ABSTRACT: We use dual-polarization C-band data collected in the Southern Ocean to examine the properties of snow observed during a voyage in the austral summer of 2018. Using existing forward modeling formalisms based on an assumption of Rayleigh scattering by soft spheroids, an optimal estimation algorithm is implemented to infer snow properties from horizontally polarized radar reflectivity, the differential radar reflectivity, and the specific differential phase. From the dual-polarization observables, we estimate ice water content $q_i$, the mass-mean particle size $D_m$, and the exponent of the mass–dimensional relationship $b_m$ that, with several assumptions, allow for evaluation of snow bulk density, and snow number concentration. Upon evaluating the uncertainties associated with measurement and forward model errors, we determine that the algorithm can retrieve $q_i$, $D_m$, and $b_m$ within single-pixel uncertainties conservatively estimated in the range 120%, 60%, and 40%, respectively. Applying the algorithm to open-cellular convection in the Southern Ocean, we find evidence for secondary ice formation processes within multicellular complexes. In stratiform precipitation systems we find snow properties and infer processes that are distinctly different from the shallow convective systems with evidence for riming and aggregation being common. We also find that embedded convection within the frontal system produces precipitation properties consistent with graupel. Examining 5 weeks of data, we show that snow in open-cellular cumulus has higher overall bulk density than snow in stratiform precipitation systems with implications for interpreting measurements from space-based active remote sensors.

KEYWORDS: Mixed precipitation; Secondary ice production; Storm environments; Radars/radar observations; Remote sensing

1. Introduction

The finding of a significant positive absorbed solar radiation bias in the Southern Ocean reported first by Trenberth and Fasullo (2010) motivated considerable research of mid- and high-latitude cloud and precipitation processes there (Lubin et al. 2020; McFarquhar et al. 2021). Regime-based studies of midlatitude clouds tend to focus on frontal systems (McCoy et al. 2019; Naud et al. 2018) and the cold air sectors of cyclones where open-cellular cumulus transitions to closed-cell stratocumulus (McCoy et al. 2017; Bodos-Salcedo et al. 2016; Naud et al. 2016). In the warm sector of cyclones, water vapor is drawn into the system from lower latitudes where it converges along the warm conveyor belt (Carlson 1998; Ralph et al. 2004) to form snow that provides energy to deepening cyclones (Browning and Pardoe 1973; Shapiro and Keiser 1990). In the cold air sectors that are dominated by transitions between open- and closed-cellular convection, the factors that control precipitation phase and extent in large-scale subsidence can influence cloud fraction and regional albedo (McCoy et al. 2017).

Space-based remote sensing remains the only viable means of acquiring global statistics on precipitation from the remote regions of the mid–high-latitude oceans. Interpretation of snow observations from CloudSat (Tanelli et al. 2008) and GPM (Skofronick-Jackson et al. 2017) is challenged by the inherent limitations of the measurements and uncertainties in the assumptions that are necessary to derive microphysical properties from the measurements. A primary challenge in interpreting CloudSat and GPM observations of snow properties are poorly constrained assumptions about certain microphysical characteristics. The bulk density $\rho_b$, for instance, is a function of the aspect ratio $\tau$ of the ice crystals and the distribution of mass within the particle size distribution (PSD) often described by a
mass–diameter ($m–D$) power-law relationship (Schmitt and Heymsfield 2010). Once assumptions critical to the bulk density are made, the choice of method to model the backscatter cross section as a function of particle size influences the interpretation of measurements. These issues are dealt with differently by the operational CloudSat (Wood et al. 2014; Cooper et al. 2017) and GPM snowfall rate estimates when they account for sampling, instrument frequency differences, and microphysical assumptions. However, they leave open how to choose the most relevant microphysical assumptions in the broad spectrum of snow that naturally occurs in the atmosphere.

This study aims to build on existing formalisms describing dual-polarization radar observables in terms of the microphysical properties of snow. We seek to determine how observations from such radars help constrain the poorly known intensive properties of snow and the microphysical properties that are sensitive to these properties. We apply the methodology to a unique dataset along the coast of East Antarctica that we collected while on board an Australian research vessel during a voyage there in 2018.

2. Method

Dual-polarization (dual-pol) radars simultaneously transmit and receive horizontal and vertically polarized microwave energy (Bringi and Chandrasekar 2001; Ryzhkov and Zrnic 2019). Transmitting and receiving in orthogonal polarizations enables observation of four quantities:

1) the liquid-equivalent horizontally ($h$) and vertically ($v$) polarized radar reflectivity factors $Z_{v,h}$ with typical linear units of mm$^6$ m$^{-3}$ (hereinafter these will be written in decibels and denoted as $Z_h$ and $Z_v$),

2) the difference in vertically polarized to horizontally polarized reflectivity factors normally expressed as $Z_{hv} = 10 \log_{10}(Z_h/Z_v)$,

3) the range accumulated difference in phase between the horizontal and vertical polarizations ($\varphi_{DP}$, usually expressed in degrees) that is due to differences in attenuation of the orthogonal polarized beams [the accumulated values of $\varphi_{DP}$ are converted to the range derivative of the phase shift ($K_{DP}$, with typical units of degrees per kilometer)], and

4) The pulse-to-pulse correlation between the orthogonally polarized backscattered energy, quantified in terms of the copolar cross-correlation factor $\rho_{HV}$.

