Specification of Monthly Frequency of Snow Cover Based on Macroscale Parameters

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ABSTRACT

The most likely number of days with snow on the ground during an individual month is expressed empirically as a function of the mean temperature and total precipitation for that month, as well as the presence or absence of a snow cover at the end of the preceding month. Although this study is of some value in the field of synoptic climatology, it was designed primarily to serve as a useful method for generating snow cover within numerical weather-prediction models which do not contain water- or snow-budget formulas.

1. Introduction

A thermodynamic numerical model has been developed for predicting monthly-mean temperature and other mean parameters over the Northern Hemisphere (Adem, 1965). It is important that this model be able to predict the albedo of the earth's surface, because this determines the fraction of short-wave radiation reaching the ground which is available for the exchange of heat and moisture between earth and atmosphere.

The principal variation of surface albedo over land is due to the presence or absence of snow on the ground, which can result in fluctuations of 100% or more in the energy exchange. In the present model, a crude first approximation to predicting snow cover is made by assuming that whenever the mean temperature of the ground falls below a certain critical value (say 32F), snow is present on the ground during that month. Higher average temperatures are associated with an absence of snow. Since the mean-monthly temperature of the ground is one of the parameters predicted in the model, this enables a prediction of albedo to be made from the average albedo of snow-covered and snow-free surfaces (Posey and Clapp, 1964).

There are several drawbacks to this system, the most obvious of which is that no mean temperature exists which is associated with a sharp cut-off between snow and no snow. It is clear that as the monthly-mean temperature at a given locality falls, the frequency (or number of days) with a snow cover will tend to increase monotonically from zero at high temperatures to near 100% of the total days at low temperatures. Therefore, the albedo should be based on the *relative frequency* of snow cover. Furthermore, there are other factors determining the snow cover frequency besides surface temperature.

The objective of the present study is to obtain a better approximation to a specification of snow cover based on the parameters predicted by or used as input to the thermodynamic model. Since these parameters are averages over periods of time (mainly a month), it was decided at the outset not to embark on an ambitious hydrodynamical approach based on the moisture and heat balances of a snow-ground surface, but rather to employ statistical methods based on empirical knowledge of the factors likely to be important. Similar studies have been made by Lamb (1955), Manley (1939) and Dickson.[1]

Although this study was made in support of a specific numerical model, it is hoped that it will be of some interest in other numerical experiments since the complete physical formulation of a snow budget will undoubtedly prove to be very difficult, and must be supplemented by empirically-derived knowledge.

2. Selection of parameters and data sources

The dependent variable is basically the number of days of snow cover during a given calendar month at a selected station. Since the total number of days varies from month to month, this parameter is expressed as a relative frequency, i. e., the percentage of total days in a given month having one inch or more of snow on the ground at a certain hour each morning. The minimum depth was chosen because it is found (Kung *et al.*, 1964) that the surface albedo rises rapidly as the depth of snow increases, until with an average depth of one inch the albedo reaches about half way between its value with no snow and its maximum value which is reached with a snow depth of four inches or more.

For the sake of avoiding repetition, this dependent variable will be abbreviated "snow frequency" in subsequent discussions. Abbreviations will also be assigned to the other variables.

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[1] Dickson, R. R., 1962: Snow cover in the United States. Unpublished. (Copy available on request from Extended Forecast Division, ESSA, Washington Science Center 5, Rockville, Md., 20852.)

Four independent variables have been selected, all of which are closely related to equivalent parameters used in the model. For the most part it is clear why these are related to the presence or absence of snow. Unless otherwise indicated, they are mean or total values for the same month during which the snow frequency is determined. These parameters are:

a. Surface air temperature ("surface temperature"). This is assumed to be highly correlated with the temperature of the uppermost layer of the ground or snow, a parameter which is carried in the model.

b. Total precipitation ("precipitation"). It would of course be better to use total snowfall, but this is not presently predicted in the model whereas total precipitation is. Fortunately, the amount of precipitation falling as snow is an empirical function of temperature.

