The Prediction of Clear Air Turbulence over Mountainous Terrain

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ABSTRACT

Recent aircraft measurements of clear air turbulence over Australia have shown that the phenomenon of CAT in a thermally stable environment is associated with a breakdown of waves, presumably gravity waves or Helmholtz waves on a stable interface, into random turbulent eddies. The energy distribution in the wavelength range in which clear air turbulence is experienced seems to follow the "$-5/3$ law" postulated by Kolmogorov's similarity hypothesis. The "$-5/3$ law" seems to extend to much longer wavelengths than previously anticipated.

Combining these results of aircraft measurements with the theory on lee waves which has been derived by the use of perturbation equations, one finds that the energy involved in standing lee waves over mountains may "cascade" down from a wavelength range of approximately 10 km to a range near 100 m which then would be experienced as clear air turbulence, provided that the energy levels are high enough to cause any responses in an aircraft.

This physical model of turbulence being "fed" by mountain waves has been used in developing a forecasting scheme of CAT over mountains. Results of a preliminary, but very encouraging, study are reported in this paper.

1. Introduction

The phenomenon of clear air turbulence (CAT), first recognized in the early 1940's as "air pockets," gained in importance with the development of fast-flying swept-wing aircraft. Several cases are on record of loss of control of aircraft, even structural damage, and of passenger and/or crew injuries as a result of CAT (Reiter, 1964; Beard, 1966). Planning of supersonic transport aircraft is considering the effects of this unexpected turbulence occurring at any flight level in the atmosphere that has been explored so far.

Prediction of CAT, of vital importance for economic and safety reasons, is faced with the difficulty of having to rely on macroscale (synoptic) data while dealing with a microscale phenomenon. Because of the vast difference in scales involved in the forecasting problem, we will never be able to pin-point isolated patches of turbulence in space and time, not even by refined forecasting techniques. We may, however, improve present forecasting skills of CAT appreciably by developing physical "models" of CAT and of mesoscale disturbances (dimensions of about 10^3–10^5 m) upon whose energy the small-scale turbulence feeds.

The present report attempts to develp such a "model" of CAT formation, which is especially applicable over mountainous terrain where orographic lee waves may form under certain atmospheric conditions (Foltz, 1966).

2. The nature of CAT

Measurements of CAT by aircraft over Australia (Burns and Rider, 1965; Reiter and Burns, 1966) revealed certain anisotropy of turbulent eddies of a size >300 m. Smaller eddies seem to be reasonably isotropic and their energy distribution as a function of wave number obeys the "$-5/3$ law" of turbulence theory, according to

$$E(k) = a\epsilon^{2/3}k^{-5/3}, \qquad (1)$$

where $E(k)$ is the wave-number dependent, one-dimensional spectral density of kinetic energy, a is a universal constant, ϵ the dissipation rate of turbulent kinetic energy, and k the wave number.

Whereas a $-5/3$ slope in a representation of $\log E(k)$ versus $\log k$ seems to hold to much longer wavelengths than 300 m, the w-component shows less energy at low frequencies (long wavelengths), at least in several of the spectra measured by aircraft. This is to be expected because negative buoyant forces active in the stable environment in which most of these flights were made, tend to suppress vertical motions at long wavelengths. Even in an unstable environment one should expect such anisotropy at long wavelengths because of the small vertical and large horizontal dimensions of the atmosphere. Schematic spectra indicating this anistropy are shown in Fig. 1 (Pinus *et al.*, 1967), the references being given in Table 1.

TABLE 3. Pilot report summaries of turbulence during lee-wave flights (Foltz, 1966).

