d describes the polarimetric radar quantities and the forward operator code that is used to generate the synthetic polarimetric observables. Our results are shown in section 3, followed by a discussion in section 4 and conclusions in section 5.

2. DSD parameterizations

The various gamma DSDs are characterized by different relations between the three DSD parameters N_0 , Λ , and μ , as well as respective formulations of a mean DSD diameter. The k th moment of the DSD is defined by

$$M(k) = \int_0^\infty D^k N(D) dD. \quad (2)$$

Its mass-weighted mean diameter D_m , also denoted as mean volume diameter in Testud et al. (2001) or

Ulbrich and Atlas (2007), is given for arbitrary gamma DSDs [see (1)] by

$$D_m = \frac{\int D^4 N(D) dD}{\int D^3 N(D) dD} = \frac{\mu + 4}{\Lambda}, \quad (3)$$

with integral bounds $D_{\min} = 0$ and $D_{\max} = \infty$ and $\mu > -4$ (see appendix A). In the polarimetric radar literature, D_m is widely used and related to the different polarimetric observables.

In current two-moment schemes, most often the zeroth and third moments of a gamma DSD are used as prognostic variables. The zeroth moment is the total number concentration N_T (m^{-3}) given by

$$N_T = \int_0^\infty N(D) dD = N_0 \frac{\Gamma(1 + \mu)}{\Lambda^{(1+\mu)}} = M(0), \quad (4)$$

which is related to N_0 [$\text{m}^{-(\mu+4)}$] in (1) via

$$N_0 = \frac{N_T}{\Gamma(\mu + 1)} \Lambda^{(\mu+1)}. \quad (5)$$

The numerator N_T relates to the mass specific number concentration q_{nr} (kg^{-1}) as follows: $N_T = \rho q_{\text{nr}}$. Following Milbrandt and Yau (2005), the mass mixing ratio of rain q_r (kg kg^{-1}) is proportional to the third DSD moment. With the density of water ρ_w (kg m^{-3}) and the density of air ρ (kg m^{-3}), we obtain for the density of rain within the air–rain mixture ρ_r (kg m^{-3})

$$\rho_r = \rho q_r = \frac{\pi \rho_w}{6} \int D^3 N(D) dD = \frac{\pi \rho_w}{6} M(3). \quad (6)$$

Another definition of a mean diameter is the mean-mass diameter D'_m (unfortunately also called mean volume diameter in some studies; e.g., Szyrmer et al. 2005; Seifert 2008; Naumann and Seifert 2016), which is derived as follows from the two prognostic moments related to N_T and q_r by (5) and (6), respectively. Since the mean mass of a drop x_m (kg) is given by

$$x_m = \frac{\rho_r}{N_T} = \frac{\rho q_r}{N_T}, \quad (7)$$

its diameter—the mean-mass diameter D'_m —is

$$D'_m = \left(\frac{6x_m}{\pi \rho_w} \right)^{1/3}. \quad (8)$$

From the definitions we also find

$$D'_m = \left[\frac{M(3)}{M(0)} \right]^{1/3}. \quad (9)$$

With D'_m the slope parameter Λ (m^{-1}) can be expressed as (see appendix A)

$$\Lambda = [(\mu + 3)(\mu + 2)(\mu + 1)]^{1/3} D_m'^{-1}. \quad (10)$$

a. DSD parameterization based on the mean-mass diameter

We now introduce the two-moment bulk microphysics parameterizations of Milbrandt and Yau (2005, hereinafter MY05), Seifert (2008, hereinafter S08), and Milbrandt and McTaggart-Cowan (2010, hereinafter MMTC10). We discuss only their application to rain, so no ice particles are considered. DSD constraints in these parameterizations are formulated as μ – D'_m relations such that μ is effectively a function of the two prognostic moments N_T and q_r . This ansatz was first used by MY05, who postulated for rain the relation

$$\mu = d_1 \tanh[d_2(D'_m - d_3)] + d_4, \quad (11)$$

with D'_m in meters and $d_1 = 19.0$, $d_2 = 600 \text{ m}^{-1}$, $d_3 = 0.0018 \text{ m}$, and $d_4 = 17.0$. MY05 studied particle sedimentation for DSDs with fixed and variable μ in one-, two- and three-moment schemes and found (11) from pairs of μ and D'_m from the three-moment simulations' sedimentation profiles.

The μ – D'_m relation proposed by S08 takes into account besides drop sedimentation also evaporation, collision–coalescence, and collisional breakup. It was derived from simulations of an idealized rain event using a one-dimensional rain-shaft model with bin microphysics with a fixed temperature of $T = 20^\circ\text{C}$ and constant relative humidity and is given as

$$\mu = \begin{cases} 6 \tanh[c_1(D'_m - D_{\text{eq}})]^2 + 1, & D'_m \leq D_{\text{eq}} \\ 30 \tanh[c_2(D'_m - D_{\text{eq}})]^2 + 1, & D'_m > D_{\text{eq}} \end{cases} \quad (12)$$

with $c_1 = 4000 \text{ m}^{-1}$, $c_2 = 1000 \text{ m}^{-1}$, and $D_{\text{eq}} = 0.0011 \text{ m}$ the so-called equilibrium mean-mass diameter. S08 is used in the current two-moment scheme of the NWP-model COSMO (Baldauf et al. 2011), in which the mean-mass diameter for rain drops is limited by $D'_m \in [0.079, 1.78] \text{ mm}$ (U. Blahak 2018, personal communication).

MMTC10 modified the MY05 relation in (11) by a modified μ – D'_m relation to

$$\mu = 11.8(1000D'_m - 0.7)^2 + 2 \quad (13)$$

[see Eq. (10) in MMTC10], which is similarly to MY05 derived from sedimentation profiles of three-moment simulations.

