

## Reply

W. L. SMITH

*National Environmental Satellite Center, ESSA, Suitland, Md.*

23 June 1967

In his comment on my note, Berkofsky fails to illustrate how  $\mu$  is related to  $\lambda$ , if indeed related at all. Although knowledge of the atmosphere's temperature profile (and hence saturation mixing ratio profile) establishes the theoretical maximum of total precipitable water which may exist, it is not clear how this information may be utilized to provide an estimate of

the actual total precipitable water which is solely dependent on the profile of dew-point temperature. In any event, the formula for total precipitable water (Smith, 1966) was derived in order to provide water vapor estimates in the absence of atmospheric soundings.

There is also serious doubt to the generality as well as the validity of Berkofsky's Eq. (7). Since it was

derived from a dynamic equation [(Eq. (2))] which is only valid for pseudo-adiabatic processes occurring as a result of induced upward vertical motions, then it follows that Eq. (7) is restricted to this same dynamic condition. Furthermore, Berkofsky's Eq. (7) is an inadequate solution for  $w_s(p)$  since the parameter  $\mu$  which he treated as an independent variable is also a function of  $w_s(p)$ .

A general and static state expression for saturation mixing ratio as a function of pressure, temperature and surface saturation mixing ratio can be derived from the well known Clausius-Clapeyron equation written for a particular atmospheric pressure level  $p$  as

$$\frac{de_s(p)}{e_s(p)} = \frac{L_v}{R_v} \frac{dT(p)}{T(p)^2} \quad (1)$$

where  $e_s(p)$  is the saturation vapor pressure at the temperature  $T(p)$  which exists at the pressure level  $p$ , and  $L_v$  and  $R_v$  are the latent heat of vaporization and gas constant for moist air, respectively. Since it follows from the equation of state for a perfect gas that the saturation mixing ratio at any pressure level  $p$  is given by

$$w_s(p) = \frac{R_d \left[ \frac{e_s(p)}{p - e_s(p)} \right]}{R_v} \approx \frac{R_d e_s(p)}{R_v p}, \quad (2)$$

where  $R_d$  is the gas constant for dry air, then substituting (2) into (1) yields

$$\frac{1}{w_s(p)} \frac{dw_s(p)}{d \ln p} - \frac{L_v}{R_v} \frac{1}{T(p)^2} \frac{dT(p)}{d \ln p} = 1. \quad (3)$$

Integrating (3) between the limits  $p$  and  $p_0$ , where  $p_0$  refers to the pressure at the earth's surface yields the following solution for saturation mixing ratio at any arbitrary pressure level:

$$w_s(p) = w_s(p_0) \left( \frac{p_0}{p} \right) \exp \left\{ \frac{L_v}{R_v T(p_0)} \left[ 1 - \frac{T(p_0)}{T(p)} \right] \right\}. \quad (4)$$

Unlike Berkofsky's Eq. (7), Eq. (4) is completely general and free of any dynamic conditions. It is interesting to note that Berkofsky's Eq. (7) and Eq. (4) above are identical only under the different assumptions that  $T(p)/w_s(p) = -L_v/R_v$ , a constant (i.e.,  $\mu=1$ ) for (7) and  $T(p)$  is a constant for (4). Although the assumption pertaining to (4) is a physically possible atmospheric situation (at least throughout the lowest few hundred millibars of atmosphere), the assumption required for (7) to achieve equivalence with (4) represents a physically impossible atmospheric state since  $T(p)/w_s(p)$  is always a positive quantity. Hence, the general validity of Berkofsky's equation is certainly questionable.

#### REFERENCE

- Smith, W. L., 1966: Note on the relationship between total precipitable water and surface dew point. *J. Appl. Meteor.*, 5, 726-727.