c. Number of days of snow cover during the last 5 days of the preceding month ("snow days"). This is related to snow frequency in the following month due to a carry-over effect. The last 5 days were chosen, rather than the last day, in order to use a representative sample with due regard to the often rapid changes in snow cover and to the fact that only one observation is available each day. This parameter is closely related to the average snow boundary during the last week of the preceding month, a factor which is now used as input data to the model.

d. Temperature at 500 mb. ("upper temperature"). This is closely akin to the "thickness" parameter used by Lamb (1955) and Dickson (*loc. cit.*) as an indicator of the mean position of the snow boundary. It is independent of surface temperature only if the monthly-mean lapse rate varies with time. It is also one of the model parameters.

There are other parameters used in the model which are clearly related to snow cover, including the radiation balance at the ground, evaporation and sensible heat transfer from the ground. These quantities were not used simply because observed values are not generally available.

The dependent snow frequency and the four selected independent variables were extracted from published climatological data summaries for 30 stations in the United States and three stations in Canada, and for the 16 years 1950 to 1965, inclusive. For each station, data corresponding to each of three cold-season months were extracted. For the most part these were the three winter months, but to assure as great a variability as possible in the snow frequencies, spring or fall months were chosen for a few of the more northern stations.

3. Statistical procedure and discussion of results

Since the relationship between the dependent and independent variables was expected (and subsequently proved) to be both non-linear and joint, it was decided to use a subjective graphical-correlation procedure which follows the methodology of Ezekiel (1930) with some minor deviations. The procedure used may be outlined as follows:

TABLE 1. Number of cases in each of 15 subclasses corresponding to 5 classes of precipitation and 3 of number of days of snow cover during last 5 days of preceding month.

	Precipitation classes (inches)					
Snow days	0–1	>1 to 2	>2 to 3	>3 to 4	>4	Total
0	285	199	157	95	133	869
1–4	129	91	82	39	35	376
5	117	80	43	18	14	272
Total	531	370	282	152	182	1517

The graphical method requires subdivision or "cross-classification" of the data several times among the independent variables. In an attempt to assure an adequate sample size within each subclass it was necessary to assume that the effect of the four variables on snow frequency is independent of month, climate or geographical position. Therefore, the data for all station-months were lumped together in one large sample consisting of 1517 cases.

"Surface temperature" was selected as the primary independent variable and plotted in scatter diagrams against "snow frequency" as the dependent variable, separately for each of 5 categories of precipitation shown in the top row of Table 1. The number of cases in each category is shown in the last row. The two curvilinear regression lines were constructed in accordance with the accepted procedures of graphical correlation, and a curve roughly bisecting these was drawn and defined as the curve of best fit. This represents a departure from accepted graphical procedure, which calls for only the regression of snow frequency on surface temperature. This variation of accepted usage was devised by the author with the thought that the curve of best fit, defined above, gives a better picture of the true physical relationship between any two variables with due allowance for random errors.

The curves of best fit, simplified to a set of connected line segments, are shown for each of the 5 precipitation categories in Fig. 1. It can be seen that a very similar type of relationship was found for all 5 precipitation groupings, except that snow frequency increases sharply as precipitation amount increases. Thus, at a surface temperature of 30F the snow frequency corresponding to more than 4 inches of precipitation (43%) is almost twice that corresponding to 0–1 inch precipitation (24%).

The curves reveal that the variation of snow frequency with precipitation is non-linear, increasing rapidly for smaller amounts and leveling off at the higher ranges of precipitation.