FIG. 1. Shown are (left) μ as a function of D'_m and (right) μ as a function of Λ , as given in S08 [see (12)], MY05 [see (11)], and MMcTC10 [see (13)], and for the constrained-gamma DSDs of Zhang01 [see (14)] and Lam15 [see (15)]. For Zhang01 and Lam15 D'_m is derived from the DSD parameters using (10). The triangle and diamond indicate the values used to discuss the corresponding S08 DSDs in Table 1.

b. Constrained-gamma DSDs

In radar meteorology the DSD parameters are often assumed to follow a constrained-gamma distribution for which μ and Λ are related by a function empirically estimated from observations. A well-known example is the relation by Zhang01 with

$$\mu = -0.0016\Lambda'^2 + 1.213\Lambda' - 1.957, \quad (14)$$

(see also Brandes et al. 2003; Zhang et al. 2003; Brandes et al. 2007) with Λ' (mm^{-1}) equal to Λ in (1) but expressed in millimeters instead of meters. Zhang01 found this relation from disdrometer observations during summer 1998 in east-central Florida. A similar but considerably shallower relation was estimated by Cao et al. (2008) for disdrometer observations in Oklahoma. Other studies use normalized gamma distributions and define a median volume diameter (e.g., Kalogiros et al. 2013).

Lam et al. (2015, in the following Lam15) derived an empirical relation for Λ' as a function of μ with

$$\Lambda' = 0.041\mu^2 + 0.310\mu + 1.740, \quad (15)$$

from a 3-yr dataset of disdrometer observations from Kuala Lumpur, Malaysia (January 1992–December 1994). Observed in an equatorial climate, this dataset includes many heavy rain events with rain rates up to 200 mm h^{-1} . Since the observations in the two datasets come from very different locations, we expect that the two relations cover a considerable range of DSDs observed in nature.

c. Behavior and dependencies of gamma DSD parameters

We now compare the μ – D'_m and μ – Λ relations discussed above. We compute the shape parameter μ using

(11)–(13) for D'_m on the interval $[10^{-6}, 0.002] \text{ m}$ with a resolution of 10^{-6} m , and Λ from μ and D'_m using (10). For the two constrained-gamma DSDs of Zhang01 and Lam15, the μ – Λ relation is directly given by (14) and (15), respectively, and D'_m is generated for discrete pairs of (Λ, μ) by inverting (10).

Figure 1 (left) shows the respective μ – D'_m relations. While μ monotonically increases with D'_m for MY05, the modifications of S08 and MMcTC10 show a non-monotonic behavior. For all three relations μ increases with D'_m in the gravitational sorting regime (i.e., when D'_m is large), resulting in a narrowing of the DSDs (Fig. 1, left). The μ – D'_m relations derived from the constrained-gamma DSDs of Zhang01 and Lam15 start at small D'_m with negative μ , which increases with increasing D'_m but only up to a maximum of about 1.1 (1.7) mm for Zhang01 (Lam15). Then for both relations D'_m slowly decreases while μ further strongly increases, leading to an ambiguous relation. The higher values of D'_m in the Lam15 relation are probably related to the stronger precipitation events in the corresponding dataset. The μ – Λ relations (Fig. 1, right) show the opposite pattern: while the empirical relations of Zhang01 and Lam15 exhibit a monotonic increase of μ with respect to Λ , the relations of MY05, S08, and MMcTC10 are ambiguous and show two branches of μ both increasing with Λ .

The relations between the two formulations of a mean diameter (D_m and D'_m) are displayed in Fig. 2 (left): D_m increases with D'_m for the MY05, S08, and MMcTC10 formulations of the DSD, while the constrained-gamma DSDs of Zhang01 and Lam15 are ambiguous, with D'_m attaining a maximum while D_m still increases. Using (3) the μ – D'_m dependency can be translated into a μ – D_m dependency (Fig. 2, right). In the MY05 relation μ increases

FIG. 2. (left) Mass-weighted mean diameter D_m against mean-mass diameter D'_m and (right) μ as a function of D_m . We compute D_m from the DSD values by (3). The triangle and diamond indicate the values used to discuss the corresponding S08 DSDs in Table 1.

monotonically also with D_m , whereas the S08 and MMcTC10 relations reveal a minimum in μ at a D_m of about 1 (2) mm for MMcTC10 (S08), and S08 even shows an ambiguous regime around $D_m = 1.9$ mm. For the two empirical constrained-gamma DSDs μ increases with D_m .

d. Polarimetric radar forward operators

To investigate the influence of the different DSD formulations on the polarimetric radar variables, we employ a radar forward operator, which computes the polarimetric variables from the DSDs accounting for electromagnetic scattering, absorption, and phase shifts. The horizontal reflectivity Z_h and the vertical reflectivity Z_v (both in $\text{mm}^6 \text{m}^{-3}$) are given by

$$Z_h = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_{D_{\min}}^{D_{\max}} |f_{\text{HH}}(\pi, D)|^2 N(D) dD, \quad (16)$$

$$Z_v = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_{D_{\min}}^{D_{\max}} |f_{\text{VV}}(\pi, D)|^2 N(D) dD \quad (17)$$

(e.g., Brandes et al. 2004), with $f_{\text{HH}}(\pi, D)$ and $f_{\text{VV}}(\pi, D)$ being the backscattering amplitudes of a raindrop at horizontal and vertical polarization, respectively, with the equivolume diameter D . The subscript HH (VV) stands for horizontal (vertical) polarization, both receiving and transmitting; λ is the radar wavelength (cm); and K_w is the dielectric constant of water. For convenience, the reflectivities are expressed in the logarithmic unit of decibels of reflectivity (dBZ) using $Z_{\text{H,V}} = 10 \log Z_{\text{H,V}}$.

The differential reflectivity Z_{DR} (dB) is the ratio between the reflected horizontal and vertical power returns (e.g., Brandes et al. 2004)

$$Z_{\text{DR}} = 10 \log \frac{Z_h}{Z_v} = Z_H - Z_V. \quad (18)$$

In (18) the dependence of $Z_{\text{H,V}}$ on N_0 via $N(D)$ cancels out as Z_{DR} is the ratio of Z_h and Z_v ; that is, Z_{DR} does also not depend of the total number concentration N_T of particles. Since Z_{DR} increases with increasing oblateness of the drops, and oblateness increases with increasing drop size (e.g., Pruppacher and Beard 1970), Z_{DR} varies with the mean drop size. The specific differential phase K_{DP} ($^\circ \text{km}^{-1}$), given by

$$K_{\text{DP}} = \frac{180\lambda}{\pi} \int_{D_{\min}}^{D_{\max}} \text{Re}[f_{\text{HH}}(0, D) - f_{\text{VV}}(0, D)] N(D) dD \quad (19)$$

(e.g., Brandes et al. 2004), increases similarly to Z_{DR} with increasing oblateness, density, and water content of the drops; but in contrast to Z_{DR} , K_{DP} is also proportional to the number concentration of the hydrometeors. For a comprehensive description of the physical interpretation of the polarimetric variables, see, for example, Kumjian (2013).