The dual-pol observables carry information on various aspects of the hydrometeor size distribution. In an early paper on the topic, Vivekanandan et al. (1994) express $K_{DP}$ in terms of the particle aspect ratio $r$, the bulk density of the hydrometeors $\rho_b$, and ice water content (IWC) $q_i$:

$$K_{DP} = (47.4/\lambda)(1 - r)^{-1/2} \rho_b^{0.03} q_i,$$

where $\lambda$ is radar wavelength. Ryzhkov et al. (1998) presented a slightly more nuanced analysis in which $q_i$ is expressed in terms of the dual-pol measurable $q_i = CK_{DP}/(1 - Z_{DP}^b)$, where $C$ contains assumptions about the bulk density of the hydrometeors as well as size distribution properties. More recently, Matrosov et al. (2020, 2017) used dual-pol measurements at frequencies up to Ka band to infer the $r$ of falling snow hydrometeors. Because of these sensitivities, dual-pol measurements have been used to examine the vertical structure of snow in the atmosphere. In addition to the papers cited above, Kennedy and Rutledge (2011) present a detailed analysis of a Colorado Front Range snow storm. They identify a unique vertical signature in the measurements associated with the dendritic ice crystal growth region near $-15^\circ C$. Matrosov et al. (2020) also identify this signature of dendritic growth in retrieval statistics. More recently, Allabakash et al. (2019) inferred dendritic growth, aggregation, and riming in a snowstorm over South Korea using dual-pol radar measurements.

Ryzhkov et al. (2020) discuss how dual-pol radar measurements can be used to improve cloud and precipitation modeling. We use their derived formalisms [see also Jung et al. (2010), Ryzhkov et al. (2011), and Buković et al. (2020)] to develop forward models of the dual-pol observables in snow. We then use an optimal estimation methodology (Maahn et al. 2020) to estimate an atmospheric state vector using an observational vector $y$ and its covariance $S_y$:

$$y = \left[ Z_h \ Z_{hv} \ K_{DP} \right],$$

$$S_y = \left[ \begin{array}{ccc} e_{Z_h}^{Noise} & e_{Z_{hv}}^{Noise} & e_{K_{DP}}^{Noise} \ e_{Z_h}^{FM} & e_{Z_{hv}}^{FM} & e_{K_{DP}}^{FM} \ e_{Z_h}^{Noise} + e_{Z_{hv}}^{Noise} + e_{K_{DP}}^{Noise} & e_{Z_h}^{FM} + e_{Z_{hv}}^{FM} + e_{K_{DP}}^{FM} \ \end{array} \right],$$

where $\epsilon$ denotes a variance, the superscripts refer to either instrument noise or uncertainty in forward modeling (FM) due to assumptions. Note that we assume the uncertainties are uncorrelated and the covariances among terms are neglected.

Using forward models described below, we define a state vector and its covariance:

$$x = \left[ q_i \ D_m \ b_m \right],$$

$$S_x = \left[ \begin{array}{ccc} e_{q_i} + e_{D_m} + e_{b_m} \ e_{D_m} + e_{b_m} \ e_{b_m} \ \end{array} \right],$$

where $q_i$ is the ice water content, $D_m$ is the mass-mean size of a snow particle size distribution, and $b_m$ is the exponent of a mass–dimensional or $m–D$ relationship for the snow hydrometeors: $m = a_m D_b^{b_m}$ (Schmitt and Heymsfield 2010); $a_m$ is assumed to be correlated with $b_m$ and, thereby, parameterized in terms of $b_m$. With a forward model described below, an optimal estimation framework (Rodgers 2000) is used to infer $x$ from $y$, allowing us to quantify $S_x$ or the state covariance and evaluate how assumptions and measurement noise contribute to uncertainty in $x$ (i.e., Wood et al. 2014) through Gauss–Newton iteration. The operative function for incremental steps in $x$ for converging on an optimal $x$ and $S_x$ is

$$\delta x = (S_x)^{-1} \left[ \begin{array}{c} K_S S_y^{-1} K_T^T \end{array} \right]^{-1} \{S_y^{-1} [y - F(x)]\},$$

or Eq. (5.8) in Rodgers (2000). The terms within the brackets are functions of the difference between the updated state
vector $\mathbf{x}$ and an initial state vector $\mathbf{x}_e$ and the difference between the observations and the forward modeled observations $F(x)$. The prior data and its covariance are derived from airborne in situ measurements collected by the University of North Dakota (UND) Citation during the OLYMPEX campaign in late 2015 (Houze et al. 2017) in snow. We construct priors by identifying 10-s aircraft observing periods when the airborne data were within 5 K of the temperature of a radar volume of interest and $Z_h$, calculated from the in situ microphysics, was within 5 dBZ of the measurement. There is an obvious difference of several orders of magnitude in the sample volume of an aircraft probe during a 10-s period when compared with the sample volume of a radar with a beamwidth on the order of a degree and range of several tens of kilometers. While we do not show it here, we implemented several averaging approaches to attempt to bring the sample volumes into better agreement. We found, in the end, that these methods had little effect on the prior statistics except to reduce the number of individual prior events. In the end we decided on the simpler approach of just using the 10-s in situ data as noted above. In the future, additional effort is needed to create prior statistics for convection and frontal systems separately. However, here we did not have sufficient measurements in mixed phase convection to accomplish this separation.

The $\mathbf{K}_s$ matrix informs the algorithm of how sensitive the measurements are to the state vector quantities. Here, we express $\mathbf{K}_s$:

$$\mathbf{K}_s = \begin{bmatrix} \frac{\partial Z_h}{\partial q_i} & \frac{\partial Z_{DR}}{\partial q_i} & \frac{\partial K_{DP}}{\partial q_i} \\ \frac{\partial Z_h}{\partial D_m} & \frac{\partial Z_{DR}}{\partial D_m} & \frac{\partial K_{DP}}{\partial D_m} \\ \frac{\partial Z_h}{\partial b_m} & \frac{\partial Z_{DR}}{\partial b_m} & \frac{\partial K_{DP}}{\partial b_m} \end{bmatrix}$$

(4)

The observational error covariance $\mathbf{S}_e$ scales the relative importance of the measurements and contributes to $\mathbf{S}_x$. We consider two sources of error in $\mathbf{S}_e$. The first is random measurement uncertainty. The second term typically dominates due to uncertainties in the forward model that accounts for assumptions made in a typically ill-constrained problem. As shown by Austin and Stephens (2001), we can write the forward model error in terms of a first-order sensitivity matrix $\mathbf{K}_f$ and the covariance of the assumptions $\mathbf{K}_b$:

$$\mathbf{S}_e = \mathbf{S}_x + \mathbf{K}_s \mathbf{S}_x \mathbf{K}_s^T,$$

(5)

where the second term on the right quantifies the forward model uncertainty.