The curves also show that snow frequency increases slowly with lowering surface temperature, remaining below 10% until a temperature somewhat above freez-

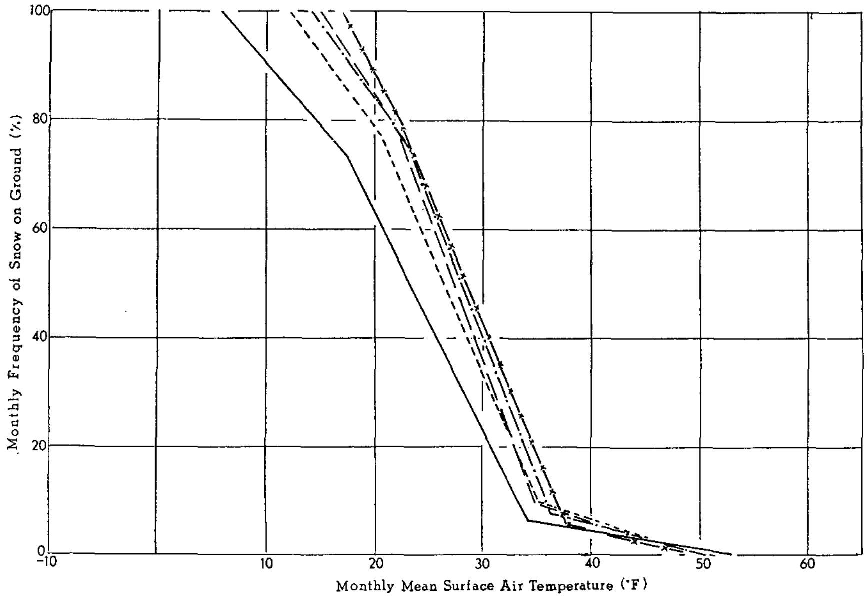


FIG. 1. Curves of best fit relating monthly-mean temperature and snow frequency for 5 precipitation categories. Solid curves, 0 to 1 inch precipitation; short dash, >1 to 2 inches; long dash, >2 to 3 inches; dash dot, >3 to 4 inches; and dash cross, >4 inches.

ing is reached when the frequency increases rapidly, leveling off at 100% at temperatures between 5 and 17F. This type of non-linear variation is not surprising. What might be somewhat unexpected is the finding that a snow frequency of 50% (half the days of a given month with and half without snow) is found at temperatures below freezing. This finding is probably due to a combination of circumstances. Two of the more likely explanations are the following: a) The mean monthly temperature is an average of the daily maximum and minimum temperatures. Therefore, at a mean temperature of 32F, the maximum temperature is likely to be 40F or higher, so that any snow cover would tend to melt during the day. b) Temperatures tend to rise in winter as a general precipitation area approaches a station in the interior of a continent. Thus, if the mean monthly surface temperature happened to be near 32F, it is possible that most precipitation would occur as rain.

This result may be compared with Manley's finding for the British Isles that a temperature of about 34F corresponds to 50% likelihood of "snow lying." The difference may be due to several factors, including the effect of the continent, just mentioned. The British stations are under strong maritime influence with more cloudiness and small diurnal temperature changes; whereas all but 4 of the 33 stations in the present study are in a continental interior. This suggests that the sample should be enlarged so that geographical and climatological sub-divisions can be made.

Another cause of the discrepancy may lie in the definition of snow on the ground. Manley considered a day with "snow lying" as one when more than half the ground is covered with snow. This probably means that the limit between snow and no snow corresponds to an average snow depth of less than 1 inch, which would lead to a greater frequency of snow days compared to the definition used here.

The next independent variable to be studied was the number of days with snow on the ground during the last five days of the previous month ("snow days"). This parameter was subdivided into three broad classes; 0, 1–4, and 5 snow days. The number of cases in each of these classes, subdivided for each of the 5 precipitation categories is shown in Table 1. In order to isolate the effect of this parameter, the influence of precipitation was partly removed by plotting the "residual snow frequency" against temperature for each of the 15 precipitation-snow days subclasses. The residual snow frequency is defined as the difference, observed snow frequency minus that estimated from the curves of Fig. 1.

The result for 0–1 inch precipitation and all classes of snow days is summarized in Fig. 2. The dashed lines are extreme-error envelopes, used as guides in constructing the curves of best fit (solid lines), which in this case are simply the curvilinear regressions of residual snow frequency on temperature. This procedure is inconsistent with that used previously in that the second regression line (regression of temperature on residual snow frequency) is not used.

