The Size Distribution of Cumulus Clouds in Representative Florida Populations

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ABSTRACT

The cumulus cloud populations of the Florida peninsula were photographed periodically and comprehensively on 19 days in August and September 1957, using multiple, high-flying aircraft. Particular sets of these photographs, called population samples, were selected which showed representative examples of the cumulus populations of customary, widespread occurrence of the various days and hours. These were analyzed to determine the size distribution characteristics of the cumuli and the time trends.

The analyses revealed 1) that the number density of the cumuli decreased nearly exponentially with increasing cloud-size, 2) that a definite maximum-size-class of the cumuli existed in the populations, which increased in size diurnally in a regular fashion, 3) that there was considerable variance, both within single populations and with the time of day, in the relationships of the vertical and horizontal dimensions of the clouds, 4) that particular modal classes of the cumuli, of intermediate size, existed in the populations which were the primary contributors to the population sky cover and cloudy volume, and 5) that group structures occurred in the populations which began forming noticeably about mid-morning and which were an important developmental feature of the cumulus convection.

The equations of a cumulus population model were developed on the results of the Florida sampling. These equations were employed to facilitate comparisons of the Florida data with other population data previously reported by Blackmer and Serebreny. They were also applied in other ways, as are pointed out.

1. Introduction

Knowledge of the size distributions of cumuli, and of the changing characteristics of these distributions with the time of day, can contribute much to a better understanding of the fundamental growth and decay processes that are important in cumulus convection, and that lead to the production of rainshowers. The clouds of the populations, except for the very largest ones which are modified substantially by latent heat influences, are the "visual tracers" of the regions of updraft motion within the general convective layer. Studies of the size distributions of cumuli are therefore revealing of the large eddy characteristics of this layer and are indicative of the possible transport mechanisms whereby sensible heat and water vapor are carried upward, from the top of the turbulent boundary layer near the ground surface, to the levels of the cumulus bases.

Knowledge of the size distribution properties of cumuli is also important in various operational problems, such as that of attempting to determine and predict the interference effects of clouds on guidance and detection systems for missiles and satellites, or in multifarious other meteorological and astronomical problems, including those involved in radio-radar propagation, or those concerned with the blocking-attenuating effects of clouds along particular lines of sight. Such information is additionally essential in problems

of the type considered recently by Lund (1965), where he attempted to explain the variability of sunshine records at particular stations in terms of the associated variability of the cloud cover.

Size distribution information for cumulus populations has been reported previously by Blackmer and Serebreny (1962) and Bradbury and Fujita (1964). Blackmer and Serebreny presented general information for two strips of clouds which lay on either side of aircraft flight paths that extended from Florida to California; Bradbury and Fujita presented data for large cumuli that were located over the San Francisco Mountains near Flagstaff, Ariz. The present study differs from these in that the cumuli considered are primarily small clouds, of the fair weather and cumulus congestus types (where latent heat influences are relatively unimportant). They are clouds that were typical and representative of the ones of common, widespread occurrence over the Florida peninsula which existed over uniform terrain and which developed and dissipated in obvious correlation with the solar cycle. The study is concerned with the determination of the size distribution characteristics of these particular clouds, as well as with the establishment of the temporal trends in the distributions that were associated with the normal development and dissipation of cumulus convection.

2. The 1957 observations

Four RF-84F photo-reconnaissance aircraft were supplied by the Tactical Air Command, USAF, and were flown over the Florida peninsula, during 19 days in August and September 1957, in a manner such as to secure complete photographic coverage of the peninsula cumulus populations at approximate 1–2 hr intervals. Four T-11 (or K-17) type cameras were carried on each aircraft, one pointed to look directly forward of the aircraft, 20° depressed, the others being mounted in the conventional trimetricon arrangement (one pointed downward, two pointed directly right and left of the fuselage axis, 30° depressed). The gross, large-scale features of the Florida cloud populations were recorded by the oblique cameras and the details of the particular overflown populations were recorded by the down-pointing cameras. Fifty-eight total sorties were flown during the 1957 study, about two-thirds of which were accomplished during the morning developmental stages of the convection, the remainder during the afternoon.

The details of the flight and observational programs of this study period, and the resultant findings con-

cerning the general cloud physics and meteorological events at the peninsula scale, have been described by Plank (1965). To summarize these events, the ones pertinent to the present paper, it may be reported that the solar-induced cumulus convection of general, widespread occurrence customarily began about 0830, all times Eastern Standard, and terminated about 1700. The sky cover over the peninsula as a whole became a maximum about 1330 and the general rainshower activity peaked approximately 1½ hr later at 1500. The clouds comprising the general widespread coverage usually consisted of fair weather cumuli, in the morning, and of collections and groups of fair weather cumuli and cumulus congestus, during the midday and afternoon. The clouds sometimes had patternform (the cumuli being organized in rows); more often they did not. Plank (1966) has reported the circumstances of the occurrence of these patterned and non-patterned populations, and their relations to the wind fields.