We calculate the polarimetric variables with the so-called T-matrix method originally formulated by Waterman (1965). Our implementation uses the T-matrix code for nonspherical particles with fixed orientation by Mishchenko et al. (1996) and Mishchenko (2000) assuming a canting angle of zero and a temperature of 290 K (Xie et al. 2016b). The particles are assumed to be oblate water ellipsoids. The refractive index is computed according to Ray (1972). For the aspect ratio of the ellipsoids, we use the relation by Brandes et al. (2002, 2003):

$$r = 0.9951 + 0.02510D - 0.03644D^2 + 0.005303D^3 - 0.0002492D^4, \quad (20)$$

with D in millimeters. In (16)–(19) we restrict the integrals to the drop diameter interval between 0.05×10^{-3} m and $D_{\max} = 8 \times 10^{-3}$ m, and use diameter increments of $D_i = 0.05 \times 10^{-3}$ m.

For S08, MY05, and MMcTC10 we compute the DSDs on the interval for D'_m between 10^{-6} and 0.00178 m with a resolution of 10^{-6} m. For Zhang01 we compute Λ between 0.8 and 20 mm^{-1} with a resolution of 0.01 mm^{-1} and μ using (14). For Lam15 we calculate Λ using (15) for μ between -1 and 15 with a resolution of 0.01 . For all DSDs, N_0 is obtained from the respective (Λ, μ) pairs and (5) with a fixed $q_{\text{nr}} = 1500 \text{ kg}^{-1}$. Finally, the T-matrix code is applied to compute the polarimetric moments for the DSDs, and the rain rate is derived from the DSDs according to appendix B.

The calculations are performed for X band (9.39 GHz), for which most results are shown later, and also for S band (3 GHz), because we later compare $Z_{\text{DR}}-D_m$ relations with an empirical relation found for S band. X band is our main focus, since our study grew out of an attempt to assimilate observations from the Bonn X-band radar (Diederich et al. 2015a,b).

We concentrate on the polarimetric variables Z_H , K_{DP} , and Z_{DR} , where Z_{DR} is most strongly related to the mean particle size. Specific attenuation at horizontal polarization A_H and specific differential attenuation A_{DP} also increase with increasing particle size, but also with increasing concentration of rain drops similar to K_{DP} and Z_H . Thus, Z_{DR} is most appropriate to estimate the mean particle size of raindrops. Accordingly, relationships for A_H and A_{DP} are not discussed in this paper. Otto and Russchenberg (2011) and Schneebeli and Berne (2012) also found strong correlations between Z_{DR} and the backscatter differential phase δ . Trömel et al. (2013) investigated the temperature dependence of the $Z_{\text{DR}}-\delta$ relationship and conclude that δ can also be used for retrievals of the mean particle size, especially in the resonance range (>3.6 mm) when Z_{DR} exceeds 1.2 dB. The use of δ as a proxy for raindrop size, however, is especially beneficial at shorter wavelengths where Z_{DR} may be compromised by differential attenuation. At C band, δ is negligible for diameters smaller than 4 mm, while at S band δ is negligible in rain over the entire size spectrum. Thus Z_{DR} remains the most used measure for raindrop sizes especially for national weather radar networks mostly operating at C and S bands.

3. Results

In the following we used a temperature $T = 290$ K and a particle concentration $q_{\text{nr}} = 1500 \text{ kg}^{-1}$, for the other parameters see section 2d. The value of Z_H increases monotonically with D'_m for the DSD formulations of

MY05, S08, and MMcTC10 (Fig. 3, top left). With respect to D_m , the S08 DSD formulation contains a small ambiguous region at $D_m = 1.9$ mm, which coincides with a bump at $D'_m = 1.1$ mm (see also Fig. 2). Because of the nonmonotonic relation between D'_m and D_m , Z_H first increases with D'_m for the constrained-gamma relations of Zhang01 and Lam15 but then decreases again, creating a range of ambiguity. For the Zhang01 constrained-gamma relation Z_H increases monotonically with D_m , whereas Z_H drops at D_m values above 2.2 mm for the Lam15 relation (Fig. 3, top right). Similar features are observed for K_{DP} , with a monotonic increase of K_{DP} with D'_m and a non-monotonic dependence on D_m at about 1.9 mm for the MY05, S08, and MMcTC10 relations. For large D'_m the values of K_{DP} become unrealistically high for the MY05, S08, and MMcTC10 relations. However, a large D'_m indicates a regime with excessive size sorting where q_{nr} is small. Thus a simultaneous occurrence of large D'_m and q_{nr} is very improbable and not relevant in practice. For the Zhang01 relation, K_{DP} increases with D_m and slightly drops again for $D_m > 2.5$ mm. The value of K_{DP} shows a much larger increase with D_m for Lam15, but dramatically drops at values between 2 and 2.5 mm (Fig. 3, bottom right). The rain rate shows a very similar dependence on D'_m and D_m as K_{DP} (Fig. 4).