Fig. 2 illustrates well the joint influence of temperature and snow days. Considering first the case of zero snow days (lower curve), it can be seen that when the temperature is relatively high (35F or above), the added factor of absence of snow on the ground at the end of the previous month makes very little difference,

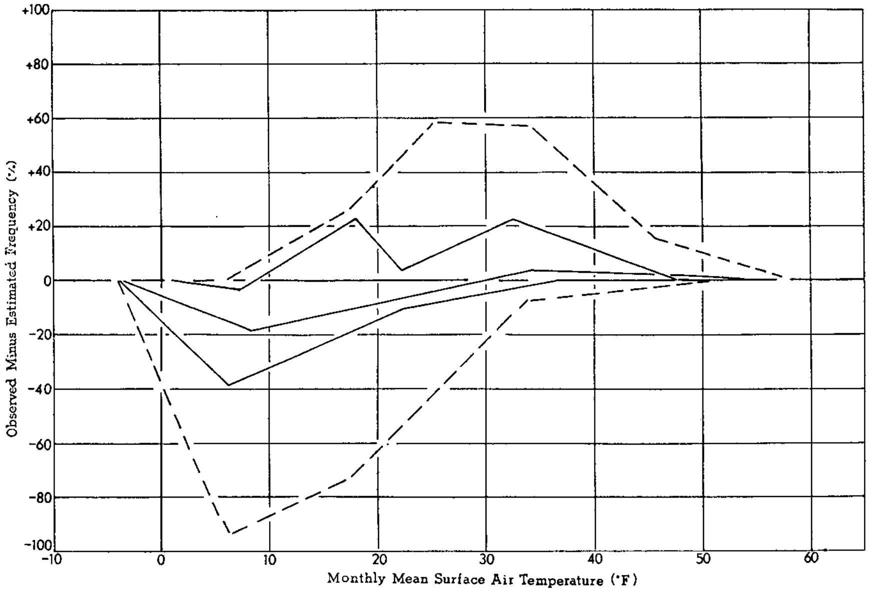


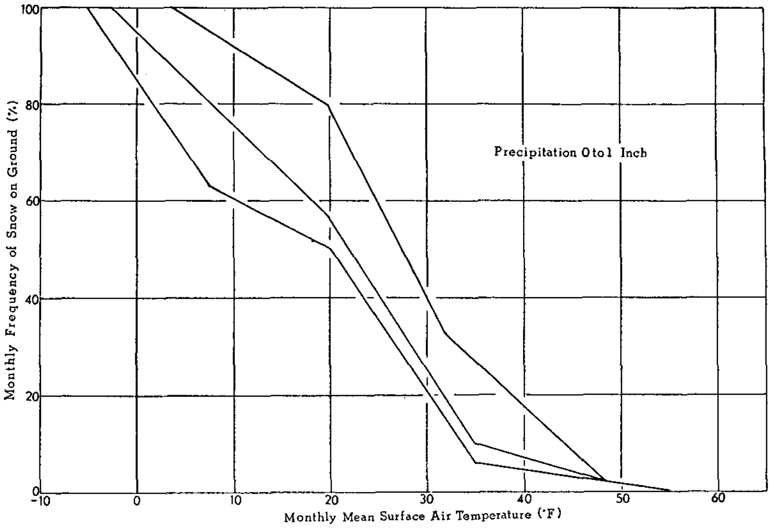
FIG. 2. Average residual snow frequency vs. monthly-mean temperature for 0 to 1 inch precipitation and all three groups of snow days. Upper solid curve, 5 snow days; middle, 1 to 4; and lower, 0. Dashed lines are extreme-error envelopes.