Various large cloud features were also observed during the study period, which should be mentioned. There were, for example, major bands of cumulus congestus

FIG. 1. Patternform cumuli photographed NNE of Tampa, Fla., on 12 August 1957 at 0944 EST. Area of bounded region is 580 mi².

FIG. 2. Non-patternform cumuli photographed SSE of Tampa, Fla., on 3 September 1957 at 0935 EST.
Area of bounded region is 880 mi².

and cumulonimbus that occasionally extended into the peninsula from the oceans. There were certain cloud groups that formed nocturnally whose influence was carried over into the day. There were long bands of clouds that paralleled the coastlines when the peninsula was dominated by sea breezes. There were "rings of clouds" that existed around the larger inland lakes, particularly during the morning and in situations where winds were light. Also, there were numerous thunderstorms that occurred over the different local areas during the midday and afternoon. These large cloud features, however, it should be noted, actually occupied only a relatively small fraction of the total surface area of the peninsula at any given time. The primary peninsula coverage was almost invariably produced by the fair weather and congestus type populations cited earlier, which are the specific subject of the present paper.

3. The sampling area requirements

Certain sampling area requirements had to be met to insure the representativeness of the population samples selected for analysis. (The samples consisted of sets of aerial photographs that showed the cumuli over various specific locales.) These requirements arose because there was a definite upper-limiting, or maximum, size class of the cumuli in the Florida populations, the constituents of which had a finite number density only over surface areas exceeding a particular threshold size.

The existence of an upper-limiting size class of cumuli in the Florida populations, and the nature of the associated sampling area requirements, may be demonstrated by reference to Figs. 1 and 2. The first photograph shows a population of patternform (cloud row) type, wherein the rows are being viewed in a direction normal to their orientation. The second shows a population of non-patternform type. The cumuli within the bounded regions in these photographs were analyzed by dividing

the regions into multiple square areas, 2 mi on a side, and by determining the largest cumulus cloud within each square and measuring its base diameter (equivalent diameter). These diameter values were then averaged, for all squares, such as to establish an "average maximum diameter" that pertained to a sampling area of 4 mi². The procedure was repeated for larger square (or rectangular) areas having 4 times the area of the first, 9 times, 16 times, and so forth, until the region itself became the sampling area.

The dashed curve in Fig. 3 reveals how the average maximum diameter of the clouds of the patternform population increased with the size of the sampling area. The solid curve shows the same for the cumuli of the non-patternform population. In both cases, the maximum increased rapidly with sampling area until the latter was of the order of 100 mi². The increase thereafter, to areas of about 300 mi², was more gradual. For areas exceeding this, the maximum remained rather constant. The figure illustrates that any photographic sample of these populations, which might be selected for purposes of size distribution analysis, for example, should provide analyzable cloud subject coverage of at least 300 mi² of surface area. Otherwise, the very largest cumuli, which are present in the cloud fields in relatively small number concentration, will be inadequately represented.

The sampling area-maximum diameter trends illustrated here were found to be typical of all of the Florida populations. However, since the absolute sizes (and spacings) of the clouds increased diurnally, as we shall demonstrate later, the particular sampling area sizes required with the morning populations were substantially smaller than those required in the afternoon. Areas of several hundred square miles were adequate in the morning, whereas areas ≥ 1000 mi² were needed in the afternoon.

FIG. 3. Variation of average maximum cloud diameter with the size of the sampling area in the patternform population of Fig. 1 (dashed curve) and in the non-patternform population of Fig. 2 (solid curve).

4. The selection of the population samples

Thirty-four population samples were selected originally for investigation but only 19 of these were analyzed in complete detail. The samples were selected for the particular days, situations, times and locales described in the following paragraphs. The sampling area requirements were considered and each sample provided analyzable cloud subject coverage of some 3000–9000 individual cumuli, variable with the population sky cover. The selections were made with the objectives 1) of investigating the size distribution characteristics and time trends in particular populations of patternform type, 2) of investigating, for comparison, the characteristics and trends in other, non-patterned populations, and 3) of establishing, for the Florida populations of usual, prevalent occurrence, the typical characteristics and trends throughout a complete diurnal cycle of the convection.

Four samples were selected, under the first of these objectives, from the photographic data of 10 August 1957, a day when patternform cumuli had existed throughout most of the morning hours over a region near Tampa. These cumuli had been photographed repeatedly by the reconnaissance aircraft at four different times, 0822, 0906, 0945 and 1055. Sets of the aerial photographs (down-pointing and oblique) were chosen which showed the cumuli over a particular inland site located 30 mi ESE of Tampa, which was common to all observational times, which had uniform surface topography, and over which, as revealed by the photographs, the cloud development had proceeded regularly and homogeneously with the solar heating. Portions of these samples (mosaics assembled from the down-pointing photographs) are shown in Fig. 5a, at the left.

