Case Studies of the Distribution of CAT in the Troposphere and Stratosphere[1]

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ABSTRACT

Two separate case studies of clear air turbulence are presented, one in the stratosphere over the Rocky Mountains, the other in the upper troposphere over the midwestern plains.

The mechanism in both situations appears to be similar. CAT occurs in strongly baroclinic zones with strong vertical wind shears and low Richardson numbers. There is a tendency for the most severe turbulence to be located at the edges of the baroclinic zones.

1. Introduction

It is now well known that CAT occurs in regions of large vertical wind shear and large horizontal temperature gradients, but it is by no means clear how strong these gradients must be. On the basis of either perturbation theory or energy considerations, the conclusion is usually reached that there exists a "critical" value of the Richardson number, defined by

$$\mathrm{Ri}=\frac{g}{\theta}\frac{\partial\theta}{\partial z}\Big/\left|\frac{\partial \mathbf{V}}{\partial z}\right|^2, \tag{1}$$

where z is height, $\mathbf{V}$ the vector wind, θ potential temperature, and g gravity. If Ri is less than the "critical" value R_c, perturbations can grow and turbulence will be produced. Theories and observations do not agree on the magnitude of R_c. Perturbation theories usually lead to values of about 0.25 or less. Observations in the atmospheric boundary layer and in the wind tunnel do not suggest a definite R_c; instead, for $\mathrm{Ri}<0.15$, turbulence is certain, and for $\mathrm{Ri}>0.50$, turbulence is absent. In between, there is a zone in which turbulence may occur; its intensity generally appears to decrease with increasing Richardson number (Webster, 1964).

There is now considerable evidence that CAT is associated with baroclinic layers (internal fronts) in which the wind shear is so strong as to overcome the stabilizing effect of the stratification [see, e.g., Reiter and Burns (1966) and Colson and Panofsky (1965)]. Presumably, when the shear is large enough that the Richardson number becomes less than critical, disturbances can grow and CAT begins. However, it has been extremely difficult to determine observationally what the "critical" Richardson number for CAT should be. There are several origins of this problem.

In the first place, it is not clear how the Richardson number in the free atmosphere should be measured. Obviously, ordinary rawinsonde soundings smooth the wind profiles too much to even estimate the positions of the baroclinic layers, much less the Richardson numbers. If, nevertheless, Richardson numbers are computed from such soundings, they will be systematically too large because the shears are too small.

On the other hand, soundings by accurate radar, such as the FPS 16, show a wealth of detail, some of which is quite persistent and has been ascribed to inertia or gravity waves. The question, then, is whether these small features should be smoothed before the Richardson numbers are computed; if such soundings are not smoothed, the Richardson numbers will tend to be quite small, and, in fact, not very accurate, because temperature soundings on the same scale are not available.

A second difficulty arises from the fact that a layer with a greater-than-critical Richardson number can be made unstable by a wave with considerable vertical amplitude, as suggested by Phillips (1967). This may, indeed, be the reason why CAT seems so much more fre-

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quent and severe in mountain areas than in the plains. For, as Phillips has shown, the Richardson number in the trough of the wave may be less than the mean Richardson number of the layer; this may also account for the often-reported spottiness of the CAT occurrences.

Colson and Panofsky (1965) suggested that, in analogy with turbulence in the surface layer, the critical Richardson number separates turbulent from laminar flow. The intensity of the turbulence, however, is proportional to the square of the wind shear across the baroclinic layer, which takes the place of just the square of the wind speed near the ground.

It is the purpose of this paper to demonstrate through a detailed synoptic analysis of two CAT situations, one in the troposphere and one in the stratosphere, that some of these characteristics are not limited to the troposphere.

2. Tropospheric study, 23–24 April 1963

Two WB-50 aircraft equipped with Doppler radar and free air thermometers made flights between Dayton and Flint commencing at 1830 GMT and terminating at 0230 GMT. The horizontal and vertical paths of both aircraft are shown in Fig. 1. The dashed lines in this figure represent the boundary of a baroclinic layer at 0000 GMT.

The airplane data were gathered at 40-mi intervals. In addition, radiosonde ascents were available at 1800 and 0000 at Dayton, Flint, and other stations.