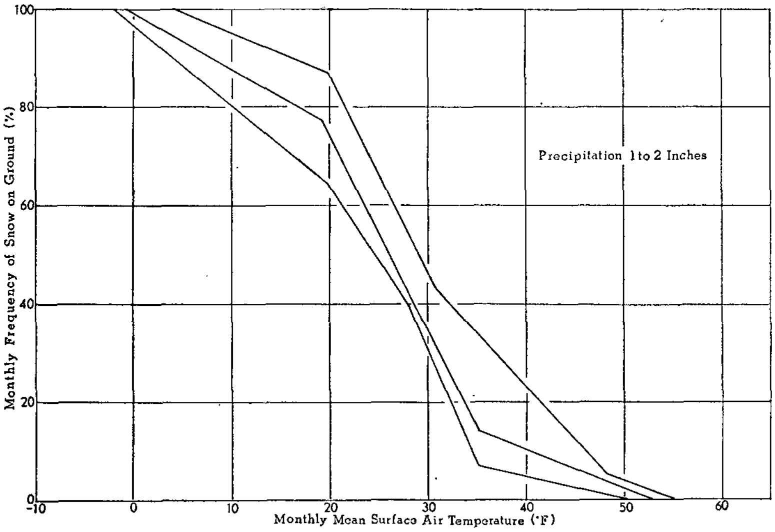
since snow frequency remains low in any case. However, as the temperature decreases below 35F, the absence of "snow days" reduces the frequency significantly below its value based on temperature and preciptation alone, attaining a maximum reduction of 40% at a temperature of 6F. With further reduction in temperatures the envelope guide lines suggest that the average residual snow frequency must return again to zero at a temperature near −4F. Undoubtedly, additional data for low temperatures would lead to modification of the envelopes, and therefore to changes in the left-hand part of the curve, but the latter is at least consistent with the available sample.

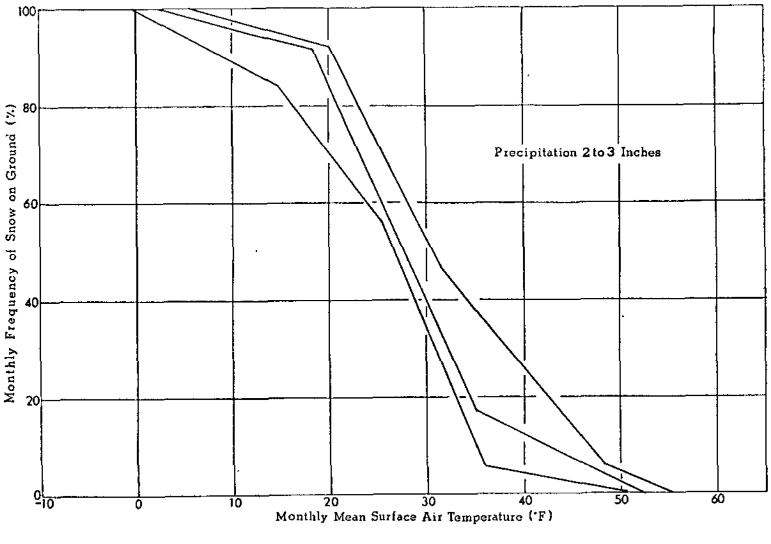
The curves of best fit for the other two snow categories illustrate the contrast between the different classes of snow days. In general, when one or more of the previous five days is free of snow, there is some reduction in the average frequency based on temperatures and precipitation alone, with a maximum at low temperatures; although this reduction is less pronounced or may be reversed as the number of snow days increases from 0 to 4. However, when all five previous days have snow (upper curve) there is an overall *increase* in the average residual snow frequency, with maxima at 32F (+24%) and at 18F (also +24%). The explanation for the peak at 32F seems clear. As the temperature decreases, the presence of snow at the end of the previous month, with its accompanying low temperature, has an increasingly important carry-over effect during the first part of the next month even though the mean temperature of that month may be above freezing. However, as the temperature drops below freezing, precipitation is more likely to be in the form of snow, causing a rapid increase in snow frequency which reduces the importance of a prior snow cover. There is no obvious explanation for the second peak at 18F although it is probably real, because a similar maximum appears in the graphs for the other four precipitation categories (not shown). The sets of curves for the five precipitation categories are fairly similar in pattern.

The fourth independent variable, 500-mb temperature, was subdivided into four class intervals (−30 to −15, −15+ to −10, −10+ to −5, and −5+ to +6F), giving an approximately equal number of cases in each interval.