Since Z_{DR} is independent of q_{nr} [i.e., N_0 cancels out in (18)] it directly relates to D_m (mm). Bringi and Chandrasekar (2001, BC01 in the following) derived an empirical relation at S band (3 GHz):

$$D_m = 1.619 Z_{\text{DR}}^{0.485}. \quad (21)$$

Like most empirical relations, the BC01 relation represents a long-term-average relationship derived from a dataset that resulted from observations during many different synoptic conditions with different dominating processes acting at different times. Figure 5 compares this empirical relation with $Z_{\text{DR}}-D_m$ following MY05, S08, and MMcTC10 as well as the constrained-gamma relations of Zhang01 and Lam15 for S band. MY05 meets the general shape of the BC01 relation but exhibits too low values. The $Z_{\text{DR}}-D_m$ relation of the Zhang01 constrained-gamma DSD roughly follows the BC01 relation. For both relations Z_{DR} increases monotonically with D_m , but Z_{DR} starts to diverge for $D_m > 2.5$ mm. The Lam15 constrained-gamma DSD, however, results in a non-monotonic relation above $D_m = 2$ mm. The S08 parameterization yields Z_{DR} values close to the Bringi relation for a D_m below 2 mm, but then becomes nonmonotonic and drops down to the MY05 relation. MMcTC10 meets the empirical relation best for medium D_m , but also underestimates Z_{DR} for large D_m . The $Z_{\text{DR}}-D'_m$ relations are also shown for completeness (Fig. 5, left).

FIG. 3. Polarimetric moments (top) Z_H and (bottom) K_{DP} as a function of (left) D'_m and (right) D_m using the DSD formulations of S08, MY05, and MMcTC10 [see (13)], and for the constrained-gamma DSDs of Zhang01 and Lam15. The polarimetric moments are derived for X-band frequency (9.39 GHz) with $q_{nr} = 1500 \text{ kg}^{-1}$ and a temperature of 290 K. The vertical line indicates the maximum D'_m or D_m that is used for rain in the two-moment scheme of COSMO-DE.

It remains unclear for which range of D_m the relations between Z_{DR} and D_m in Fig. 5 should be considered valid. The vertical line in Fig. 5 indicates the maximum D_m that is used for rain in the two-moment scheme of the NWP model COSMO-DE. However, Bringi and Chandrasekar (2001) and Bringi et al. (1998) fit their D_m – Z_{DR} relation to values of D_m of up to about 3.5 mm, and Lam et al. (2015, their Table 5) report observed D_m values of up to 4.95 mm. This calls for more extensive studies also of the empirical relation between Z_{DR} and D_m .

Table 1 displays the parameters and polarimetric moments obtained for two values of the μ – D'_m relation of S08 indicated with a triangle and a diamond in Figs. 1 and 2. Both parameter sets lead to similar Z_H and Z_{DR} despite rather different D_m , D'_m , and q_{nr} . The first case (triangle) has a smaller Λ , which results in a wider DSD tail and a smaller μ , which moves the mode of the DSD toward smaller D and thus results in a narrower DSD. For the second case (diamond), the tail is

steeper due to a larger Λ resulting in smaller drops, whereas μ is larger, which makes the DSD wider around its mode. Thus in certain constellations similar polarimetric moments may relate to different DSD regimes. In such cases an inversion would not provide a unique solution, and additional information is needed (e.g., on the prevailing microphysical processes). Not properly accounting for such ambiguities may have profound impacts on data assimilation.

4. Discussion

Given the sensitivity of both the polarimetric forward operators and the bulk microphysics schemes to the DSD, the differences in the DSD parameter relations are clearly an issue to be solved. The sampling of disdrometers may cause part of the difference. For example, an undercatchment of large drops may lead to size sorting being less well represented in the statistical analyses of disdrometer relations [see Kumjian (2012),

FIG. 4. Rain rates as a function of (left) D'_m and (right) D_m using DSD formulation of S08, MY05, and MMcTC10, and for the constrained-gamma DSDs of Zhang01 and Lam15. The rain rates are derived for X-band frequency (9.39 GHz) with $q_{nr} = 1500 \text{ kg}^{-1}$ and a temperature of 290 K. The vertical line indicates the maximum D'_m or D_m that is used for rain in the two-moment scheme of COSMO-DE.

who compared MMcTC10 with a constrained-gamma relation]. For the two-moment bulk schemes, the mean-mass diameter D'_m is connected to size sorting. However, observations suggest that large mass-weighted mean diameters D_m , which are related to large D'_m , are not always associated with size sorting (cf. Kumjian 2012). Thus it seems disputable whether all cases of large D'_m should be interpreted as cases of size sorting, which limits the potential of the resulting gamma DSD to yield reliable polarimetric observables.

Both for the Zhang01 and Lam15 DSDs, $D'_m < 1.1 \text{ mm}$ concurs with $\mu \leq 0$, which is not reflected in the relations from the two-moment bulk microphysical models at all,

which in these models rules out certain DSDs that do occur in nature. The dependencies of μ – D'_m obtained from Zhang01 and Lam15 are ambiguous; that is, one value of D'_m can be associated with different μ values (and hence two different DSDs). Accordingly, a unique diagnostic μ – D'_m relation is not possible with a constrained-gamma relationship following either Zhang01 or Lam15. As a consequence, it is not straightforward to use the Zhang01 or Lam15 relation in a numerical model with a two-moment bulk microphysics scheme. For example, Morrison and Milbrandt (2015) used the similar relation of Cao et al. (2008) and had to restrict the calculation to $\mu > 0$.

FIG. 5. Polarimetric moment Z_{DR} as a function of (left) D'_m and (right) D_m using DSD formulation of S08, MY05, and MMcTC10, and for the constrained-gamma DSDs of Zhang01 and Lam15. Additionally, denoted by BC01, the empirical relation in (21) of Brangi and Chandrasekar (2001) is shown in the right panel. For consistency with the BC01 relation the moment Z_{DR} is derived for S-band frequency (3 GHz) with $q_{nr} = 50 \text{ kg}^{-1}$ and a temperature of 290 K. The vertical line indicates the maximum D'_m or D_m that is used for rain in the two-moment scheme of COSMO-DE.

TABLE 1. DSD parameters and polarimetric moments for two DSDs using the μ - D'_m relation of S08.