The forward model we use to populate $F(x)$ is based on Rayleigh scattering by soft spheroids and follows the methodology described in Ryzhkov et al. (1998, 2011, 2020) and Jung et al. (2010). With a rule of thumb that soft spheroid Rayleigh scattering models are valid for $D < \lambda/2$ (Leinonen et al. 2017), we assume that this simple scattering approximation is valid for most snow up to X band, although the largest snowflakes may violate this assumption for X band. Following Posselt and Mace (2014), we assume that the hydrometeor size distribution (herein, the PSD) can be approximated by a modified gamma distribution function, $N(D) = N_0(D/D_0)\alpha \exp(-D/D_0)$, where $N_0$ is a constant of proportionality (cm$^{-3}$), $D_0$ is a characteristic size (cm) that controls the logarithmic slope of $N$ with $D$, and $\alpha$ controls the breadth of the distribution (note that $\alpha$ is typically referred to as the shape parameter and is represented with the symbol $\mu$ in the rainfall literature). Unless we specify otherwise, all expressions and relationships hereinafter use cgs units. See Posselt and Mace (2014) for derivation of the state vector quantities from the $N(D)$ expression.

We consider the size $D$ to be the maximum dimension of a hydrometeor. Oblate spheroids, assumed here, have an axis of rotation about the minor axis $rD$ and are assumed oriented in the horizontal plane with a mean canting angle of 0 with some standard deviation $\phi$ of that angle. Thus, horizontally polarized radar energy will be oriented perpendicular to the axis of rotation but parallel to the major axis of the spheroid.
In contrast, the vertically polarized energy will be oriented along the rotation axis, which constitutes the oblate spheroid’s minor axis. The radar observables reduce to integrals of scattering functions over the PSD. Following the notation of Ryzhkov et al. (1998),

\[ z_{h,v} = \frac{10^{12} \lambda^4}{\pi K^2} \int \sigma_{h,v} N(D) dD \]  

and

\[ K_{\text{DP}} = \frac{10^5 \times 1800 \lambda}{\pi} \int \text{Re}(f_h - f_v) N(D) dD, \]

where \(10^{12}\) and \(10^5\) convert from cgs units to the standard units of \(z\) and \(K_{\text{DP}}\); \(K\) is the frequency-dependent dielectric constant of water; the complex scattering amplitudes \(f\) and the backscatter cross sections \(\sigma\) that are a function of \(f\) are derived in van de Hulst (1957). To account for variations in the ice crystal canting, we adopt the formalisms for \(\sigma_{h,v}\) and \(\text{Re}(f_h - f_v)\) that are derived in Jung et al. (2010) their Eqs. (3)–(5) that assume \(\phi = 0\).

We solve the integral forward model equations by approximating the scattering functions in the integrands by power laws that allow for an analytic solution of the form

\[ z_{h,v} = a_{h,v} 10^{12} \lambda^4 N_0 D_0^{(a+b_{h,v}+1)} \Gamma(a + b_{h,v} + 1) \]  

and

\[ K_{\text{DP}} = a_{\text{DP}} 10^5 \times 1800 \lambda N_0 D_0^{(a+b_{\text{DP}}+1)} \Gamma(a + b_{\text{DP}} + 1), \]

where \(a_{h,v,k}\) and \(b_{h,v,k}\) are the prefactors and exponents of power laws. Figure 1 demonstrates the validity of a power-law

![Figure 2](image-url)
approximation for these quantities. This methodology is like that presented by Ryzhkov et al. (1998). With the assumed modified gamma PSD function, we express the unknowns in Eq. (7) as follows:

\[
D_0 = \frac{D_m}{\alpha + b_m + 1} \quad \text{and} \quad N_0 = \frac{q_i}{a_m + D_0^{\frac{1}{\alpha + b_m}} \Gamma(\alpha + b_m + 1)},
\]

allowing us to express the forward model equations in terms of the parameters in the state vector \( \mathbf{x} \).

The forward model equations for the state vector \( \mathbf{x} \) using the 3 dual-pol measurable in \( \mathbf{y} \) are not fully constrained by the measurements. Therefore, additional assumptions about \( \alpha, r \), the standard deviation of the canting angle, and the \( m-D \) prefactor \( a_m \) are necessary. Analysis of \( m-D \) relationships in the literature suggests that \( a_m \) and \( b_m \) are correlated (i.e., Mitchell et al. 1996). Xu and Mace (2017) used that correlation to develop an algorithm to derive the most likely \( m-D \) relationship in aircraft-observed ice PSDs. Here we use a regression of \( a_m \) and \( b_m \) found by compiling \( m-D \) relations reported in the literature: \( a_m = 10^{3.4(4 - b_m)} \), and we assume an uncertainty of 25\% in that estimate based on a regression analysis of \( a_m \) as a function of \( b_m \) (e.g., Xu and Mace 2017).

We also assume the statistics of \( \alpha \) using in situ data collected during OLYMPEX where we found a mean value of \( \alpha \sim 2 \) with a standard deviation of 1.5 like the assumptions in the GPM snow algorithm (Greco et al. 2016). The aspect ratio \( r \) is a quantity that we estimate using a heuristic approach. For a particular sample volume, we solve the algorithm for the state vector assuming aspect ratios of 0.8, 0.6, 0.4, and 0.2 with a prior uncertainty of 0.2 in that estimate. We identify which assumed aspect ratio agrees mostly closely with the measurements and assume that value as the most likely aspect ratio.