A rather elaborate procedure was followed in analyzing the data. Isentropic charts from 295 to 335K potential temperature were analyzed for wind speed and Montgomery stream function at intervals of 30 ergs gm^{-1} for the United States and Canada. The purpose of this analysis was to supplement the aircraft wind and temperature data in the cross section between Flint and Dayton. Flint-Dayton-Nashville cross sections, prepared in the manner outlined in the remainder of this section, at 1800 and 0000 GMT, are shown in Figs. 2 and 3.

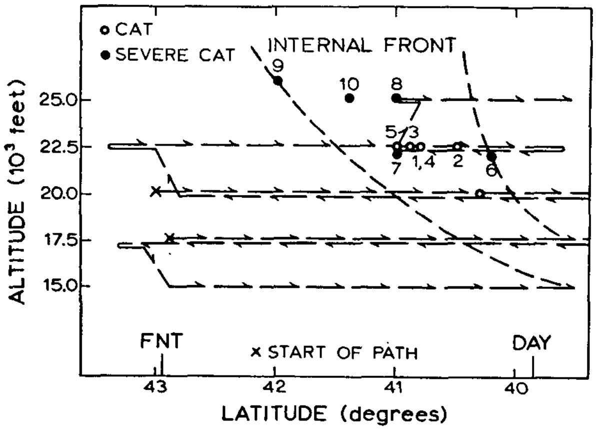

Fig. 1. Paths of aircraft in tropospheric study with the boundaries of internal front (baroclinic zone) shown by dashed lines. Numbers refer to turbulence reports.

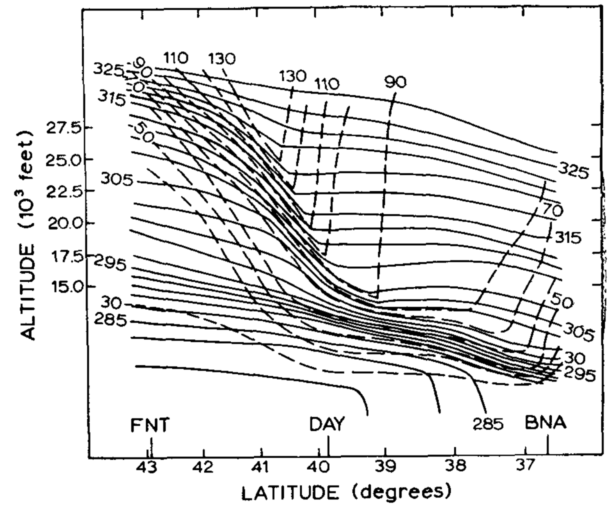

Fig. 2. Isentropic cross section for 1800 GMT 23 April 1963, Flint to Nashville. Solid lines are isentropes at intervals of 2K; dashed lines are isotachs at intervals of 10 kt.

Considerable effort was devoted to locating the baroclinic layer as accurately as possible. To facilitate determination of the boundaries of the layer, the winds and temperatures were plotted as a function of time and latitude. Separate graphs were constructed from the observations of each aircraft. Discontinuities in the spatial and temporal variation of wind and temperature can easily be detected on these graphs. The discontinuities marked boundaries of the baroclinic layer, and from the graphs these discontinuities could be traced in time and space.

After the analysis of the aircraft data was completed, it was easiest to analyze cross sections for two different times, one cross section corresponding to the 1800 GMT sounding on 23 April, the other 6 hr later. The aircraft data were divided so that data closest to the 1800 GMT sounding were plotted on that cross section, and the values closest to the 0000 GMT sounding were plotted on the other. Isentropes and isotachs were analyzed for each time period separately, making use of both the aircraft and radiosonde data. In addition, graphs of the winds and temperatures as a function of latitude were consulted in order to determine the location of the boundaries of the internal front.

In constructing the cross section in regions outside the range of the aircraft data, the thermal wind equation was used in a subjective way. Inside the region of aircraft data the chief analysis problem was reducing the data to a common time and locating the boundaries of the internal front as accurately as possible.