It was not possible at this stage to continue subdividing the data by cross-classification so as to eliminate the joint effect of the other variables, because this would require $5\times3\times4$ or 60 subclasses, with an average of only 25 cases in each subclass. Therefore, it was assumed at first that the second residual of snow frequency (i.e., observed snow frequency minus that estimated after allowing for surface temperature, precipitation, and snow days) has no joint relationship with precipitation and snow days. In accord with this assumption, the second residual snow frequency was plotted against surface temperature only, for each of the 4 classes of upper temperature. The results (not shown) were completely negative, a zero correlation being indicated between these parameters. Therefore, a possible joint effect of precipitation and snow days was tested by indicating on the scattergrams only those cases having >2, $\leqslant 3$ inch precipitation and 5 snow days. This also produced negative results, and it was concluded that upper temperature gives no further reduction in the residual snow frequency after the other


Precipitation 0 to 1 Inch
Monthly Frequency of Snow on Ground (%)
100
80
60
40
20
0
-10
0
10
20
30
40
50
60
Monthly Mean Surface Air Temperature (°F)


Precipitation 1 to 2 Inches
Monthly Frequency of Snow on Ground (%)
100
80
60
40
20
0
-10
0
10
20
30
40
50
60
Monthly Mean Surface Air Temperature (°F)


Precipitation 2 to 3 Inches
Monthly Frequency of Snow on Ground (%)
100
80
60
40
20
0
-10
0
10
20
30
40
50
60
Monthly Mean Surface Air Temperature (°F)

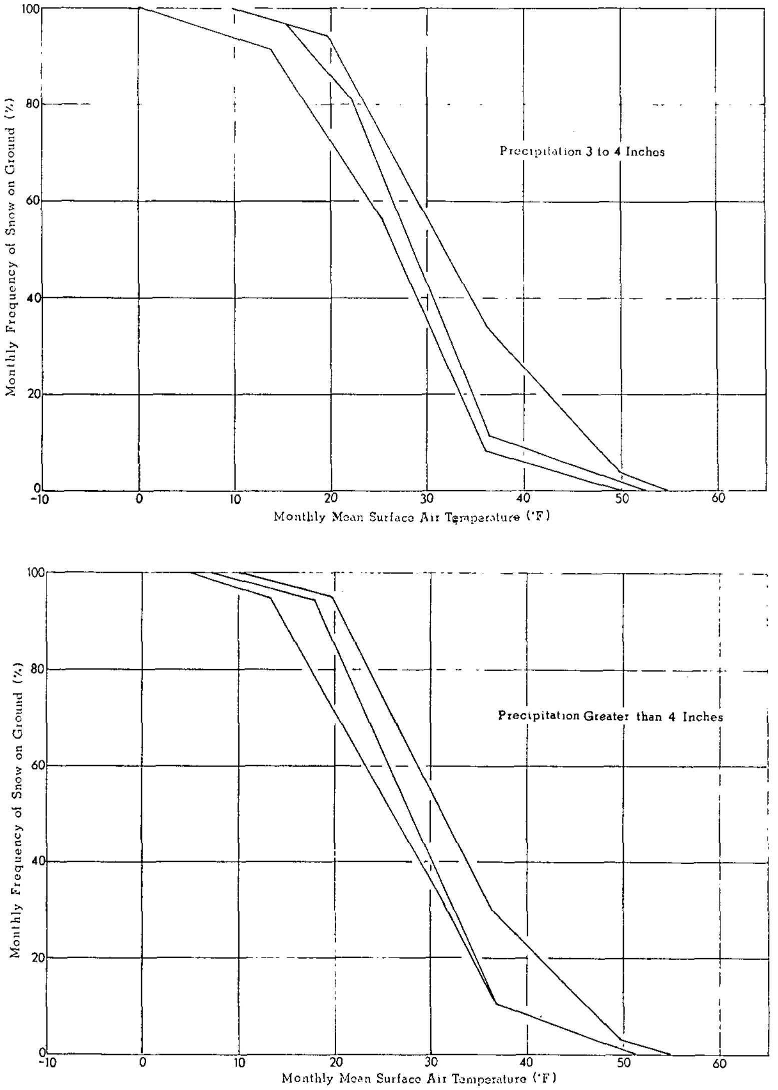


FIG. 3. Set of five graphs relating snow frequency to monthly-mean temperature for the 5 precipitation and 3 snow-day class intervals. Upper curve, 5 snow days; middle, 1 to 4; and lower, 0.

three independent variables have been accounted for.