Turbulence degree*	Flight conditions** AC No.	AC %	BC No.	BC %	IC No.	IC %	I&O No.	I&O %	NC No.	NC %	Total No.	Total %	Per cent in each turbulence category
					All pilot reports								
N	102	43	48	20	7	3	13	6	65	28	235	100	51
L	35	27	41	31	14	11	15	12	25	19	130	100	28
L/M	6	15	17	41	4	10	5	12	9	22	41	100	9
M	4	10	14	36	6	16	4	10	11	28	40	100	9
M/S	0	0	0	0	0	0	2	25	6	75	8	100	2
S	1	33	2	67	0	0	0	0	0	0	3	100	1
											457		100
	Pilot reports above 15,000 ft over West, and above 10,000 ft over East												
N	60	55	6	5	4	4	9	8	33	28	112	100	59
L	29	42	8	12	6	9	10	15	15	22	68	100	35
L/M	2	40	0	0	1	20	1	20	1	20	5	100	3
M	1	20	0	0	2	40	1	20	1	20	5	100	3
M/S	0	0	0	0	0	0	0	0	0	0	0	100	0
S	0	0	0	0	0	0	0	0	0	0	0	100	0
											190		100

* Turbulence categories: N = none; L = light; M = moderate; S = severe.

** Flight conditions: AC = above clouds; BC = below clouds; IC = in clouds; I&O = in and out of clouds; NC = no clouds.

As has been mentioned earlier, the model is based on the assumption that the $-5/3$ slope in a log-log representation of spectral density extends over a wavelength range much larger than the inertial subrange (see Fig. 1). Fig. 4 shows such extensions of the $-5/3$ slope range for various degrees of CAT.

In making this assumption, we have to be aware of the following limitations. The spectra in Fig. 1, which seem to justify this assumption, are based on measurements of the u-component (i.e., the component of flow measured along the aircraft trajectory). The kinetic energies in lee waves estimated in the preceding section and entered into Fig. 4, however, pertain to the w-component. Under stable stratification, we have to assume that negative buoyant forces will render the spectral density of w smaller than that of u and v in the domain of lee-wave lengths. Thus, the computed kinetic energies in w may be accompanied by even larger perturbation motions in u and v. On the other hand, because of the energy consumed by the buoyant forces, the slope of the spectrum curves between lee-wave lengths and CAT wavelengths may be steeper than indicated in Fig. 4. These two uncertainties should tend to offset each other at least partially. The assumption of a $-5/3$ slope, as in Fig. 4, might therefore be justified reasonably well.

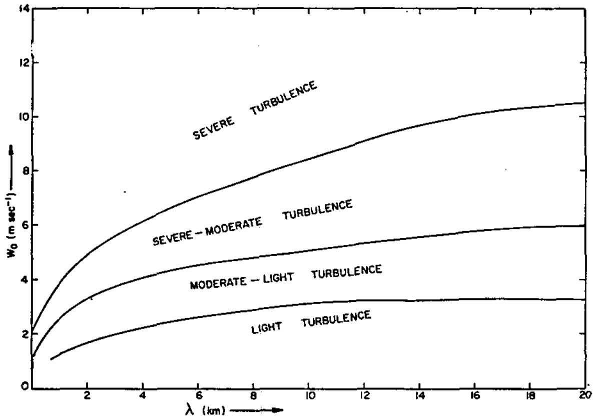

FIG. 5. Estimate of intensity level of CAT from maximum vertical velocity w_0 in lee waves of wavelength λ (Foltz, 1966).

Based upon this extrapolation of slope, we may forecast from the kinetic energy per unit area of lee waves, which according to Eq. (12) is essentially a function w_0 and wavelength λ, the intensities of CAT to be expected with the appropriate amount of this mesoscale energy available for dissipation. Fig. 5 shows a nomograph which permits easy estimates of CAT intensity level.

5. Correlation between lee waves and CAT

Special pilot reports over the Rocky Mountains and over the eastern United States were collected during seven lee-wave cases observed principally by satellite between February and May 1966. Table 3 shows the results categorized according to turbulence intensities and flight conditions.

The correlation between observations and computations obtained by the method described above is shown in Tables 4 and 5. The first of these two tables indicates

TABLE 4. Verification of turbulence forecasts for western United States (Foltz, 1966). See Table 3 for turbulence categories.

Forecast	Observed L&N	L/M	M	M/S	S
L&N	42	3	2	0	0
L/M	24	7	5	0	0
M	0	1	2	1	0
M/S	0	0	0	0	0
S	0	0	0	0	0

TABLE 5. Verification of turbulence forecasts over western United States and over eastern United States above 10,000 ft MSL (Foltz, 1966). See Table 3 for turbulence categories.