Richardson numbers calculated from the ratios of finite differences of wind speed ΔV and potential temperature $\Delta\theta$ over a constant increment of height, $\Delta z = 500$ m, were used. For this study, it could be assumed that $\Delta V/\Delta z = |\Delta V/\Delta z|$, i.e., the wind speed difference over Δz is the same as the magnitude of the vector difference. This assumption is valid only when turning of the wind

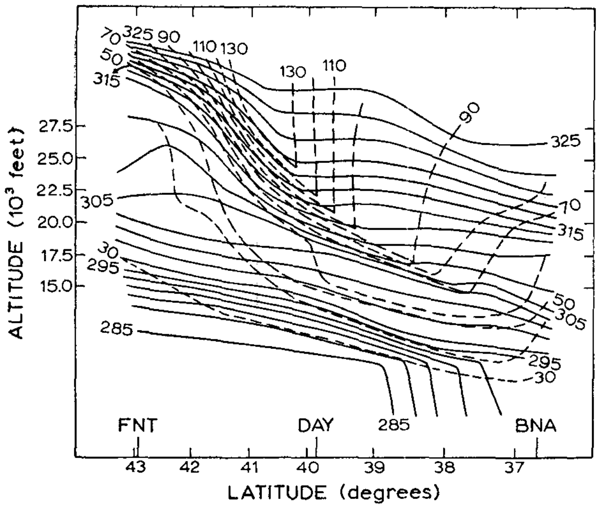

FIG. 3. Same as Fig. 2, except 6 hr later.

with height is negligible, and the observations show that the wind turned no more than 5° through the entire baroclinic zone. Geostrophic Richardson numbers R_g were also computed by estimating wind shear from isobaric temperature gradients.

One of the most obvious features shown on the cross sections is the metamorphosis of the baroclinic layer between 1800 and 0000 GMT. At 1800 GMT, the cross section in Fig. 2 shows a zone of isentropes that slope downward to the south uniformly with height. This "front" is connected with a stable layer, whose upper surface is at 550 mb. It is the westward extension of a cold front that lies along the east coast. On the 0000 GMT cross section in Fig. 3, the slope of the upper front is about the same as it is on the 1800 GMT cross section. However, the lower end of the front has disassociated itself and lifted away from the stable layer at 550 mb. The apparent lifting of this isentropic zone away from the lower stable layer is the result of the advection of a different structure into the 0000 GMT cross section at the 550-mb level. The internal front slopes upward to the west. This explains why the westerly winds brought a higher base into the 0000 GMT cross section.

TABLE 1. Time, altitude and intensity of turbulence reports and Richardson numbers.

Report no.	Time (GMT)	Altitude (ft)	Intensity	Ri	R_g
1	2240	22,300	moderate	0.30	0.54
2	2340	22,300	moderate	0.52	0.54
3	2350	22,300	moderate	0.23	0.45
4	0000	22,400	moderate	0.23	0.54
5	0010	22,500	moderate	0.36	0.39
6	0025	22,000	severe	0.44	0.88
7	0025	22,000	severe	0.30	0.36
8	0218	25,000	severe	0.37	0.28
9	0224	26,000	severe	7.70	1.09
10	0225	25,000	severe	0.71	0.20
12	2015	19,700	moderate	0.56	1.10

TABLE 2. Richardson numbers at time of aircraft passage where turbulence was not reported.

Report no.	Ri	R_g	Report no.	Ri	R_g
	15,000 ft			20,000 ft	
4	22.00	12	3	5.9	14
5	2.40	53	4	1.2	0.87
6	6.70	35	5	0.78	0.96
7	6.50	50	11	0.65	0.60
8	6.60	13	12	4.0	1.4
12	0.38	1.1	13	5.7	1.5
13	0.26	298	14	6.0	1.6
14	0.26	298	15	4.1	18
	17,500 ft			22,500 ft	
3	2.4	11	6	1.0	0.51
4	3.1	5.6	7	0.33	0.54
5	2.3	0.51	13	0.77	0.48
6	1.6	1.1	14	0.24	0.45
11	1.5	0.51	15	0.29	0.57
12	4.3	4.2	16	0.25	0.42
13	6.1	3.6		25,000 ft	
			17	0.77	0.35
			18	0.25	0.44
			19	0.77	0.35

The average actual Ri at aircraft points at various levels between, and including, 17,500 and 22,500 ft are shown in Fig. 4. The points at the 15,000- and 25,000-ft levels are eliminated from this figure because the analysis is somewhat uncertain at these levels, i.e., they lie at the boundaries of the aircraft data.