For comparison with these samples of the patternform type populations, three other samples were chosen for the day of 16 August 1957, a day when non-patterned clouds were present over this same site (as that of 10 August) whose development throughout the morning hours had also been regular and homogeneous. Sets of photographs were selected for the three aircraft pass times, 0957, 1030 and 1138. Mosaics illustrating portions of these samples are presented in Fig. 6a.

The other 27 samples were selected, not for any single, particular day, but from within the totality of the available photo-reconnaissance data for the entire 19 days (some 150,000 photographs from 58 sorties). Several years of effort were directly or indirectly involved in these selections, which were accomplished as follows: First, the photographs of each sortie were inspected, together with all of the nephanalysis and meteorological analysis information that was available for the sortie times (Plank, 1965). Three sets of photographs were then chosen for each hour of observation on each day. The first set, consisting of three subsets, showed cumulus populations of typical appearance and average sky cover which existed, respectively, over the northern, middle

FIG. 4. Mosaics showing the hour-by-hour appearance of representative Florida cumulus populations of non-patternform type. The mosaics of the middle column show populations of average (typical) sky cover; those of the left-hand column populations of small (less than typical) sky cover; those of the right hand column populations of large (greater than typical) sky cover.

and southern interior regions of the peninsula. The second set, also composed of three sub-sets, showed populations over these same regions that were of typical appearance but which had "smaller than average" sky cover. The third set showed populations that had "larger than average" sky cover. Care was taken in all selections to insure that the local terrain underlying the clouds had uniform surface characteristics, i.e., that it was devoid of lakes, large swamp areas, places of industrial activity, etc.

These photo sets, approximately 450 in all, were grouped by observational hour and sky cover category. Single-set selections were then made from within each group, on a visual appearance basis, as to which best represented the Florida populations in general. Thus, 27 sets, or population samples, were chosen finally, one for each sky cover category and each one-hour period of the day. Mosaics showing portions of the down-pointing portions of these samples are presented in Fig. 4.

Detailed size distribution analyses were performed for only 12 of these 27 samples. The samples of the average sky cover category, of the middle column of Fig. 4, were analyzed, as were, in addition, the 0900–1000, 1200–1300 and 1500–1600 samples of the large sky cover category, illustrated at the right. The rest of the samples could not be handled within the available time.

5. The analyses

Photogrammetric analyses were performed on these 19 identified samples (of 10 and 16 August, plus the typical populations) to establish 1) the base altitudes of the cumuli, 2) their horizontal dimensions at the base level, 3) their vertical thickness-base diameter relationships, 4) their size-class contributions to the total sky cover of the populations, and 5) their size-class contributions to the total cloudy volume. Information about the horizontal dimensions and number densities of the clouds was obtained from analyses performed on the down-pointing aerial photographs; information about the vertical dimensions was secured from analyses of the oblique photographs.

The cloud base altitudes were determined first. This knowledge was essential to provide the photographic scaling information that permitted measurements of the other parameters. The altitudes were determined, for some 20–30 individual cumuli of each sample, by measuring the "shadow displacement distances" of particular, base-level cloud features (seen in the down-pointing photographs) from the shadows of the features as seen on the ground. The altitudes were triangulated from these displacement distances through knowledge of the solar elevation angle at the time and the camera optics. Other independent information about the average base altitudes was secured from measurements of the parallax mismatch that occurred between cloud and

TABLE 1. Diameter classes employed in the sizing and counting of the cumuli.

Diameter range of the cumuli (mi)	Diameter-width of classes employed (mi)	Number of classes in range
0.0–0.1	0.05	2
0.1–0.4	0.1	3
0.4–1.2	0.2	4
1.2–2.8	0.4	4

terrain subjects that were doubly portrayed in successively-taken, overlapping-pairs of the photographs.

The measure of cloud size was specified next. The measure adopted, the "equivalent diameter" (hereafter referred to merely as the diameter), was defined to be the diameter of that circle, or disk, which had an area equivalent to the plan-projected area of the particular cloud of the photograph. This diameter was presumed to be that of the horizontal cross section of the cloud at its base level. Such presumption was based on the observation that most Florida cumuli, of the smaller sizes that were not subject to important wind shearing effects, generally tapered upward from base to top, so that the horizontal section most likely to be projected during down-pointing photography was the base section.

It was also necessary, prior to any actual work on the samples, to specify the diameter classes that would be employed in the number counting. The objective was to establish classes that were of sufficient width, diameterwise, so that the clouds of the different sizes could be discriminated visually from the photographs by the analyst. The objective, at the same time, was to avoid classes of excessive width, such as would seriously degrade the size-number resolution. A problem also existed in that the total diameter range of the cumuli of the populations was quite large, and that it varied diurnally by a factor of three or so. Hence, although it was desirable to employ classes of relatively narrow width to resolve the distribution details of the smallest cumuli, these same narrow classes could not be employed over the total diameter range of the clouds, without requiring excessive numbers of classes and prohibitive analysis effort. The classes adopted, a compromise of these divergent requirements, are indicated in Table 1. Four diameter ranges of the cumuli were specified and the widths of the counting classes employed in each were increased in geometric progression from the first range to the last.