Figure 2 shows the sensitivity of the dual-pol observables to the state vector quantities where the rates of change essentially quantify the first-order derivatives in the \( \mathbf{K} \) matrix [Eq. (4)]. The state vector parameters constrain the observables to varying degrees. The top row of Fig. 2 illustrates that \( Z_d \) and \( K_{dp} \) are sensitive in different ways to a combination of \( D_m \) and \( q_i \) when \( b_m \) is held constant while \( Z_q \) is insensitive to \( q_i \) at constant \( b_m \). The insensitivity of \( Z_d \) to \( q_i \) illustrates, for instance, how we could use a combination of \( Z_d \) and \( K_{dp} \) to derive \( q_i \) and \( D_m \) assuming \( b_m \) or \( r_b \) (i.e., Ryzhkov et al. 1998). Allowing \( b_m \) to vary at an \( r_b \) of 0.2 while holding \( q_i \) fixed, we find that the all the observables are sensitive to changes in \( b_m \) and \( D_m \) while \( K_{dp} \) has a nonmonotonic response at higher value of \( b_m \). Similarly, holding \( D_m \) fixed at a value of 0.2 cm,
the $Z_h$ and $Z_{dh}$ are found to be sensitive to both $b_m$ and $q_i$, while $Z_{dh}$ does not vary with $b_m$ for fixed $D_m$.

Uncertainties in the retrieved state vector parameters are quantified through the state parameter covariance matrix

$$S_x = (S_x^{-1} + K_x S_x^{-1} K_x)^{-1}.$$  

To evaluate the single-pixel uncertainties in the algorithm, we examine the magnitudes of the terms in $S_x$ using in situ data collected by the UND Citation in snow during OLYMPEX. We take 10-s averaged snow PSDs where we also have a measurement of bulk condensed ice mass and, therefore, an estimate of the $m-D$ relationship. We forward calculate the dual-pol observables and use the simulated measurements as input to the inversion algorithm, with the results then being compared with the observed microphysics. Then, we randomly perturb the observables according to their instrumental and forward modeled uncertainties for each PSDs and invert the perturbed observations to estimate the state vector. The simulated measurements are perturbed 10 times for each PSD resulting in a dataset with approximately $10^5$ members (Fig. 3a). The theoretical variances given along the diagonal of $S_x$ agree with the bootstrapping method with the fractional uncertainties in $S_x$ a bit more pessimistic (by about 30%) than what we find in the perturbation method (Fig. 3b). If we do not account for the forward model errors (Fig. 3b), we retrieve $q_i$, $D_m$, and $b_m$ to within a standard deviation of approximately 40%, 35%, and 15%, respectively. However, when we do account for the forward model errors, we obtain a more realistic characterization of the algorithm uncertainties. The ice water content uncertainty increases to ~120% or just over a factor of 2. Similarly, we retrieve $D_m$ to ~60% and $b_m$ to ~40%. Given the large volumes of data typically sampled by scanning dual-pol radars, the large datasets still convey useful information assuming random and uncorrelated errors. Hereinafter for convenience we refer to this method as the dual-polarization snow retrieval (DPSR) algorithm.

3. Results

We examine the extent to which dual-polarization radar measurements can characterize the processes involved in snow formation in shallow convection and frontal systems. For this, we use the OceanPol C-band radar on the Research Vessel (R/V) Investigator (RVI) collected during January and February 2018 (Mace et al. 2021). The Australian Marine National Facility OceanPol radar is a scanning, dual-polarization, Doppler, 1.3° beamwidth radar operating at C-band (EEC DWSR-2501DP model). Dual-polarization measurements are obtained using simultaneous transmission and reception from two individual horizontal and vertical receiver channels. The OceanPol antenna control system ingests high-precision inertial motion unit data from the ship at 10 Hz in real time to steer the radar beam to maintain a programmed azimuth and elevation direction. The accuracy of this stabilization has been found to produce a pointing accuracy better than 0.1°, even in the heavy seas typical of the Southern Ocean. Doppler measurements are automatically corrected in real time for the Doppler component induced by ship velocity components.
Dual-polarization moments are also corrected using the statistical corrections proposed in Thurai et al. (2014). Calibration of $Z_h$ has been achieved using the so-called GPM volume matching method outlined in Warren et al. (2018) and validated using a three-way-intercomparison exercise between collocated ground-based, ship-based, and GPM satellite observations (Protat et al. 2022). Once calibrated, the minimum detectable signal of the radar has been estimated as 10 dBZ at 100-km range. The $Z_{dr}$ measurements have been calibrated using $Z_h-Z_{dr}$ scatterplots, assuming that $Z_{dr}$ is $\sim$0.1 dB at the lowest measured reflectivities in light rain. The $Z_{dr}$ and $\rho_{HV}$ have been corrected in areas with low signal-to-noise ratio using the technique outlined in Schuur et al. [2003, their Eqs. (5) and (6)], and $K_{dp}$ is estimated from the measured differential phase shift ($\varphi_{DP}$) over a 2-km running window in range by using a linear regression-based method described in Bringi and Chandrasekar (2001). The OceanPol data used in this work used a series of 14 elevations ranging from 0.7° to 32°, with an azimuthal resolution of 1°. This volumetric sequence was repeated every 6 min.

**Fig. 5.** MERRA-2 reanalysis that depicts the large-scale circulation for the 9 Feb 2018 case study. The center of the red-outlined circle denotes the approximate location of RVI.

### a. Shallow convection

Precipitation plays a fundamental role in the self-organization of open-cellular mesoscale convective systems (MCC) at all latitudes. Feingold et al. (2010) discuss how convective processes cause energy to be redistributed vertically through phase changes and horizontally through the formation of cold pools. Interaction among cold pools of adjacent convective clusters then engenders the formation of new cycles of convection. Lasher-Trapp et al. (2021) examine the microphysical properties of newly formed ice hydrometeors at the tops of shallow convection measured during the Socrates campaign (McFarquhar et al. 2021). During Socrates, in situ and remote sensing data were collected in low-level clouds south of Tasmania in the range from $-50^\circ$ to $-60^\circ$ latitude. Specifically, Lasher-Trapp et al. sought to understand the mechanisms by which the number of ice crystals encountered at the tops of shallow convective towers could exceed, by orders of magnitude, the concentration of ice-nucleating particles (INP) that are active at the temperatures of shallow convection encountered ($\sim$ >$-20^\circ$C).
there. Similar results were reported by Huang et al. (2017) using aircraft data collected near Tasmania in the region where Mossop et al. (1970) discovered ice multiplication processes. See Korolev and Leisner (2020) for a recent review of the family of secondary processes that may be active in producing ice hydrometeors in mixed-phase shallow convection. Lasher-Trapp et al. (2021) concluded that not only are secondary ice processes critical to the precipitation processes in shallow convection but that an additional necessary condition was the occurrence of multiple cumulus updrafts in complexes that allowed for the relofting of previously generated ice crystals. The previously generated ice crystals caused ice multiplication (i.e., Hallett and Mossop 1974) as the ice crystals interacted with supercooled liquid water in updrafts.