The average Ri for each level in the figure is larger at 1800 than at 0000 GMT. The level of the greatest incidence of turbulence reports, 22,500 ft, has the lowest average Ri at both time periods, decreasing from 0.69 to 0.39 during the 6-hr period between soundings.

Because many of the aircraft data collection points lay within the layer at both time periods (1800 and 0000 GMT), it was decided that averages of the Ri at these points should be computed for both times so that the change of these Ri with time could be appraised. These

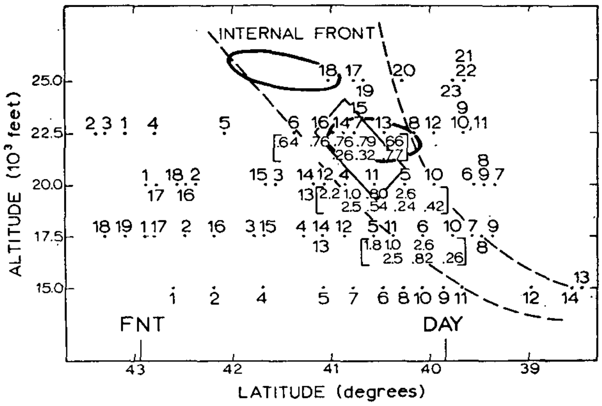

FIG. 4. Comparison of turbulence reports and Richardson numbers. Areas with turbulence reports indicated by solid curves. Upper number in the brackets is Ri at 1800 GMT; lower number refers to 0000 GMT. Boundaries of internal front (baroclinic zone) are indicated by dashed lines. Numbers without brackets refer to aircraft data points.

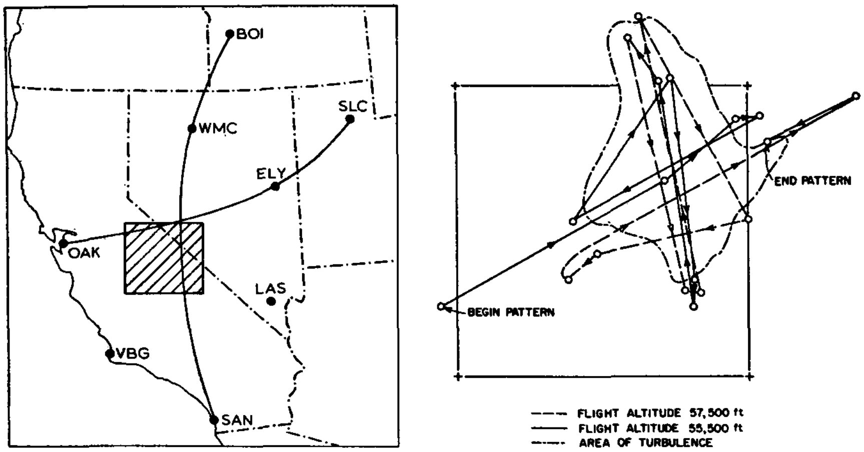

FIG. 5. Left: general location of study. Curving solid lines indicate position of cross sections through the turbulent area marked with slanting lines. Square indicates region shown in detail on right. Right: flight paths used in study.

points are also located in the portion of the baroclinic zone where the analysis is most reliable. Points that lie within the front at both time periods are contained within the boxed portion of the zone shown in Fig. 4. The average Ri at 1800 GMT is 0.76 decreasing to 0.32 at 0000 GMT, a low value that is generally believed to indicate turbulence.

Fig. 1 shows that the severe turbulence occurs at or near the boundaries of the internal front more frequently than moderate CAT. This is similar to what McConathy (1966) and Anderson (1957) have found. Also the turbulence parameters used by Endlich and Mancuso (1964) favor turbulence along the boundaries of the internal fronts.