Symbol	Δ	$\diamond$
q_{nr} (kg m^{-3})	146	66
N_0 ($\text{mm}^{-\mu-1} \text{m}^{-3}$)	2675.392	4878.291
Λ (m^{-1})	3322.979	4314.442
μ (-)	1.510	4.215
D'_m (mm)	1.027	1.428
D_m (mm)	1.658	1.904
Z_H (dBZ)	32.06	33.97
Z_{DR} (dB)	1.505	1.512
ρ_{HV} (-)	0.921	0.9932
K_{DP} ($^\circ \text{km}^{-1}$)	0.1001	0.1569
R (mm h^{-1})	1.866	2.503

For the assimilation of polarimetric information as well as for the exploitation of radar polarimetry for model evaluation and the improvement of parameterizations it is mandatory that gamma DSD parameterizations used in NWP yield plausible (observed) polarimetric moments. Both the S08 and MMcTC10 parameterizations produce unexpected and possibly nonphysical behavior in the synthetic polarimetric moments with respect to their sensitivity to the mass-weighted mean diameter D_m . Both MY05 and MMcTC10 result in too-low Z_{DR} values compared to the empirical relation of Bringi and Chandrasekar (2001). S08 meets the empirical Z_{DR} - D_m relation overall well but shows an unrealistic anomaly for large mass-weighted mean diameters. Such deviations from monotony are not in line with expectations from observations (cf., e.g., Kumjian 2013). The resulting ambiguities also need special attention in data assimilation schemes. A thorough uncertainty assessment is recommended prior to the assimilation of polarimetric moments into bulk microphysics schemes (e.g., Cao et al. 2010; Wen et al. 2018).

Few alternatives to a fixed μ or diagnostic μ - D'_m relation exist for gamma DSD parameterizations in two-moment schemes. One suggestion is the usage of gamma DSDs with finite maximum diameter (Ziemer and Wacker 2012); thus future work should analyze the behavior of polarimetric moments under this model. Three-moment schemes, as investigated in Milbrandt and McTaggart-Cowan (2010), allow for the computation of all three gamma DSD parameters from the three prognostic moments and have proven to yield both more realistic size sorting and polarimetric moments, but are currently still computationally too expensive to be used in NWP.

Most empirical relations between DSD parameters, DSD moments, and polarimetric variables are based on datasets that comprise a multitude of synoptic situations and hence microphysical processes. Thus the existence

of a universal DSD parameterization is questionable at least. We may expect that the impact of prevailing microphysical processes on DSDs require spatiotemporal adjustments of a number of parameterizations to be defined. Several authors discussed these impacts on the DSD and the polarimetric variables (e.g., Bringi et al. 2003; S08; Kumjian and Ryzhkov 2012; Barthes and Mallet 2013; Trömel et al. 2013; Xie et al. 2016a,b). A possible pathway to solve the resulting parameterization problems may be the use of different relations when different microphysical processes (e.g., size sorting as a particular aspect of non-steady rain) are active. Polarimetric information may help to determine situations with active size sorting, thus providing information on the evolution of the precipitation processes. Measurements need to be stratified accordingly, in order to estimate such non-steady empirical relations.

5. Conclusions

This study compares some existing DSD parameterizations used in numerical weather prediction and polarimetric radar meteorology. Striking differences are identified between the DSD parameter relations of the two-moment bulk microphysics models such as MY05, S08, and MMcTC10, and the empirical relations as derived from disdrometer measurements such as Zhang01 and Lam15. For example, Zhang01 and Lam15 show a monotonic increase of μ with Λ , while MY05, S08, and MMcTC10 exhibit two branches of μ both increasing with Λ . This also results in a large variety of relations between mean-mass diameter D'_m and mass-weighted mean diameter D_m , as well as μ and D_m . Diverging results are evident also within the groups of similar DSD formulation concepts. Since all these relations reduce the three-parameter gamma DSD space onto a two-parameter space, the space of possible relations between the DSD parameters and prognostic moments of rain is reduced as well—potentially excluding the representation of some microphysical processes.

The differences in the DSD formulations result in significantly different polarimetric variables as derived using a T-matrix code. Particularly striking are the differences for differential reflectivity Z_{DR} , which is a promising variable for data assimilation because of its information on the mean particle size. A particular drawback of the relation by S08 is that similar values for Z_H and Z_{DR} may be obtained with different DSDs, which would make the inverse problem of DSD parameter estimation ambiguous. Our study calls for a thorough assessment of uncertainties related to DSD parameter estimations, which is a prerequisite

for any successful assimilation of radar polarimetric information into numerical weather prediction models.

Acknowledgments. This study was conducted with support from TRR 32/3 (www.tr32.de) “Patterns in Soil-Vegetation-Atmosphere Systems: Monitoring, Modeling, and Data-Assimilation” (Simmer et al. 2015) funded by the Deutsche Forschungsgemeinschaft (DFG). We are grateful to Dr. Axel Seifert (DWD, Deutscher Wetterdienst) and three anonymous reviewers for their very valuable comments on the subject.

APPENDIX A

Computation of Λ , D'_m , and D_m from DSD Moments

The formula for Λ given μ and D'_m [see (10)] can be derived as follows. Here, we abbreviate by $c := \pi\rho_w/6$. Using Milbrandt and Yau [2005, their formula (2)] we obtain

$$M(3) = \frac{N_T}{\Lambda^3} \frac{\Gamma(\mu+4)}{\Gamma(\mu+1)}, \quad (\text{A1})$$

$$\Leftrightarrow \Lambda = \left[\frac{N_T}{M(3)} \frac{\Gamma(\mu+4)}{\Gamma(\mu+1)} \right]^{1/3}, \quad (\text{A2})$$

$$\stackrel{(6)}{\Leftrightarrow} \Lambda = \left(\frac{cN_T}{\rho q_r} \right)^{1/3} \left[\frac{\Gamma(\mu+4)}{\Gamma(\mu+1)} \right]^{1/3} \\ \Leftrightarrow \Lambda = D_m^{-1} [(\mu+3)(\mu+2)(\mu+1)]^{1/3}, \quad (\text{A3})$$

where we use the definition (9). An equivalent formula in terms of the mean mass x_m is given as $\Lambda = [c(\mu+3)(\mu+2)(\mu+1)x_m^{-1}]^{1/3}$.