Figures 4–6 document an MCC event observed by instruments on the RVI operating near 62°8S and 137°8E on 9 and 10 February 2018. The Aqua MODIS image (Fig. 4) from 0605 UTC 9 February 2018 shows an MCC pattern with relatively sparse cloud cover that persisted over the ship for approximately 36 h. The MERRA-2-derived large-scale meteorology (Fig. 5) shows that the open-cellular clouds existed within a region of cold air advection to the northeast of a surface low pressure center. The surface system was associated with an upper-level trough along 120°E. Southerly flow near 100°E from continental Antarctica is evident in a cold air outburst series of roll clouds that transition into closed cells and then break up into an open-cellular pattern downstream where they then passed within the scan pattern of the Ocean-Pol radar on the RVI. Surface temperatures decreased following the frontal passage at the ship, and open-cellular MCC was observed first at 1000 UTC with cloud tops near 4 km. The cell complexes, hereinafter referred to as squalls, passed over the ship at irregular intervals. The period between 1000 and 1400 UTC was the most active period at the RVI, with the passage of four distinct squalls. The 1600 UTC sounding (Fig. 6) is conditionally unstable up to a subsidence inversion near 600 hPa. The inversion roughly coincides with the cloud tops observed by the Bistatic Radar System for Atmospheric Studies (BASTA; Delanoe et al. 2016; Protat and McRobert 2020) W-band radar (Fig. 7) that was on a stabilized platform (Filisetti et al. 2017). Ascending W-band Doppler velocities exceeding
3 m s\(^{-1}\) are seen near the tops of several cells between 1000 and 1200 UTC (not corrected for particle motion). At the ship, the passage of each cell complex brought an increase in surface winds from 20 to 30–40 kt (1 kt \(\approx 0.51\) m s\(^{-1}\)) associated with distinct cold pools where surface temperature minima near freezing were observed (not shown). The cold-pool temperature minima were \(-1\) K colder than the local SSTs. Observers on the ship reported that aggregate snow was observed much of the time during squalls, with embedded periods of graupel. A Micro Rain Radar (MRR) operating at 24 GHz (not shown) observed reflectivities during brief periods of graupel in excess of \(+35\) dB\(Z_e\).

Figure 8 depicts the C-band observables at 1.5-km height during the period when the line of echoes that run roughly from northwest to southeast passed over the RVI between 1000 and 1400 UTC. C-band \(Z_a\) in the cores of the squalls was on the order of \(+35\) dB\(Z_e\), in agreement with the MRR observations. In the analysis that follows, we use \(\rho_{HV} > 0.95\), \(Z_{dr} > 0\), and \(Z_h > 15\) dB\(Z_e\) as an indicator of where the radar sample volumes were suitable for application of the DPSR algorithm. As we show, while there is correlation among the dual-pol observables, there is also independent variability in those observables that allows us to derive the snow microphysical properties. The maximum values of \(K_{DP}\) found associated with \(\rho_{HV} > 0.95\) (a threshold that we require for analysis) are not qualitatively higher than what has been documented in other studies where active snow growth processes were occurring (Allabakash et al. 2019; Oue et al. 2015).

Horizontal cross sections of gridded OceanPol \(Z_h\) (Fig. 9) illustrate the three-dimensional structure of the open-cellular cumulus precipitation features. The multiple squalls southeast of the ship location had passed over the ship in the previous hour (Fig. 7). The multicellular structure of these complexes is evident with several cores of high reflectivity in each of the squalls. The fractional coverage of precipitation decreases with height and focuses on the higher reflectivity cores that extend to just above 4 km. The vertical structure of the C-band dual-pol observables (Fig. 10) shows a vertical region between 2.5 and 3 km with the highest \(Z\) values. Above 3 km, there is a noticeable shift in \(Z_h\) to lower values. We also note that below 2 km, \(Z_h\) tends to shift to lower values with a broader distribution. Recall, however, that overall precipitation coverage increases nearer to the surface, suggesting some
spatial diffusion of the precipitation formed at higher levels. The $Z_{dr}$ is uniformly distributed between 0.5 and 1 dB, with some suggestion that the distribution narrows in the 2.5–3-km vertical region. The $K_{DP}$, on the other hand, has a minimum between 2.5 and 3 km. The $K_{DP}$ distributions broaden below, suggesting an increase in oriented planar ice that the ship observers noted as aggregate snow.

We applied the DPSR algorithm to the OceanPol data observed between 0400 UTC 9 February and 0000 UTC 10 February 2018 and collected the results into vertically resolved distributions of the retrieved quantities. The results (Fig. 10) reveal a well-defined vertical structure of the snow microphysics in the shallow convection observed on this day. The $D_m$ has a bimodal population of sizes that overlap in the 2–3-km region. Below this height, the snow has a mean $D_m$ near 4 mm, increasing toward the surface. The vertical bimodality in $D_m$ is associated with bimodality in the opposite sense in the total number concentration of snow particles per unit volume $[N_p = N_0 D_0 \Gamma(\alpha + 1)]$ expressed throughout in inverse liters; $N_p$ has a modal value near 2 L$^{-1}$ in the lowest 1 km. Above 2 km, where we observe a mode of smaller $D_m$, there tends to be higher $N_p$ with modal values between 10 and 50 L$^{-1}$. The bulk density of the snow is expressed as

$$\rho_b = \frac{6a_m D_m^{b_m-1} \Gamma(\alpha + b_m + 1)}{\pi r \Gamma(\alpha + 1)};$$

(9)

$\rho_b$ of the snow below 2 km gradually decreases toward the surface as the mean $q_p$ decreases. This intriguing vertical structure is not unique to this case study. We have examined open-cellular cumulus observed on 22 January 2018 in the Southern Ocean and data collected by a C-band radar on the coast of Norway during a cold air outbreak on 28 March 2020. While each event has differences in the magnitudes of quantities, precipitation coverage, and depth of the convective features, a similar vertical structure is found with high and small $N_p$ and $D_m$ values, respectively, near the echo tops of the layers where $\rho_b$ is highest.