In Fig. 1, most of the turbulence occurs at 22,500 and 25,000 ft, with only one report of turbulence at 20,000 ft. Thus, there is a general agreement between the magnitude of the Ri and the reports of turbulence at different altitudes, such that turbulence occurrences are most frequent at higher levels where Ri's are the smallest.

A possible cause for the existence of the most severe turbulence near the edges of the front may be that turbulence initially develops in the interior parts of the zone, where the wind shears are originally largest. As turbulence continues, mixing reduces the gradients of wind and temperature near the center of the zone while frontogenesis increases the gradients near the boundaries where turbulent mixing has not occurred as frequently. Thus, the net effect over a period of time is to increase the gradients along the boundaries and hence cause the greatest vertical wind shear and most of the severe CAT to occur there.

Table 1 lists conditions at all data points where turbulence was reported along with the actual and geostrophic Richardson numbers. Seven of the eleven cases have Ri <0.50. Two more differ from 0.50 by less than the estimated error of measurement. At point 10, the Richardson number of 0.71 is unreliable because the observation occurs near the edge of the data, where $\Delta V/\Delta z$ is not accurately determined. The horizontal gradient $\Delta\theta/\Delta n$ is more accurate, so that the geostrophic Richardson number of 0.20 is possibly more reliable than the actual Ri. Similarly, the observation at point 9 is uncertain. The large value of Ri there is probably due to the fact that the point was actually located just outside the baroclinic zone. In summary, all observations in Table 2 are reasonably consistent with a "critical" Ri of 0.50.

For comparison, Table 2 shows some statistics of the nonturbulent points. All but 8 of the 32 cases have a Richardson number above 0.50. Many nonturbulent points outside of the zone are not included in Table 2; for these, the Ri were quite large. Points 12, 13 and 14 at 15,000 ft, which have low Ri and are not turbulent, occur near the edge of the detailed data, where $\Delta V/\Delta z$ is uncertain. The geostrophic Richardson numbers R_g exceed unity in agreement with the lack of turbulence. Probably, the vertical distance between isotachs in the figure is much too small. Points 14, 15 and 16 at 22,500 ft are surrounded by turbulence reports, but no turbulence is reported there. Either the CAT there was spotty, or the pilots failed to report turbulence. Finally, point 18 at 25,000 ft again occurred near the limit of the data.

In summary then, the hypothesis that R_c is near 0.50 is reasonably consistent also with results obtained at the nonturbulent points. However, this estimate is based on the hypothesis that isentropes and isotachs are uniformly spaced within the zone, a hypothesis which cannot be tested within the limitations of the present observations.

3. Stratospheric study, 29 August 1966

The case study of stratospheric CAT is based on the flight of a high altitude research airplane from Edwards

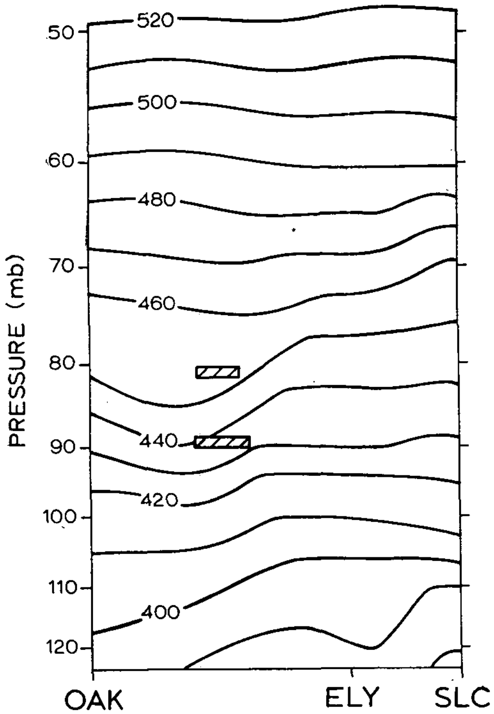

FIG. 6. Vertical cross section for 0000 GMT 30 August 1966, Oakland to Salt Lake City, showing isentropes, and turbulent areas indicated by dashed lines.

Field, Calif., from 1745 to 2104 GMT, 29 August 1966. The airplane was instrumented with an aircraft turbulence measuring system which records actual velocities in the Cartesian x, y and z components (x along the flight path); see, e.g., Crooks *et al.* (1967) and Dutton (1967).