With the establishment of these classes, the cumuli shown in the down-pointing photographs were sized and counted by a technique analogous to that conventionally employed to size and count raindrop signatures on dyed filter paper. Contact size (9"×9") prints of the original aircraft negatives were employed which had been carefully printed to optimize their contrast resolution for cloud subjects. These were analyzed over a light table, which enhanced the visual discernibility of

FIG. 5a. Number density, sky cover and volume histograms, and corresponding equation plots, for the population samples of 10 August 1957.

the cloud details of smallest albedo. The size classification was accomplished using a transparent overlay (template) on which concentric circles had been inscribed demarking the diameter boundaries of the classes. This overlay was applied successively to each cloud of the photographs and the equivalent diameters were established, on an area comparison basis, by determining which pairs of the concentric circles had areas bounding those of the clouds.

This size classification technique worked satisfactorily for the small and medium size cumuli of the populations, of diameter $\lesssim 1$ mi. These appeared as discrete, distinctly-separated entities in the photographs and their shapes, in plan projection, were approximately circular. The technique did not work particularly well, however, for various of the larger, irregularly-shaped clouds. The size class of these clouds couldn't always be determined by simple comparative reference to the concentric circles and their areas sometimes had to be measured by other means.

The very smallest cumuli of the population samples were identified and counted to the extreme limits of the photographic resolution. This, for cloud subjects, was approximately 50 ft. Many clouds of this limiting size were actually found to exist in the populations of the early morning hours.¹ With the populations of later occurrence, however, the minimum was somewhat larger, of the order of 100 ft at midday and about 100–200 ft in the afternoon. There were large uncertainties in the number counts of these smallest cumuli which depended on the diligence of the analyst in searching out the clouds and on his subjective judgment as to what constituted a cloud (as opposed to its being a part of some larger cloud). The uncertainties also depended on the exposure-development settings employed in the printing of the photographs. An estimated 10–30%

¹ Some of these smallest cumuli were probably the just formed nuclei of clouds that subsequently developed into larger size. Mostly, however, they were cumuli that occurred around the immediate peripheral vicinity of other pre-existent clouds.

error in number count might well have resulted from this cause alone.

Particular analysis problems arose with the populations of the midday and afternoon. Numerous group structures were present in these populations which had to be treated in a special way. There were two types of structures. First, there were "open spaced groups," in which the constituent clouds were clustered together with a spacing between that was appreciably smaller than that of the other surrounding clouds. These groups were not especially difficult to handle in the analyses, since the component clouds were distinctly separated from one another and could be sized and counted as individual entities. The special problems arose with the group structures of the second type, with the "base connected groups," wherein the contained cumuli were interjoined and interconnected to one another, usually and primarily at the cloud base level. (Certain such groups can be seen in Fig. 4, in the mosaics of the afternoon hours of the last column.) It was sometimes difficult to identify the true interconnection boundaries that separated the primary constituent clouds of these groups and occasional subjective judgements had to be made. There were relatively few such cases, fortunately, so that the overall sampling results were not materially affected. Moreover, the number of these group structures within the populations in general was usually an order of magnitude smaller than that of the separated, non-grouped cumuli.

FIG. 5b. Thickness-diameter data, and corresponding equation plots, for the population samples of 10 August 1957.

In conjunction with these analyses just described, other analyses were additionally performed on the oblique photographs of the population samples. These were conducted to establish the vertical dimensions of representative numbers of the clouds of different base-diameter size. Photogrammetric measurements were made to determine the thickness-diameter relationships of about 25 individual cumuli of each of the samples, spanning the observed cloud size range and including certain of the component clouds of group structures, when such were present. The techniques of these measurements, involving the use of Canadian grids and other types of perspective grids, have been given by Plank (1965).

6. Results

The results of these analyses are illustrated in Figs. 5-8 and in Table 2. The data of these figures will be described in the present section and we will explain how the second and third sets of histograms shown in in Figs. 5a-8a (subsequently referred to as the "a" figures) were derived from the first. The curves which have been fitted to the histograms will not be discussed at this time. These are considered in Section 8.

The first histograms of the "a" figures reveal how the number density N of the cumuli of the populations² varied with the equivalent diameter D . These number densities have been normalized to pertain to a common surface area of reference of 100 mi² and to a common class width of 0.05 mi, which was the smallest of the class widths actually employed in the analyses. The histogram scales are common throughout all of the figures.

The total numbers N_T of the clouds of all sizes within the populations per 100 mi² of surface area, were also determined. These totals are listed in Table 2.

The diagrams of Figs. 5b-8b (referenced hereafter as the "b" figures) show the vertical dimensions h , of representative individual clouds of the population samples, cross plotted against the equivalent base-diameter dimensions D . Each point represents a set of measurements performed on a single cloud. Collectively, the points of any given diagram illustrate how the h vs D dimensions of the cumuli varied across the diameter range of the population.

The results so far mentioned were the only ones that stemmed directly from the analyses. From these, however, with several assumptions, it was possible to obtain the higher-moment histograms showing the distribution of the sky cover and the cloudy volume.