The usual definition of the mass-weighted mean diameter D_m is the ratio of the fourth and third DSD moments:

$$D_m = \frac{\int D^4 N(D) dD}{\int D^3 N(D) dD}. \quad (\text{A4})$$

For a gamma DSD, the indefinite integral over the k th moment of the gamma DSD is computed as

$$\int_0^\infty D^k N(D) dD = \int_0^\infty N_0 D^k D^\mu \exp(-\Lambda D) dD \\ = N_0 \frac{\Gamma(\mu+k+1)}{\Lambda^{(\mu+k+1)}}, \quad (\text{A5})$$

if $\mu+k+1 > 0$. Here, Γ is the gamma function.

Using this equality, for D_m we get

$$D_m = \frac{N_0 \frac{\Gamma(\mu+5)}{\Lambda^{(\mu+5)}}}{N_0 \frac{\Gamma(\mu+4)}{\Lambda^{(\mu+4)}}} \quad (\text{A6})$$

$$= \frac{1}{\Lambda} \frac{\Gamma(\mu+5)}{\Gamma(\mu+4)} \quad (\text{A7})$$

$$= \frac{1}{\Lambda} \frac{\Gamma(\mu+4)(\mu+4)}{\Gamma(\mu+4)} \quad (\text{A8})$$

$$= \frac{\mu+4}{\Lambda}, \quad (\text{A9})$$

where we used the functional equation for the gamma function, $\Gamma(x+1) = x\Gamma(x)$. Since we apply (2) for the fourth and third moments, it is required that $\mu > -4$ and the integral bounds are $D_{\min} = 0$ and $D_{\max} = \infty$.

APPENDIX B

Computation of Rain Rates

The rain rate (mm h^{-1}) is defined as

$$R = 6\pi \times 10^{-4} \int_0^\infty D^3 v(D) N(D) dD, \quad (\text{B1})$$

where $v(D)$ is the terminal fall speed of drops (m s^{-1}).

Parameterizations of the terminal fall speed may differ; a well-known approximation [used, e.g., by Zhang et al. (2001)] is the power law

$$v(D) = 3.778 D^{0.67}, \quad (\text{B2})$$

for D in millimeters. Inserting (B2) into (B1) results in

$$R = 6\pi \times 10^{-4} \times 3.778 \times \int_0^\infty D^{3.67} N(D) dD \quad (\text{B3})$$

$$= N_0 \times 6\pi \times 10^{-4} \times 3.778 \times \frac{\Gamma(3.67+\mu+1)}{\Lambda^{(3.67+\mu+1)}}. \quad (\text{B4})$$

REFERENCES

- Baldauf, M., A. Seifert, J. Förstner, D. Majewski, M. Raschendorfer, and T. Reinhardt, 2011: Operational convective-scale numerical weather prediction with the COSMO model: Description and sensitivities. *Mon. Wea. Rev.*, **139**, 3887–3905, <https://doi.org/10.1175/MWR-D-10-05013.1>.
- Barthes, L., and C. Mallet, 2013: Vertical evolution of raindrop size distribution: Impact on the shape of the DSD. *Atmos. Res.*, **119**, 13–22, <https://doi.org/10.1016/j.atmosres.2011.07.011>.