Forecast	Observed L&M	 L/M	 M	 M/S	 S
L&N	106	4	2	0	0
L/M	30	7	5	0	0
M	0	1	2	1	0
M/S	0	0	0	0	0
S	0	0	0	0	0

TABLE 6. CAT forecast comparisons (Foltz, 1966). See Table 3 for turbulence categories.

Date	CAT forecasts using rawinsonde data w_0 (m sec^{-1})	 Forecast CAT	CAT forecasts using satellite data w_0 (m sec^{-1})	 Forecast CAT
3-7-66	1.2	N-L	1.6	N-L
3-7-66	2.0	N-L	3.6	N-L
3-7-66	1.9	N-L	3.6	N-L
3-7-66	5.0	M	6.5	M
3-9-66	4.1	L-M	4.1	L-M
3-9-66	3.1	L-M	4.1	L-M
3-9-66	4.1	L-M	8.0	M
4-26-66	2.0	N-L	4.0	N-L
4-26-66	3.9	L-M	6.4	M
4-26-66	0.4	N-L	2.7	N-L
2-14-66	1.9	N-L	2.7	N-L
2-14-66	3.2	N-L	7.5	M
2-14-66	0.7	N-L	2.8	N-L
2-14-66	0.6	N-L	7.3	M

that 51 cases were forecast correctly, 34 cases were "near misses," meaning that they were off by only one category of turbulence. Only 2 cases were off by more than one category. If we include one-half of the "near misses" into the "acceptable forecasts" on the grounds that there are more categories of CAT in Table 4 than are normally included in forecasts of this phenomenon (namely, light, moderate and severe), the forecasting procedure applied here yielded 78% positive results. Table 5 indicates an even better average in "acceptable forecasts" (85%).

It should be pointed out, though, that the term "forecasting" is used here in the sense of short-period CAT warning, and not as prediction over time intervals of synoptic scale. The proposed method of estimating CAT occurrences over mountains depends on energy computations from *existing* mountain waves. Since lee-wave patterns usually vary relatively slowly with time, CAT estimates made with the above procedure should be valid for several hours. Forecasts might even be extended in time if the variability of atmospheric parameters leading to mountain-wave formation were properly taken into account in the prediction procedures outlined above. No such attempts have been made as yet, although they should prove to be entirely feasible.

Estimates of lee-wave lengths may be made either from radiosonde data using the outlined computational procedure, or even more conveniently from satellite photographs wherever wave clouds are in evidence. This opens the possibility of estimating CAT over mountainous terrain with only sparse upper-air information. From $\lambda = 2\pi/k$, k may be obtained and substituted into the exponential term of Eq. (11). Assuming that, with small ℓ_s^2

$$L^2 \cos^2\alpha \cong \frac{4\pi}{\lambda^2}, \tag{13}$$

Eq. (11) now becomes

$$w_0 \cong \frac{2\pi hb}{e^{kb}} U_1 \frac{4\pi^2}{\lambda} [\tan^2\alpha (n\pi - \alpha - \tan\alpha)^{-1}]. \tag{14}$$

Mountain height h and half-width b may be evaluated from topographical maps, while U_1, the wind speed at gradient-wind level, may be estimated from synoptic maps. Measurements of lee-wave length λ are easily obtained from satellite photographs. This leaves α as the only unknown. However, from Fig. 3 we see that the denominator in the last fraction of expression (14) attains a maximum at $\alpha \cong 65°$ and $n = 1$, rendering a value of unity for this fraction. This tends to maximize the estimated values of w_0.

Comparisons between CAT computations made from radiosonde data and those made from satellite data using the simplified method just outlined, are given in Table 6. Agreement between the two forecasting schemes is very satisfactory.

