On the Computer Calculation of Vapor Pressure and Specific Humidity Gradients from Psychrometric Data

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Accurate evaluation of small vapor pressure differences is required in micrometeorological investigations of the flux of water vapor in the atmospheric boundary layer. Psychrometric techniques are often used, as these have the advantage that vapor pressure differences can be determined directly from measured differences of dry- and wet-bulb temperature (Slatyer and McIlroy, 1961). In the calculation, the slope of the saturation vapor pressure curve is required at the

mean wet-bulb temperature. Extraction of this factor from standard meteorological tables, while satisfactory in some applications, is often inappropriate to the methods of automatic computation. So, too, is its derivation from the internationally accepted formulation for saturated vapor pressure due to Goff and Gratch (1946) as given by List (1958). This note describes procedures which facilitate accurate computation.

Upon differentiation with respect to z , the psychro-

metric formula,

$$e = \epsilon_{t'} - A p (1 + 0.00115 t') (t - t'), \quad (1)$$

yields

$$\frac{de}{dz} = \left(\frac{\partial \epsilon}{\partial t'} \right)_{(t')} \cdot \frac{dt'}{dz} - A p (1 + 0.00115 t') \left(\frac{dt}{dz} - \frac{dt'}{dz} \right) - 0.00115 A p (t - t') \frac{dt'}{dz} - (1 + 0.00115 t') (t - t') A \frac{dp}{dz}. \quad (2)$$

(See List of Symbols for definition of terms). Neglecting the last two terms (introducing a negligible error), the above equation can be written in finite difference form as

$$\frac{\Delta e}{\Delta z} = (S + K) \frac{\Delta t'}{\Delta z} - K \frac{\Delta t}{\Delta z}, \quad (3)$$

where

$$S = \left(\frac{\partial \epsilon}{\partial t'} \right)_{(t')},$$

and

$$K = A p (1 + 0.00115 t').$$

Here, the gradient of the chord, $\Delta \epsilon / \Delta t'$, has been approximated by the gradient of the tangent at the average wet-bulb temperature, $(\partial \epsilon / \partial t')_{(t')}$, which introduces errors of the order of 0.1% for $\Delta t' = 2.0^\circ\text{C}$ and errors of about 0.5% for $\Delta t' = 5.0^\circ\text{C}$. Measurement of t' at one point, together with measurement of $\Delta t'$ gives (t') .

The value of S can be calculated with quite sufficient accuracy from an approximation $E(t')$ to the saturation vapor pressure made by Tetens (1930), and transformed to a more convenient form by Murray (1967). Thus,

$$E(t') = \alpha \exp[\beta t' / (t' + \gamma)], \quad (4)$$

where $\alpha = 6.1078$, and β and γ are 21.875 and 265.50, respectively, over ice and 17.269 and 237.30 over

water. Differentiation with respect to temperature gives

$$\left(\frac{dE}{dt'} \right) = \frac{\sigma}{(t' + \gamma)^2} \exp[\beta t' / (t' + \gamma)], \quad (5)$$

where $\sigma = \alpha \beta \gamma = 35,473$ over ice and 25,029 over water.

The accuracy of this latter expression for the slope can be gaged from Table 1, where values calculated from it at certain temperatures are compared with those calculated from the Goff-Gratch formula, with the slope of the tangent to the curve at t' being approximated by the gradient of the chord spanning the temperature interval 0.2°C . The last column lists the percentage error of the Tetens formula. The magnitude of this error over the whole temperature range is never $> 0.2\%$ and rarely $> 0.1\%$.

Thus, very small errors are introduced when the computation of vapor pressure differences follows the procedures outlined above. These errors decrease in magnitude with decreasing wet-bulb temperature difference $\Delta t'$.

In micrometeorology there is often a requirement for specific humidity gradients. Differentiation of the expression which relates vapor pressure to specific humidity yields

$$\frac{dq}{dz} = \left(\frac{de}{dz} \right) \frac{0.622 p}{(p - 0.378e)^2} - \left(\frac{dp}{dz} \right) \frac{0.622e}{(p - 0.378e)^2}.$$

In many applications the magnitude of the second term is negligible compared with the first term. When this is so the first term can be usefully approximated to yield a finite difference form

$$\frac{\Delta q}{\Delta z} = \frac{\Delta e}{\Delta z} \cdot \frac{0.63}{p}. \quad (6)$$

The accuracy of the approximation may be gaged from Table 2, where the percentage error involved in its application is listed for a range of vapor pressures, at atmospheric pressures of 900 and 1000 mb.

The demonstrated accuracy and the inherent simplicity of formulation make the approach outlined in this note well suited to the automatic computation of humidity differences.

TABLE 1. Comparison between the slope of the saturation vapor pressure curve derived from the Goff-Gratch and Tetens formulas. Values for $t' \leq 0.0^\circ\text{C}$ are with respect to an ice surface.

Wet-bulb temperature t' ($^\circ\text{C}$)	Goff-Gratch $\frac{\Delta \epsilon}{\Delta t'} \text{ [mb } (^\circ\text{C})^{-1}]$	Tetens $\frac{dE}{dt'} \text{ [mb } (^\circ\text{C})^{-1}]$	Tetens error (%)
-20.0	0.09902	0.09905	-0.03
-10.0	0.23058	0.23083	-0.11
0.0	0.50292	0.50322	-0.06
10.0	0.82226	0.82279	-0.07
20.0	1.44771	1.44730	0.03
30.0	2.43543	2.43340	0.08
40.0	3.93310	3.93026	0.07
50.0	6.12284	6.12414	-0.02

TABLE 2. Errors involved in approximating $0.622 p / (p - 0.378e)^2$ by $0.63/p$.

Vapor pressure e (mb)	Error at $p = 900$ mb (%)	Error at $p = 1000$ mb (%)
5	-0.87	-0.91
10	-0.45	-0.53
15	-0.01	-0.14
20	0.40	0.24
25	0.82	0.62
30	1.24	0.99
35	1.67	1.36

List of Symbols

- A psychrometric constant equal to 0.000660 when pressures are in consistent units and temperatures in $^{\circ}\text{C}$
- E pressure in mb of water vapor in a saturated atmosphere at temperature t' as rendered by an approximation formula due to Tetens
- e pressure in mb of water vapor in the atmosphere
- e_s pressure in mb of water vapor in a saturated atmosphere at temperature t'
- p atmospheric pressure
- q specific humidity
- t dry-bulb temperature in $^{\circ}\text{C}$
- t' wet-bulb temperature in $^{\circ}\text{C}$

- $\langle t' \rangle$ average wet-bulb temperature over a given wet-bulb temperature difference
- z distance within the temperature field

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