² Reference to N as being the cloud number density, although convenient, is not strictly correct. There are two reasons. First, the surface area of reference used herein is 100 mi², not 1 mi². Second, the clouds of the populations, except during the early morning and mid-morning hours, were not randomly distributed over the ground surface. Instead, as mentioned, they were frequently clustered together in various types of group structures.

From the definition of equivalent diameter, it follows that the summed, plan-projected area of the clouds within a diameter class, of mid-diameter D_i , having a number density N_i , is

$$A_i = \pi N_i D_i^2 / 4, \quad (1)$$

and that the sky cover (contribution) of the class may be specified as

$$S_i = A_i / A_r = \frac{\pi N_i D_i^2}{4 A_r}, \quad (2)$$

where A_r is the surface area of reference, taken to be 100 mi² in our case.

The second histograms of the "a" figures show the distributions of S_i with D for the different population samples.

The total sky cover of the populations is given by

$$S_T = \sum_{i=1}^{D_m} S_i, \quad (3)$$

where the separate S_i contributions of the various classes are summed from the first class, of near-zero diameter, to the last class, which contains the cumuli of maximum diameter. The particular values of S_T for the different samples are listed in Table 2.

From knowledge of the N vs D variations of the populations (the data of the first histograms), combined with the h vs D data of the "b" figures, it was possible to establish the approximate distribution of cloudy volume. The Florida cumuli, as mentioned, were observed to taper upward from base to top. Moreover, as revealed

FIG. 6a. Same as Fig. 5a except for 16 August 1957.

FIG. 6b. Same as Fig. 5b except for 16 August 1957.

by the "b" figures, there were substantial variations, both within single populations and from population to population, in the h/D ratios of the clouds. To account for these facts, it was assumed that the cumuli had the approximate three-dimensional shape of inverted, truncated, right-paraboloids of revolution, of diameter D across the truncated section of the base, and of height h from truncation plane to apex. The volume of any single such paraboloidal cloud is

$$v = \pi D^2 h / 8; \quad (4)$$

the summed volume within any diameter class is

$$V_i = \pi N_i D_i^2 h_i / 8; \quad (5)$$

and the total volume contained in all classes is

$$V_T = \sum_{i=1}^{i=D_m} V_i. \quad (6)$$

The third histograms of the "a" figures show the distributions of V_i with D for the various samples, the volume totals being given in Table 2. The h_i values used in the computations were established from the "b" figures, by determining the average h values corresponding to the particular D_i values of the classified N data.

7. Discussion

The first histograms reveal that the number density of the cumuli, in virtually all of the samples, decreased more or less exponentially with increasing cloud base diameter. This can be seen by reference to the dashed lines shown superimposed over the histograms, which are the exponential lines of best fit (discussed in Section 8). The decrease of N with D is seen to be most nearly exponential with the population samples of the morning hours of active thermal heating, and less so with the samples of the afternoon (Fig. 7a) or with those of large sky cover (Fig. 8a). The rate of decrease of N with D (the slope of the histograms as plotted) was largest with the population samples of earliest occurrence, of the 0800–0900 period, and smaller later on. It varied diurnally in a regular fashion, both in the pattern-form populations of 10 August and in the nonpatterned

TABLE 2. Observed and analytically determined values of population parameters.

Sample Date	Time (EST)	Cloud base altitude h_b (ft)	Minimum cloud diameter d (ft)	Maximum cloud diameter D_m (mi)	Total number of cumuli N_T (no. per 100 mi ²)	Population sky cover S_T	Total cloud volume V_T (mi ³ per 100 mi ²)	Maximum group diameter G_m (mi)
10 Aug. 1957	0822	650	50	0.33	1480	0.084	0.62	None
	0906	1800	50	0.70	1044	0.208	2.88	1.2
	0945	2200	50	1.30	858	0.280	9.13	3.2
	1055	3000	75	1.70	770	0.215	9.38	3.2
16 Aug. 1957	0957	2550	50	0.85	901	0.096	1.35	1.6
	1030	3000	50	0.70	1530	0.201	2.44	1.6
	1138	3600	75	0.65	1562	0.173	2.53	1.6
Typical sky cover	08–09	2200	50	0.50	1645	0.062	0.58	0.9
	09–10	2300	50	0.70	1665	0.180	2.76	0.9
	10–11	2700	50	1.30	1056	0.262	8.25	1.7
	11–12	3000	75	1.30	1226	0.309	7.94	3.0
	12–13	3650	75	1.60	1138	0.349	12.06	2.6
	13–14	3500	100	2.10	513	0.477	26.76	4.2
	14–15	4100	150	2.40	668	0.309	26.40	5.2
	15–16	4500	200	2.50	609	0.185	10.02	3.1
Large sky cover	16–17	4600	150	1.65	439	0.072	3.28	2.2
	09–10	1800	50	0.90	2002	0.421	9.16	2.4
	12–13	4000	75	2.50	1724	0.642	40.29	4.5
	15–16	4500	100	2.50	1720	0.290	16.21	3.2

FIG. 7a. Same as Fig 5a except for the average sky cover samples illustrated in Fig. 4.