6. Conclusions and outlook

This preliminary study of the correlation between CAT and mountain waves indicates that a physical model of energy decay from lee-wave lengths to short CAT wavelengths, based upon an admittedly crude extrapolation of turbulent spectra valid in the isotropic sub-range, seems to describe observed CAT occurrences over mountains very well. Further supporting evidence for the validity of this model may be gained from the fact that the centers of the regions with highest CAT probability are not located over the mountain crests, but downstream from the mountains. As may be seen from Fig. 6, the centers of maximum probability of CAT are found farther downstream with decreasing order of severity This agrees well with Fig. 1 and Eq. (1) according to which the dissipation rate of turbulence energy is smaller with light CAT than with severe CAT.

The present study was carried out only for a limited period of time. Further statistical evidence, preferably from different regions of the world, should be used to test the forecasting procedure.

Last, but not least, sophisticated aircraft measurements of atmospheric turbulence over a wide range of the spectrum would help in testing the assumption that energy is "cascading" from lee-wave lengths to CAT eddies. With more reliable spectrum measurements available in the region between wavelengths of 2 and 30 km, preferably in all three components of

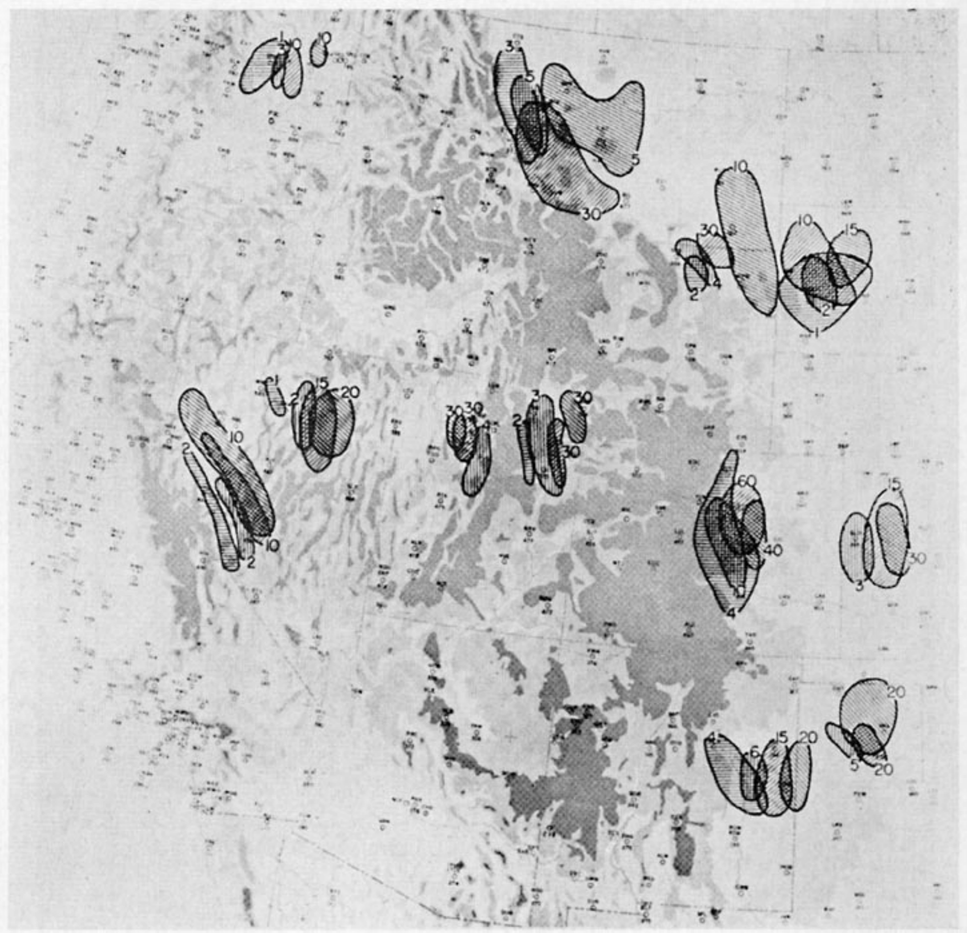

FIG. 6. Areas of most frequent occurrence of severe, moderate to severe, moderate, and light to moderate CAT. Isopleths are given in terms of number of occurrences per 18 month, per 3600 (n mi)^2 area (Foltz, 1966).

motion, one should be able to treat the mesoscale properties of atmospheric flow more adequately than is the case now

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