Figure 11 shows an east–west cross section through the feature along $-62.35^\circ$S at 1126 UTC that further illustrates the cellular nature of the snow squall. The $K_{dp}$ suggests a strong gradient near 1 km above the surface. Observations (not
shown) from the RVI show this height as cloud base through most of the day. The $Z_h$ reaches a maximum near 35 dBZ below 1 km, and $Z_{dr}$ is a maximum in association with the maximum $Z_h$ and decreases toward the east. The observations, taken together, suggest an interesting structure to this shallow convective feature. The upper portions of the feature tend to be composed of smaller $D_m$ but higher $r_b$. The maxima in $N_s$ that we find near the top of the precipitation echo is $\approx 50$ L$^{-1}$, where $r_b$ approaches 0.3 g m$^{-3}$. This region of high $N_s$ is also the region of the largest $q_i$ along this transect, reaching a peak near 1.5 g m$^{-3}$. Below, the $q_i$ is smaller, although $D_m$ reaches a peak near 5 mm. The vertical structure appears somewhat sheared with the high $q_i$ values slanting to the east (right) in this plot.

The three-dimensional time-varying structure of the open-cellular clouds observed on 9 February suggests a possible interpretation of the vertical statistics in Fig. 10. The snow squalls that tend to have time scales of several hours and spatial scales of tens of kilometers are composed, at any given time, of multiple updrafts where ice crystals are smaller, denser, and higher in number than adjacent regions. The smaller and higher-density hydrometeor vertical columns tend to be flanked by lower-density regions of higher $D_m$ snow. The vertical statistics suggest that these high $N_s$ regions of dense ice crystals are preferentially near the tops of the radar echoes and tend to have the highest values of $q_i$. This interpretation of the OceanPol measurements and the measurement statistics are consistent with the recent findings of Lasher-Trapp et al. (2021) that infer the necessity of multiple updrafts in such cumulus complexes. Snow entrained into a new updraft interacts with supercooled liquid cloud water, thereby providing the ingredients for ice multiplication processes (Korolev and Leisner 2020). We note that the $N_s$ values we infer in the high $N_s$ columns are on the order of tens per liter—similar to the concentrations found in aircraft penetrations of mixed-phase convection over the Southern Ocean reported in Lasher-Trapp et al. (2021) and Huang et al. (2017). Thus, while it is not possible to definitively state with this data...
that the high $N_s$ regions are associated with updrafts where secondary ice processes are ongoing. The vertical and horizontal structure that we infer from the dual-pol data is consistent with that interpretation.

**b. Stratiform precipitation**

> The convergence of water vapor within midlatitude cyclones essentially controls large-scale precipitation (Carlson 1998). As air from lower latitudes glides quasi-isentropically upward to colder temperatures along a convergent lower-midtropospheric flow known as the warm conveyor belt (WCB), precipitation forms primarily as snow in the mixed-phase region. Field and Wood (2007) examined the water balance of midlatitude cyclones. They found that precipitation along the WCB scales with vapor transport and the vapor transport scales with temperature approximately according to the Clausius–Clapeyron relationship. A doubling of cyclone intensity is found to be much less effective at increasing precipitation in comparison with a warmer storm since water vapor increases at $\sim 7^\circ \text{K}^{-1}$ (Field and Wood 2007). Field and Wood (2007) found a similar scaling relationship between temperature and water vapor as Field and Wood (2007). However, the amount of visible brightening (i.e., albedo increase) of the cyclone due to increased cloud cover from the higher vapor transport depended upon the efficiency with which precipitation removed water from the WCB in the models. Models that had a less efficient precipitation process.

FIG. 11. Vertical cross section along an east–west line through the feature near $-62.4^\circ$S outlined by the box in Fig. 8: (a) $\rho_b$, (b) $D_m$, (c) IWC (or $q_i$), (d) $\rho_{wv}$, (e) $Z_{bh}$, (f) snow number density, or $N_s$, (g) radial velocity, (h) $K_{dp}$, and (i) $Z_{dr}$.
tended to brighten more with warming and, thereby, had more robust negative climate feedback (higher albedo with climate warming). It is, therefore, germane to constrain the precipitation efficiency in midlatitude cyclones observationally since models tend to disagree on this efficiency. Dual-pol radar data collected over the remote oceans is uniquely able to constrain those processes and could, with more in-depth analysis and more data, address the precipitation efficiency in midlatitude storms.

We provide two examples of the vertical structure of snow within a Southern Ocean frontal system that demonstrates the precipitation processes from the passage of a midlatitude cyclone over the ship on 25 January 2018 when the RVI was at 58.8°S and 139.9°E (Fig. 13). Radar echoes in a roughly east–west band moved over the ship from the north with light stratiform precipitation during the day after 1400 UTC 25 January. The melting layer throughout the day was near 500 m. An active period of precipitation occurred just prior to the end of the stratiform precipitation at the ship near 0000 UTC 26 January (Fig. 14). A region of note is the line of higher $Z_h$ along a narrow band that is oriented southwest–northeast between 58.5° and 58°S and 138° and 139°E. Another region of note is the broad area of higher $Z_{dr}$ southwest of the ship where $Z_t$ values approach 30 dBZ with maxima near 35 dBZ. These features tended to have high values of $r_{HV}$. The band to the north has higher values of $K_{DP}$ oriented along the band while $K_{DP}$ is smaller in the high $Z_h$ and $Z_{dr}$ region to the southwest of the ship.