FIG. 7b. Same as Fig. 5b except for the average sky cover samples illustrated in Fig. 4.

populations of the other samples. This was one of the characteristic features of the Florida convection.

In association with these diurnal trends in the size distribution of the clouds, the total diameter range of the populations, given by the D_m values of Table 2, increased from about 0.5 mi, at earliest observation, to about 1.5 mi by noontime, and to about 2.5–3.0 mi by mid-afternoon. The range decreased progressively thereafter, as the populations dissipated during the late afternoon. The base altitudes of the cumuli (the h_b values of Table 2) also increased diurnally, on the average.

The second histograms of the "a" figures, those of the morning hours, show that a maximum sky cover occurred within the populations in association with a particular size class of the cumuli of an intermediate diameter. This diameter (modal diameter) increased with time, approximately in accord with the increases in D_m . It increased from about 0.15 mi, for the 0800–0900 samples, to about 0.3–0.6 mi, for the samples of

the noontime period. The values, in most cases, were about equal to $D_m/3$.

These histograms also reveal that the distribution of sky cover in the morning populations was of essentially uni-modal form whereas the distribution in the afternoon was generally of bi-modal form, being most pronounced in the late afternoon. One mode was associated with cumuli of a relatively small size, of diameter approximately equal to $D_m/6$, the other with clouds of a much larger size, of diameter just slightly smaller than D_m itself. The shift in the form of the distribution across the midday period is explainable in terms of the different structural composition of the morning and afternoon populations. The cumuli of the morning were distributed and spaced more or less uniformly over the ground surface and relatively few group structures were present. The cumuli of the afternoon, in contrast, were frequently clustered together into group structures of the two types mentioned. The largest clouds of the popula-

tions, which commonly were constituents of the groups, tended to occur in preferential association with the smallest clouds, which existed in tremendous number concentration around the immediate peripheral vicinity of the groups. There was a relative deficit of intermediate-size cumuli, relative to the numbers that were present in the morning populations.

The third histograms of the "a" figures show that the cloudy volume of the populations was contained primarily in cumuli of an intermediate size, during the morning and mid-day, and in clouds of the very largest size, during the afternoon. There was a progressive shift of the volume mode toward larger cloud size throughout the morning, from an initial diameter of about $0.4 D_m$, to a noontime diameter of about $0.6 D_m$. Then, shortly past noon, as indicated by the histograms of Figs. 7a and 8a, the modal diameter suddenly jumped upward to D_m itself. This jump was probably associated with the group structure situation discussed above.

The volume distribution within the Florida populations was partially dependent, of course, on the h/D ratios of the cumuli, and on the variability of these ratios with D . The data of the "b" figures, although adequate to demonstrate ratio differences between the populations sampled, are not extensive enough to reveal any distinctive diurnal trends. Hence, to obtain information about such possible trends, 81 additional population samples (partial samples) were selected from the photo-reconnaissance data, following the selection

criteria outlined previously. An average h/D ratio was determined for each sample, which pertained to just one particular size class of the cumuli, containing clouds of diameter approximately equal to $D_m/2$. These averages, plus those for the original 19 samples, are shown plotted vs the time of day in Fig. 9. The values scattered widely, it is seen, reflecting important daily differences. A general diurnal trend is apparent, though, wherein the clouds prior to 0900 were "somewhat flattened," with ratios averaging about 0.9, those of the 0900–1200 period were "relatively tall," with ratios ranging from 1.0 to 2.0, and those of the afternoon, after 1430, were once again "comparatively flat." Few photo-reconnaissance sorties were flown between 1200 and 1430, which accounts for the lack of data during this period. The flattening of the clouds in the middle and late afternoon corresponded to the vespertalis stage of the convection, as conventionally recognized.

The physical reasons for the types of ratio variability illustrated here and in Figs. 5b–8b were undoubtedly 1) variations of atmospheric stability, of the lower atmosphere into which the clouds were building, 2) variations in the upward flux of sensible heat, which varied diurnally with the solar radiation, and 3) variations in the latent heat amounts released within the cumuli, which presumably increased with the cloud size.

Although it is beyond the scope of the present paper to get deeply involved in the subject of cloud groups, it may be pointed out that such structures, of the second,

FIG. 8a. Same as Fig. 5a except for the large sky cover samples of the 0900–1000, 1200–1300 and 1500–1600 EST periods illustrated in Fig. 4.