- Brandes, E. A., G. Zhang, and J. Vivekanandan, 2002: Experiments in rainfall estimation with a polarimetric radar in a subtropical environment. *J. Appl. Meteor.*, **41**, 674–685, [https://doi.org/10.1175/1520-0450\(2002\)041<0674:EIREWA>2.0.CO;2](https://doi.org/10.1175/1520-0450(2002)041<0674:EIREWA>2.0.CO;2).
- , —, and —, 2003: An evaluation of a drop distribution-based polarimetric radar rainfall estimator. *J. Appl. Meteor.*, **42**, 652–660, [https://doi.org/10.1175/1520-0450\(2003\)042<0652:AEOADD>2.0.CO;2](https://doi.org/10.1175/1520-0450(2003)042<0652:AEOADD>2.0.CO;2).
- , —, and —, 2004: Comparison of polarimetric radar drop size distribution retrieval algorithms. *J. Atmos. Oceanic Technol.*, **21**, 584–598, [https://doi.org/10.1175/1520-0426\(2004\)021<0584:COPRDS>2.0.CO;2](https://doi.org/10.1175/1520-0426(2004)021<0584:COPRDS>2.0.CO;2).
- , K. Ikeda, G. Zhang, M. Schönhuber, and R. M. Rasmussen, 2007: A statistical and physical description of hydrometeor distributions in Colorado snowstorms using a video disdrometer. *J. Appl. Meteor. Climatol.*, **46**, 634–650, <https://doi.org/10.1175/JAM2489.1>.
- Bringi, V. N., and V. Chandrasekar, 2001: *Polarimetric Doppler Weather Radar: Principles and Applications*. Cambridge University Press, 636 pp., <https://doi.org/10.1017/CBO9780511541094>.
- , —, and R. Xiao, 1998: Raindrop axis ratios and size distributions in Florida rainshafts: An assessment of multiparameter radar algorithms. *IEEE Trans. Geosci. Remote Sens.*, **36**, 703–715, <https://doi.org/10.1109/36.673663>.
- , —, J. Hubbert, E. Gorgucci, W. Randeu, and M. Schoenhuber, 2003: Raindrop size distribution in different climatic regimes from disdrometer and dual-polarized radar analysis. *J. Atmos. Sci.*, **60**, 354–365, [https://doi.org/10.1175/1520-0469\(2003\)060<0354:RSDIDC>2.0.CO;2](https://doi.org/10.1175/1520-0469(2003)060<0354:RSDIDC>2.0.CO;2).
- Cao, Q., G. Zhang, E. Brandes, T. Schuur, A. Ryzhkov, and K. Ikeda, 2008: Analysis of video disdrometer and polarimetric radar data to characterize rain microphysics in Oklahoma. *J. Appl. Meteor. Climatol.*, **47**, 2238–2255, <https://doi.org/10.1175/2008JAMC1732.1>.
- , —, —, and —, 2010: Polarimetric radar rain estimation through retrieval of drop size distribution using a Bayesian approach. *J. Appl. Meteor. Climatol.*, **49**, 973–990, <https://doi.org/10.1175/2009JAMC2227.1>.
- Diederich, M., A. Ryzhkov, C. Simmer, P. Zhang, and S. Trömel, 2015a: Use of specific attenuation for rainfall measurement at X-band radar wavelengths. Part I: Radar calibration and partial beam blockage estimation. *J. Hydrometeorol.*, **16**, 487–502, <https://doi.org/10.1175/JHM-D-14-0066.1>.
- , —, —, —, and —, 2015b: Use of specific attenuation for rainfall measurement at X-band radar wavelengths. Part II: Rainfall estimates and comparison with rain gauges. *J. Hydrometeorol.*, **16**, 503–516, <https://doi.org/10.1175/JHM-D-14-0067.1>.
- Jung, Y., M. Xue, and G. Zhang, 2010: Simulations of polarimetric radar signatures of a supercell storm using a two-moment bulk microphysics scheme. *J. Appl. Meteor. Climatol.*, **49**, 146–163, <https://doi.org/10.1175/2009JAMC2178.1>.
- Kalogiros, J., M. N. Anagnostou, E. N. Anagnostou, M. Montopoli, E. Picciotti, and F. S. Marzano, 2013: Optimum estimation of rain microphysical parameters from X-band dual-polarization radar observables. *IEEE Trans. Geosci. Remote Sens.*, **51**, 3063–3076, <https://doi.org/10.1109/TGRS.2012.2211606>.
- Kessler, E., 1969: *On the Distribution and Continuity of Water Substance in Atmospheric Circulations*. Meteor. Monogr., No. 32, Amer. Meteor. Soc., 84 pp.
- Khain, A. P., and Coauthors, 2015: Representation of microphysical processes in cloud-resolving models: Spectral (bin) microphysics versus bulk parameterization. *Rev. Geophys.*, **53**, 247–322, <https://doi.org/10.1002/2014RG000468>.
- Kumjian, M. R., 2012: The impact of precipitation physical processes on the polarimetric radar variables. Ph.D. dissertation, University of Oklahoma, 327 pp.
- , 2013: Principles and applications of dual-polarization weather radar. Part I: Description of the polarimetric radar variables. *J. Oper. Meteor.*, **1** (19), 226–242, <https://doi.org/10.15191/nwajom.2013.0119>.
- , and A. V. Ryzhkov, 2012: The impact of size sorting on the polarimetric radar variables. *J. Atmos. Sci.*, **69**, 2042–2060, <https://doi.org/10.1175/JAS-D-11-0125.1>.
- Lam, H. Y., J. Din, and S. L. Jong, 2015: Statistical and physical descriptions of raindrop size distributions in equatorial Malaysia from disdrometer observations. *Adv. Meteor.*, 2015, 253730, <https://doi.org/10.1155/2015/253730>.
- Lupidi, A., C. Moscardini, F. Berizzi, and M. Martorella, 2011: Simulation of X-band polarimetric weather radar returns based on the Weather Research and Forecast model. *Proc. 2011 IEEE RadarCon*, Kansas City, MO, IEEE, 734–739, <https://doi.org/10.1109/RADAR.2011.5960635>.
- Marshall, J. S., and W. M. K. Palmer, 1948: The distribution of raindrops with size. *J. Meteor.*, **5**, 165–166, [https://doi.org/10.1175/1520-0469\(1948\)005<0165:TDORWS>2.0.CO;2](https://doi.org/10.1175/1520-0469(1948)005<0165:TDORWS>2.0.CO;2).
- Milbrandt, J., and M. Yau, 2005: A multimoment bulk microphysics parameterization. Part I: Analysis of the role of the spectral shape parameter. *J. Atmos. Sci.*, **62**, 3051–3064, <https://doi.org/10.1175/JAS3534.1>.
- , and R. McTaggart-Cowan, 2010: Sedimentation-induced errors in bulk microphysics schemes. *J. Atmos. Sci.*, **67**, 3931–3948, <https://doi.org/10.1175/2010JAS3541.1>.
- Mishchenko, M. I., 2000: Calculation of the amplitude matrix for a nonspherical particle in a fixed orientation. *Appl. Opt.*, **39**, 1026–1031, <https://doi.org/10.1364/AO.39.001026>.
- , L. D. Travis, and D. W. Mackowski, 1996: T-matrix computations of light scattering by nonspherical particles: A review. *J. Quant. Spectrosc. Rad.*, **55**, 535–575, [https://doi.org/10.1016/0022-4073\(96\)00002-7](https://doi.org/10.1016/0022-4073(96)00002-7).
- Morrison, H., and J. A. Milbrandt, 2015: Parameterization of cloud microphysics based on the prediction of bulk ice particle properties. Part I: Scheme description and idealized tests. *J. Atmos. Sci.*, **72**, 287–311, <https://doi.org/10.1175/JAS-D-14-0065.1>.
- , G. Thompson, and V. Tatarskii, 2009: Impact of cloud microphysics on the development of trailing stratiform precipitation in a simulated squall line: Comparison of one- and two-moment schemes. *Mon. Wea. Rev.*, **137**, 991–1007, <https://doi.org/10.1175/2008MWR2556.1>.
- Naumann, A. K., and A. Seifert, 2016: Evolution of the shape of the raindrop size distribution in simulated shallow cumulus. *J. Atmos. Sci.*, **73**, 2279–2297, <https://doi.org/10.1175/JAS-D-15-0263.1>.
- Otto, T., and H. W. Russchenberg, 2011: Estimation of specific differential phase and differential backscatter phase from polarimetric weather radar measurements of rain. *IEEE Geosci. Remote Sens. Lett.*, **8**, 988–992, <https://doi.org/10.1109/LGRS.2011.2145354>.
- Pruppacher, H. R., and K. Beard, 1970: A wind tunnel investigation of the internal circulation and shape of water drops falling at terminal velocity in air. *Quart. J. Roy. Meteor. Soc.*, **96**, 247–256, <https://doi.org/10.1002/qj.49709640807>.
- Ray, P. S., 1972: Broadband complex refractive indices of ice and water. *Appl. Opt.*, **11**, 1836–1844, <https://doi.org/10.1364/AO.11.001836>.
- Ryzhkov, A., M. Pinsky, A. Pokrovsky, and A. Khain, 2011: Polarimetric radar observation operator for a cloud model with