The vertical distribution of microphysical quantities as a function of temperature derived from the 2200–2300 UTC period is shown in Fig. 15. The vertical structure from this region of stratiform precipitation is distinctly different from the vertical structure in the open-cell convection (Fig. 10). We find a minimum in $D_m$ near 263 K with a noticeable increase in $D_m$ toward warmer temperatures. This temperature region also denotes a transition in snow number by a factor of 2–5. At the warmer temperatures, we find about 1 L$^{-1}$ concentration of snow, whereas at colder temperatures above approximately 1.5 km, the $N_s$ are larger. The distribution of $b_m$ broadens at warmer temperatures, and similar broadening in $\rho_b$ occurs at

![Fig. 12. Three-color MODIS visible image from 0520 UTC 25 Jan 2018. The RVI position is marked by the yellow-outlined circle.](image-url)
temperatures warmer than 265 K below 2 km. We examine
two transects along the dashed lines in Fig. 14.

The southern transect (line 1 in Fig. 14) passes north to
south through the region of relatively high $Z_h$ and $Z_{dr}$ but
lower $K_{DP}$ at 1.5 km. The derived microphysics along line 1
(Fig. 16) suggests that the 1.5 km level is in a region of vertical
transition in microphysical properties. Between 2 and 1 km,
the snow exhibits a marked increase in $D_m$ from 2 to 5 mm
and a decrease in $N_s$ from 10 to 1 L$^{-1}$. However, unlike the
shallow convection where $N_s$ and $r_b$ are correlated, we
find that $r_b$ increases as the snow number decreases toward the
surface. Overall, $q_i$ increases as the snow particles sediment.
These vertical tendencies of increasing mean size and decreas-
ing number are consistent with aggregation of hydrometeors
that were newly formed in the region between 2 and 3 km. There, we
find higher $N_s$ and smaller $D_m$. However, the simultaneous increase in $r_b$ and $q_i$ also suggests that riming may be occurring simultaneously
with aggregation.

We find a very different structure in the vertical distribution
of microphysics along the northern transect or line 2 in Fig. 14
(Fig. 17). Line 2 is bookmarked by two deeper columns of
precipitation that extend to an altitude of 4 km. The $D_m$ is
higher in these vertical columns, mainly because of higher $Z_h$.
However, we also find alternating columns of high and low $r_b$
that are correlated with alternating columns of high and low
$N_s$ and $q_i$. The eastern column has the more interestingly
varied pattern with a high $Z_h$ core ($>35$ dBZ), high $Z_{dr}$
and low $K_{DP}$. The DPSR algorithm interprets this combina-
tion of measurables as large and high-density $D_m$ with lower
$N_s$—indicative perhaps of graupel. Overall, the cellular nature
along this line is suggestive of a narrow line of convection em-
bedded within the frontal system.

c. Bulk microphysical properties

We return to the initial motivation for this study. The snow
bulk density $p_b$ [Eq. (9)] is well known to be a significant
linchpin in quantitative retrievals of snow microphysics that
use nonpolarimetric radars that cannot constrain these char-
acteristics. This includes CloudSat and GPM. As an example,
Hammonds et al. (2014) used in situ snow observations
collected during the Colorado Airborne Multi-Phase Cloud

![Fig. 13. MERRA-2 reanalysis that depicts the large-scale circulation for the 25 Jan 2018 case study. The center of the
red-outlined circle denotes the approximate location of RVI.](image)
Study (CAMPS) where direct measurement of the bulk ice mass simultaneously with measurements of the PSD allowed for constraint of the $m$–$D$ relationship in snow over the Rocky Mountains. Using forward calculations, they found that natural variability in $m$–$D$ resulted in an uncertainty in $Z_h$ of more than 5 dB. Similar results were reported in Mace and Benson (2017). This is a startling level of forward model error that is not typically accounted for in snow retrievals and, as concluded by Skofronick-Jackson et al. (2019), helps explain the differences between CloudSat and GPM algorithms.

Ideally, one might consider that the $m$–$D$ relationship in nature would vary as a function of the environmental conditions within which the snow evolves. For instance, Moisseev et al. (2017) related liquid water path to $p_b$, and used Doppler velocity as a constraint (Kneifel and Moisseev 2020). A limited number of studies have successfully attempted to investigate this sensitivity in various types of ice hydrometeors including tropical anvil clouds (Mascio et al. 2017) and snow (Szyrmer and Zawadzki 2010; Mason et al. 2018). Ideally, a retrieval algorithm would recognize the environmental conditions and draw from an observational database of such quantities. If done correctly, the uncertainties would be carried forward to account for forward model uncertainties in the algorithm as a function of the atmospheric state. While ground-based zenith pointing data may eventually provide this information, the capacity for dual-pol radar measurements to provide such a database is significant given the volumes typically sampled by them and their increasing use in operational networks (e.g., Matrosov 2020).

As an example, we take the measurements collected by the OceanPol radar over the Southern Ocean during January and February 2018 and segregate the data into open-cellular and stratiform precipitating systems. This separation is accomplished simply by examining the radar imagery time series and classifying periods by inspection of the horizontal structure of the radar echoes. Periods that might have been ambiguous or contained both types of precipitation were excluded. If microphysical differences are found, more automated techniques, such as that proposed in Lang et al. (2022), could be used to classify the large-scale conditions and feed that information to DPSR. All the requirements noted above in analysis of the case studies were also applied. Since the Southern