FIG. 8b. Same as Fig. 5b except for the large sky cover samples of the 0900-1000, 1200-1300 and 1500-1600 EST periods illustrated in Fig. 4.

base-connected type, were present in the 19 originally discussed samples which had equivalent diameters ranging upward to the G_m values listed in Table 2. These largest groups were some 2-3 times as large as the largest individual clouds. The groups occurred over a size range which overlapped that of the individuals. This is illustrated in Fig. 10, where size distribution information concerning groups is presented for the population samples of 10 and 16 August 1957. Group structures existed in virtually all of the Florida populations but they were most numerous and sizeable in the populations of the late morning and afternoon, and in those of large sky cover. The number density of the groups in single populations decreased more or less exponentially with increasing group size, as with the individual cumuli. These features can be seen by comparing the histograms of Fig. 10 with those of Figs. 5a and 6a.

FIG. 9. Time variation of the thickness-diameter (h/D) ratios determined for 100 Florida population samples. The ratio values plotted are averages for cumuli of mid-diameter size that existed in the samples. The particular ratio values pertaining to the 19 fully analyzed samples of the text discussion are denoted by the crosses.

The size distribution characteristics of the Florida cumuli and group structures did not appear to be materially influenced by the patternform state of the populations. The diagrams and tabulated results for the cloud-row samples of 10 August reveal no obvious differences that clearly exceed the variance range of the other non-patterned samples.

To summarize the analytical findings of the study regarding the possible mechanics of the Florida convec-

FIG. 10. Histograms illustrating the number density distribution of cumulus group structures in the particular population samples of 10 and 16 August 1957. The total numbers of the groups, of all sizes per 100 mi^2 of surface area, are indicated above the histograms.

tion, it appeared as if the convection operated in a manner such as to continually favor and enhance the development potentialities of the largest cumuli of the populations. Many of these largest clouds, soon after the initial formation of the populations, began to be surrounded by other clouds and to be commingled into group structures. One might presume that the cumulus elements of these groups were better able, together than singly, to withstand the deleterious effects of entrainment erosion and thereby develop more efficiently with the surface heating into structures of progressively larger size. The integrated latent heat releases within the group elements might also have induced local circulations around the groups to make them partially self-sustaining and to further augment their chances for continued accelerated growth.

8. The distribution equations

In various problems involving cumulus clouds, such as cited in the introduction, it would be advantageous to have analytical formulae available to provide approximate, first-order descriptions of the geometric characteristics of the cloud populations. Such equations would provide a standard of reference for future observational studies and would serve an immediate utilitarian purpose in numerous theoretical and applied problems concerning the assessment of the distributed and integrated effects of the clouds, or of cloud associated parameters. Such problems, for example, include those of estimating the albedo of cumulus populations for various sky cover conditions and times of day. Cumulus population models that have been employed in the past are extremely crude, the cumuli generally being assumed to be of cubical shape and of one common size. The size distribution of the clouds within the populations is not taken into account, nor is consideration given to the actual cloud shapes or to the fact of the variable thickness-diameter ratios.

As emphasized before, the numbers of the clouds in single Florida populations were found to decrease in a nearly exponential manner with increasing cloud base diameter D , approximately as specified by the distribution function

$$n = Ke^{-\alpha D}, \quad d < D < D_m, \quad (7)$$

where n is the number density of the clouds per unit surface area, K the "effective number density" of the clouds in the first infinitesimal interval beginning at zero size, and α the coefficient of the exponential number density decrease. The equation is descriptive only between $D=d$, where d is the minimum cloud size (observed to be of the order of 50–200 ft), and $D=D_m$, where D_m is the maximum observed size, ranging daily and diurnally from about 0.5 to 3.0 mi. Since no clouds smaller than d exist in the populations, the constant K cannot be interpreted physically.

The total number density of the cumuli, of the clouds of all diameter sizes in the populations, is given by

$$n_t = \int_d^{D_m} n dD = \frac{K}{\alpha} (e^{-\alpha d} - e^{-\alpha D_m}). \quad (8)$$

The distribution function of the sky cover is readily obtained from (7). In accord with our definition of equivalent diameter, the base area of any single cloud is $\pi D^2/4$ and it follows immediately from (7) that the sky cover within the populations is distributed as

$$s = \frac{\pi D^2}{4} n = \frac{\pi K D^2}{4} e^{-\alpha D}, \quad d < D < D_m, \quad (9)$$

with the total sky cover being given by

$$S_T = \int_d^{D_m} s dD = \frac{\pi K \chi}{2\alpha^3}, \quad (10)$$

where

$$\chi = -e^{-\alpha D_m} \left[\frac{(\alpha D_m)^2}{2} + \alpha D_m + 1 \right] + e^{-\alpha d} \left[\frac{(\alpha d)^2}{2} + \alpha d + 1 \right]. \quad (11)$$

The αd terms of this last equation may be ignored, since it can be assumed without material effect that $d=0$, rather than $d=50$ –200 ft, as is actually the case. The equation then simplifies to

$$\chi \cong 1 - e^{-\alpha D_m} \left[\frac{(\alpha D_m)^2}{2} + \alpha D_m + 1 \right]. \quad (12)$$

This parameter χ , in physical terms, is the ratio of 1) the partial area under the s curve from $D=0$ to $D=D_m$, the portion containing clouds, to 2) the total area under the s curve from $D=0$ to $D=\infty$.