The airplane flew, in effect, a triangular path, and the pilot was asked to explore in detail in CAT areas encountered at two separate altitudes, 55,500 and 57,500 ft (approximately 90 and 80 mb, respectively). Fig. 5 shows the general location of study and the detailed flight paths in the CAT regions. The figure indicates also the two lines along which vertical cross sections were constructed (San Diego to Boise and Oakland to Salt Lake City). The cross sections were constructed only on the basis of 0000 GMT data under the assumption that the stratospheric patterns move slowly.

The vertical cross sections are shown in Figs. 6 and 7. The isentropic surfaces in these sections are based not only on the observations at the stations located along the lines indicated on the cross sections, but also on previously analyzed isotherm charts for the western United States at 100,70 and 50 mb. Further, the thermal wind relation was used to determine the slopes of the isentropes at each station. Nevertheless, the details of the charts are subject to considerable uncertainty.

In both cross sections, baroclinic layers are indicated in the CAT regions. Of course, the airplane only flew at two levels, so that the vertical extent of the CAT is uncertain. Nevertheless, the large horizontal temperature gradients along both cross sections suggest strong wind shears in the CAT areas.

Geostrophic Richardson numbers in the general CAT volume are shown in Fig. 8. Since these Richardson numbers depend on the square of the horizontal temperature gradient, which is strongly affected by the lack of reliability in the analysis of the potential isotherms, the precise values of these numbers are subject to doubt. Nevertheless, the general pattern is quite clear; in the CAT regions, the Richardson numbers appear to be generally smaller than outside of these regions.

It is not possible to determine "critical" Richardson numbers from Fig. 8. However, the critical value of 0.50 inferred from the tropospheric analysis is not inconsistent with the stratospheric data, particularly if it is realized that the geostrophic numbers in each cross section are computed from the component of the temperature gradient only in the plane of the cross section, so that the actual Richardson numbers may be considerably lower.

4. Summary

In summary, it appears that severe or moderate CAT occurs both in the upper troposphere and the lower stratosphere in strongly baroclinic layers. Mountains can probably increase the probability and perhaps intensity of CAT, but are not necessary for its occurrence.

The "critical" value of Richardson numbers indicated in this study is in the neighborhood of 0.50; however, the actual limiting value may be lower because of data processing and reliability problems or because the ac-

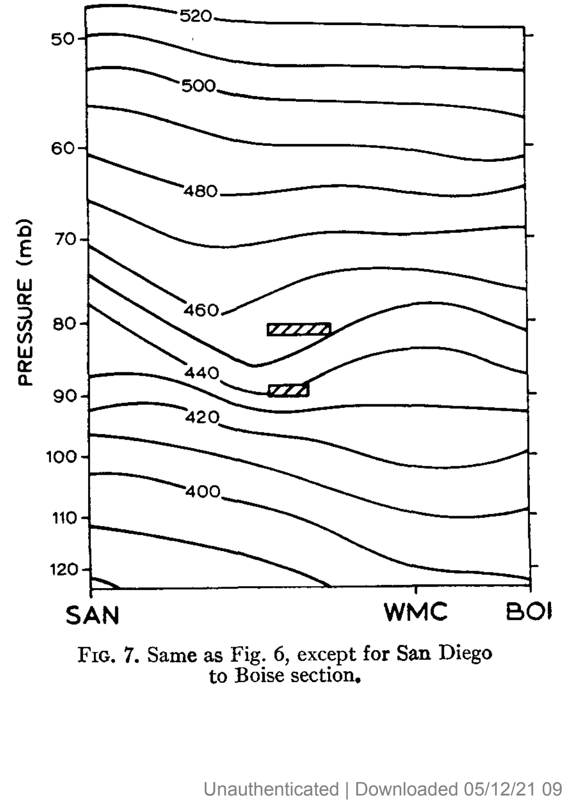

FIG. 7. Same as Fig. 6, except for San Diego to Boise section.