It must be stressed that this conclusion in no way contradicts the results of Lamb and Dickson, but merely suggests that mean-monthly surface and upper-air temperatures are so highly correlated that they are not independent.

A set of graphs similar to Figs. 1 and 2 constitute the end product of the present study. These may be combined into a set of 5 graphs, one for each class of precipitation; each contains three curves corresponding to the three classes of snow days. These are shown in somewhat smoothed form in Fig. 3.

4. Quantitative test of skill

The skill of estimates of snow frequency obtained from the curves of best fit was tested using the dependent sample only. ("Skill" is defined here as the accuracy of the estimates compared to that of climatology or the average frequency of the sample.) This was done separately for each individual station, with one exception. For each of the other 32 stations, estimates of snow frequency based on observed surface temperature and precipitation only, as well as on these two parameters plus snow days, was correlated with the observed frequency. The resulting correlation coefficients, standard errors, bias and observed standard deviations for each station will not be shown here, but only the average value of each statistic, listed in columns 2–8 in Table 2. These indicate a reasonably high degree of skill and show some improvement when the parameter "snow days" is added to the other two variables.

Since precipitation is a parameter predicted with

TABLE 2. Averaged statistics relating estimated and observed snow frequency for 32 stations in the United States and Canada and for 3 cold-season months. The symbols P_0, P_{tp}, P_{tps}, P_{tp_n}, $P_{tp_{ns}}$ are, respectively, observed snow frequency and estimated frequency based on temperature and precipitation; temperature, precipitation, and snow days; temperature and normal precipitation; and temperature, normal precipitation, and snow days. Column headings are correlation coefficient times 100 ($r\times100$), standard error with bias removed (σ_e), mean error (B) and observed standard deviation (σ_{P_0}). The latter three statistics are also given as percent.

	P_0 vs. P_{tp}				P_0 vs. P_{tps}			P_0 vs. P_{tp_n}			P_0 vs. $P_{tp_{ns}}$		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Column heading	σ_{P_0}	$r\times100$	σ_e	B	$r\times100$	σ_e	B	$r\times100$	σ_e	B	$r\times100$	σ_e	B
Average statistic	25.3	73.3	15.2	0.6	77.3	13.8	0.2	71.5	15.8	0.8	75.8	14.3	0.7

dubious skill in the present form of the thermodynamic model, the correlations were repeated after replacing observed precipitation by the corresponding normal values. (Actually, monthly averages for the 16-yr sample were considered as the normals.) The mean statistics, shown in columns 9–14, show a surprisingly small reduction in skill as compared with those using observed precipitation. This is obviously due to the fact that the largest part of precipitation variability is that *between* rather than *within* stations, i.e., precipitation for the dry, interior stations is much smaller on the average than that for the southern or east coast stations.

5. Conclusions

The result of this empirical study may be regarded as a fair second approximation to estimates of monthly snow frequency based on large-scale parameters. The most obvious step leading to further improvement is to look into possible geographical or seasonal variations, but this would require a much larger sample of data. Aside from this, one could attempt to develop a more physical or theoretical approach based on combined water- and heat-budget formulas, but it is doubtful if this approach would be profitable if data are confined to macroscale time-averaged parameters.

The family of curves in Fig. 3 are intended for use along with maps of normal snow-cover probability (Dickson and Posey, 1967), to estimate the anomaly of snow-cover frequency for individual months. However, it should be noted that due to the non-linear joint character of the relationships, one cannot expect to derive normal snow-cover probability from the graphs using normal values of the independent parameters.

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