- spectral microphysics. *J. Appl. Meteor. Climatol.*, **50**, 873–894, <https://doi.org/10.1175/2010JAMC2363.1>.
- Schinagl, K., C. Rieger, C. Simmer, S. Troemel, and P. Friederichs, 2018: Effects of a gamma DSD with variable shape parameter on polarimetric radar moments. *19th Intl. Radar Symp. (IRS 2018)*, Bonn, Germany, IEEE, <https://ieeexplore.ieee.org/document/8448252>.
- Schneebeli, M., and A. Berne, 2012: An extended Kalman filter framework for polarimetric X-band weather radar data processing. *J. Atmos. Oceanic Technol.*, **29**, 711–730, <https://doi.org/10.1175/JTECH-D-10-05053.1>.
- Seifert, A., 2008: On the parameterization of evaporation of raindrops as simulated by a one-dimensional rainshaft model. *J. Atmos. Sci.*, **65**, 3608–3619, <https://doi.org/10.1175/2008JAS2586.1>.
- Simmer, C., and Coauthors, 2015: Monitoring and modeling the terrestrial system from pores to catchments: The transregional collaborative research center on patterns in the soil–vegetation–atmosphere system. *Bull. Amer. Meteor. Soc.*, **96**, 1765–1787, <https://doi.org/10.1175/BAMS-D-13-00134.1>.
- Szyrmer, W., S. Laroche, and I. Zawadzki, 2005: A microphysical bulk formulation based on scaling normalization of the particle size distribution. Part I: Description. *J. Atmos. Sci.*, **62**, 4206–4221, <https://doi.org/10.1175/JAS3620.1>.
- Testud, J., S. Oury, R. A. Black, P. Amayenc, and X. Dou, 2001: The concept of “normalized” distribution to describe raindrop spectra: A tool for cloud physics and cloud remote sensing. *J. Appl. Meteor.*, **40**, 1118–1140, [https://doi.org/10.1175/1520-0450\(2001\)040<1118:TCOND>2.0.CO;2](https://doi.org/10.1175/1520-0450(2001)040<1118:TCOND>2.0.CO;2).
- Trömel, S., M. R. Kumjian, A. V. Ryzhkov, C. Simmer, and M. Diederich, 2013: Backscatter differential phase—Estimation and variability. *J. Appl. Meteor. Climatol.*, **52**, 2529–2548, <https://doi.org/10.1175/JAMC-D-13-0124.1>.
- Ulbrich, C. W., 1983: Natural variations in the analytical form of the raindrop size distribution. *J. Appl. Meteor. Climatol.*, **22**, 1764–1775, [https://doi.org/10.1175/1520-0450\(1983\)022<1764:NVITAF>2.0.CO;2](https://doi.org/10.1175/1520-0450(1983)022<1764:NVITAF>2.0.CO;2).
- , and D. Atlas, 2007: Microphysics of raindrop size spectra: Tropical continental and maritime storms. *J. Appl. Meteor. Climatol.*, **46**, 1777–1791, <https://doi.org/10.1175/2007JAMC1649.1>.
- Wacker, U., and A. Seifert, 2001: Evolution of rain water profiles resulting from pure sedimentation: Spectral vs. parameterized description. *Atmos. Res.*, **58**, 19–39, [https://doi.org/10.1016/S0169-8095\(01\)00081-3](https://doi.org/10.1016/S0169-8095(01)00081-3).
- Waterman, P., 1965: Matrix formulation of electromagnetic scattering. *Proc. IEEE*, **53**, 805–812, <https://doi.org/10.1109/PROC.1965.4058>.
- Wen, G., H. Chen, G. Zhang, and J. Sun, 2018: An inverse model for raindrop size distribution retrieval with polarimetric variables. *Remote Sens.*, **10**, 1179, <https://doi.org/10.3390/rs10081179>.
- Xie, X., R. Evaristo, C. Simmer, J. Handwerker, and S. Trömel, 2016a: Precipitation and microphysical processes observed by three polarimetric X-band radars and ground-based instrumentation during HOPE. *Atmos. Chem. Phys.*, **16**, 7105–7116, <https://doi.org/10.5194/acp-16-7105-2016>.
- , —, S. Troemel, P. Saavedra, C. Simmer, and A. Ryzhkov, 2016b: Radar observation of evaporation and implications for quantitative precipitation and cooling rate estimation. *J. Atmos. Oceanic Technol.*, **33**, 1779–1792, <https://doi.org/10.1175/JTECH-D-15-0244.1>.
- Zängl, G., D. Reinert, P. Rípodas, and M. Baldauf, 2015: The ICON (ICOsahedral Non-hydrostatic) modelling framework of DWD and MPI-M: Description of the non-hydrostatic dynamical core. *Quart. J. Roy. Meteor. Soc.*, **141**, 563–579, <https://doi.org/10.1002/qj.2378>.
- Zhang, G., J. Vivekanandan, and E. Brandes, 2001: A method for estimating rain rate and drop size distribution from polarimetric radar measurements. *IEEE Trans. Geosci. Remote Sens.*, **39**, 830–841, <https://doi.org/10.1109/36.917906>.
- , —, E. A. Brandes, R. Meneghini, and T. Kozu, 2003: The shape–slope relation in observed gamma raindrop size distributions: Statistical error or useful information? *J. Atmos. Oceanic Technol.*, **20**, 1106–1119, [https://doi.org/10.1175/1520-0426\(2003\)020<1106:TSRIOG>2.0.CO;2](https://doi.org/10.1175/1520-0426(2003)020<1106:TSRIOG>2.0.CO;2).
- , J. Sun, and E. A. Brandes, 2006: Improving parameterization of rain microphysics with disdrometer and radar observations. *J. Atmos. Sci.*, **63**, 1273–1290, <https://doi.org/10.1175/JAS3680.1>.
- Ziemer, C., and U. Wacker, 2012: Parameterization of the sedimentation of raindrops with finite maximum diameter. *Mon. Wea. Rev.*, **140**, 1589–1602, <https://doi.org/10.1175/MWR-D-11-00020.1>.