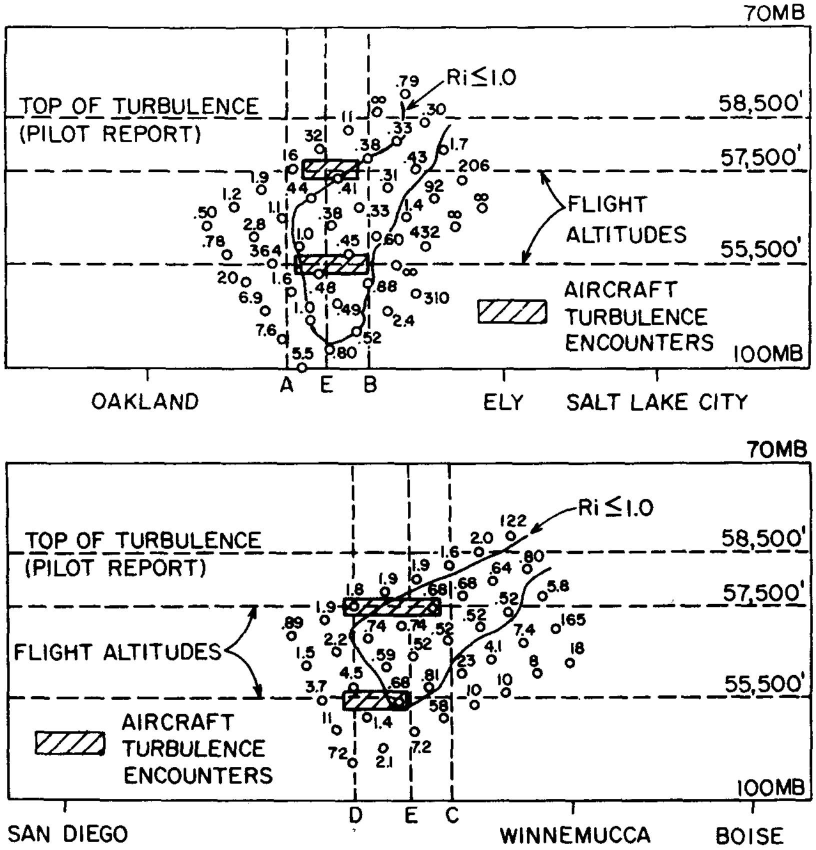

FIG. 8. Vertical distribution of geostrophic Richardson numbers in both cross sections, shown with turbulent regions.

tual isentropic surface may be irregularly spaced due to inertia waves, whereas they appear more or less uniformly spaced in the cross sections.

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REFERENCES

Anderson, A. D., 1957: Free-air turbulence. *J. Meteor.*, **14**, 477–494.

Colson, D., and H. A. Panofsky, 1965: An index of clear-air turbulence. *Quart. J. Roy. Meteor. Soc.*, **91**, 507–513.

Crooks, W. M., F. M. Hoblit, D. T. Prophet, *et al.*, 1967: Project HICAT. An investigation of high altitude clear air turbulence. Tech. Rept. AFFDL-TR-67-123, Vols. I, II, III, Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio.

Dutton, J. A., 1967: Belling the CAT in the sky. *Bull. Amer. Meteor. Soc.*, **11**, 813–820.

Endlich, R. M., and R. L. Mancuso, 1964: Clear-air turbulence and its analysis by use of rawinsonde data. Final Rept., Contract No. Cwb-10624, Stanford Research Institute, 55 pp.

McConathy, D., 1966: A case study of CAT in the Rockies. Final Rept., Contract No. Cwb-10926, Dept. of Meteorology, The Pennsylvania State University, 27 pp.

Phillips, O. M., 1967: The generation of clear-air turbulence by the degradation of internal waves. *Atmospheric Turbulence and Radio Wave Propagation*, Proc. Intern. Colloquium, 15–22 June 1965, Moscow, Nauka, 130–135.

Reiter, E. R., and A. Burns, 1966: The structure of clear-air turbulence derived from "TOPCAT" aircraft measurements. *J. Atmos. Sci.*, **23**, 206–212.

Webster, C. A. G., 1964: An experimental study of turbulence in a density-stratified shear flow. *J. Fluid Mech.*, **19**, 221–245.