![Fig. 14. As in Fig. 8, but for 2340 UTC 25 Jan 2018. The numbered lines show cross sections depicted in Figs. 16 and 17, below.](image-url)
Ocean is a rather cloudy and stormy part of the world, we found roughly 20 million and 5 million volumes that were classified as stratiform and cumuliform, respectively, from the 5-week dataset. In Fig. 18 we present the frequency distributions of the snow bulk density and exponent of the m–D relationships derived from DPSR. Since we do not directly retrieve the aspect ratio (see methods), we do not show it here. However, aspect ratios, on average, were much lower than typically assumed (0.6) and were in the 0.2–0.3 range in agreement with the results of Matrosov (2020) with no apparent systematic differences between convective and stratiform snow. Overall, we find that stratiform snow has a \( \rho_b \) near 0.09 g cm\(^{-3}\) with a standard deviation of ~30%. The \( b_m \) in stratiform snow has a mean value near 2.25 with a hint at bimodality. Snow from open-cellular systems has a similar degree of variability but is shifted to higher \( \rho_b \) (mean of 0.13 g cm\(^{-3}\)) with a mean \( b_m \) value near 2.3. While more detailed investigation is warranted including a consideration that structural aspects of the algorithm might result in these differences, we speculate that the typically higher liquid water paths in shallow convection would likely be a significant part of the explanation for these differences. The bimodality evident in the \( b_m \) of snow in stratiform conditions may also be due to the existence of liquid water in embedded convection. From a snow retrieval point of view, such differences in bulk density are significant and would result in forward model radar reflectivity errors of several dB. The range of forward model error from, say, the lowest quartile to the upper quartile of these distributions would result in forward model errors in radar reflectivity that exceed 5 dB.

4. Summary and conclusions

Motivated by the need to better understand the properties and processes associated with ice-phase precipitation at high latitudes, we use dual-polarization radar measurements collected from a research vessel in the Southern Ocean interpreted with the formalisms of Ryzhkov et al. (2020) and Jung et al. (2010) in an optimal estimation algorithm. The dual-polarization measurables of horizontally polarized radar reflectivity \( Z_h \).
differential reflectivity $Z_{dr}$, and specific differential phase $K_{DP}$ uniquely depend upon the snow particle size distribution characteristics such as total condensed water $q_i$, mass-mean particle size $D_m$, and bulk density $\rho_b$. Interpreting data from spaceborne radars (CloudSat and GPM) in retrieval algorithms typically requires assumptions about $\rho_b$. Therefore, we include in our state vector the $\rho_b$ represented as the exponent to the mass–dimensional power law, the mass mean size $D_m$, and the ice water content $q_i$. The inversion problem is underconstrained, with many more unknown parameters than independent information. To close the set of forward model equations, we make assumptions about various quantities, including the shape factor of an assumed modified gamma distribution of snow particles, the functional form of the mass–dimensional power-law prefactor, the standard deviation of the ice crystal canting angle, and the particle aspect ratio.

In addition to the observational error, the retrieval uncertainties depend upon assumptions that characterize their influence on uncertainty in the forward model. Overall, we find that the observables are all constrained by the state vector parameters, except that $Z_{dr}$ is not dependent on $q_i$. The lack of dependence of $Z_{dr}$ on $q_i$ illustrates, for instance, how a combination of $Z_{dr}$ and $K_{DP}$ inform derivations of $q_i$ and $D_m$ assuming $b_m$ or $\rho_b$ (i.e., Ryzhkov et al. 1998). The codependence of the observables to the state parameters illustrates why a Bayesian inversion algorithm like we describe here is helpful to derive the state vector quantities since no dual-pol observable maps uniquely to any of the state parameters. By considering several leading sources of uncertainty, we demonstrate theoretically and empirically that the single-pixel uncertainties in $q_i$, $D_m$, and $b_m$ are approximately 120%, 60%, and 40%, respectively.
We examine several case studies of shallow mixed-phase convection and of an active occluded frontal system using data collected by the C-band OceanPol radar on the Australian R/V Investigator from a voyage into the Southern Ocean in 2018. We find that the convective complexes were composed of multiple individual convective cells with highly variable microphysical properties within the cellular structures. This multicellular structure is consistent with surface observations at the ship of both graupel and aggregate snow. Interestingly, the dual-pol observations implied strong gradients in snow particle number where the $r_b$ tended to increase and the $D_m$ decreased. These regions with high snowflake number tended also to be regions of maximum $q_i$ but not maxima in $Z_h$. The microphysical structure that we infer is consistent with the recent findings of Lasher-Trapp et al. (2021) who found in situ evidence for secondary ice production in the tops of cold Southern Ocean shallow cumulus.

We find that frontal system demonstrated wide variations in properties. In a particular case study, we show an embedded convective line 50 km north of a region of active stratiform snow. The stratiform region had generally lower values of $K_{DP}$ than in the convective region, although the values of $Z_h$ and $Z_m$ only differed in the vertical distribution. We note an evolution in snow microphysics in the stratiform region below 3 km where ice crystal number decreased rapidly from $5 - 10$ to $1 - 2$ and $D_m$ increased from 1 to 5 mm. If this were a case of pure aggregation, we would expect $r_b$ to decrease. However, we found bulk density and $q_i$ increasing consistent with riming and aggregation processes perhaps co-occurring. In the convective line to the north, we noted possible evidence for graupel in deep cellular structures.

This study demonstrates that dual-pol radar measurements in snow collected over the remote oceans can serve as a means of illuminating the processes that are important in
high-latitude mixed-phase precipitating systems in regions where observations have traditionally been scarce. Because the measurements are sensitive to the $\rho_b$ of snow, the measurements can also serve as a pathway for parameterizing the uncertain assumptions that cause snow retrievals from space-based radars to be highly uncertain and to disagree with one another. Our initial results suggest significant variability in the $\rho_b$ and $b_m$ between stratiform and cumuliform snowing systems in the 2018 RVI OceanPol dataset. The $\rho_b$ in cumulus tend to be higher by ~30%. We speculate that higher $\rho_b$ in snow derived from open-cell cumulus when compared with snow in stratiform systems may be due to the presence of increased liquid water path. Such differences in bulk properties should be accounted for in retrieval algorithms applied to nadir and zenith pointing active and passive microwave systems including CloudSat and GPM.

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Data availability statement. All OceanPol and BASTA radar data are publicly available in the Australian Unified Radar Archive (AURA: http://www.openradar.io/). The OceanPol Weather Radar Dataset is available through the National Computing Infrastructure (https://doi.org/10.25914/5fc4975c7dda8).

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