The distribution function of the cloud volume was derived using the assumption of Eq. (4) that the cumuli had the three-dimensional shape of inverted, truncated, right paraboloids of revolution. Combining (4) with (7), the distribution function can be written as

$$v = \frac{\pi D^2 h}{8} n = \frac{\pi K D^2 h}{8} e^{-\alpha D}, \quad d < D < D_m. \quad (13)$$

It was further assumed that the h/D ratios of the cumuli, and the fact of the variation of these ratios with the diameter size of the clouds, could be described approximately as

$$\frac{h}{D} = v \left(\frac{D}{D_m} \right)^\beta. \quad (14)$$

The descriptivity of this equation is illustrated in Figs. 5b-8b, where curves, corresponding to particular solutions, are shown fitted to the thickness-diameter data of the Florida samples. The values of ν and β for the different populations are listed in Table 3. The overall fit of the curves to the data is seen to be quite satisfactory and it is apparent that the equation is equally capable of describing populations containing entirely "tall clouds" ($h/D > 1.0$), or containing entirely "short-flat clouds" ($h/D < 1.0$). Additionally, dependent on the sign of β , it is descriptive of populations wherein the h/D ratios increase with D , or vice versa.

The average values of ν and β for the 19 samples, as indicated in Table 3 at the bottom, were close to 1.0 and 0.0, respectively. Hence, the cumuli of these samples were on the average about as tall vertically as they were wide horizontally across their bases.

With (14) incorporated in (13), the volume distribution function for cumulus populations of differing ν and β becomes

$$v = \left(\frac{\pi K \nu}{8 D_m^\beta} \right) D^{3+\beta} e^{-\alpha D}, \quad d < D < D_m, \quad (15)$$

and the volume total, of the clouds of all sizes,

$$v_t \cong \int_0^{D_m} v dD = \left(\frac{\pi K \nu}{8 D_m^\beta} \right) \int_0^{D_m} D^{3+\beta} e^{-\alpha D} dD, \quad (16)$$

where, because the difference is negligible, the lower limit of integration is taken to be $D=0$, rather than $D=d$.

On performance of the integration, for β assumed to be variable just between -1 and $+1$, there results³

$$v_t \cong \left(\frac{\pi K \nu}{8 D_m^\beta \alpha^{4+\beta}} \right) \{ \rho \chi - e^{-\alpha D_m} [\sigma (\alpha D_m)^3 + \omega (\alpha D_m)^4] \}, \quad (17)$$

where χ is as defined in (12), and where

$$\rho = \Gamma(4+\beta), \quad (18)$$

$$\sigma = (1+\beta)^2, \quad (19)$$

and

$$\omega = 0, \quad \text{for } \beta \leq 0 \\ \omega = \beta(1+\beta)^4/16, \quad \text{for } \beta > 0 \quad (20)$$

The expression $\Gamma(4+\beta)$ in (18) is the Gamma function of $4+\beta$.

To convert the distribution functions (7), (9) and (15), wherein the implied diameter bandwidths are infinitesimals, into distribution equations having a finite bandwidth ϵ , centered about D , we may integrate the functions across such ϵ interval to obtain, first, from (7),

³ This integration result is exact for the particular β values -1 , 0 , $+1$ and is approximate for intermediate β 's, holding to within 1% of the true integral values as determined by numerical integration using Simpson's method (over the Florida observed ranges of α and D_m).

TABLE 3. Empirically established values of ν and β .

Sample	Time (EST)	ν	β
10 Aug. 1957	0822	1.04	0.00
	0906	0.57	-0.44
	0945	1.08	-0.23
	1055	1.02	-0.35
16 Aug. 1957	0957	0.95	0.00
	1030	0.53	-0.49
	1138	1.21	0.56
Typical sky cover	08-09	0.89	0.00
	09-10	0.97	0.00
	10-11	0.93	-0.10
	11-12	0.80	0.00
	12-13	0.85	-0.13
	13-14	0.93	-0.10
	14-15	1.20	0.00
	15-16	1.20	0.40
	16-17	1.30	0.30
Large sky cover	09-10	0.98	0.00
	12-13	0.93	0.30
	15-16	0.76	-0.30
Averages		0.955	-0.031

the *number distribution equation*

$$n_f = K \int_{D-\epsilon/2}^{D+\epsilon/2} e^{-\alpha D} dD = K \delta_n e^{-\alpha D}, \quad d < D < D_m, \quad (21)$$

where

$$\delta_n = \frac{2}{\alpha} \sinh \frac{\alpha \epsilon}{2}; \quad (22)$$

second, from (9), the *sky cover distribution equation*

$$S = \frac{\pi K}{4} \int_{D-\epsilon/2}^{D+\epsilon/2} D^2 e^{-\alpha D} dD = \left(\frac{\pi K \delta_s}{4} \right) D^2 e^{-\alpha D}, \quad d < D < D_m, \quad (23)$$

where

$$\delta_s = \frac{2}{\alpha^3 D^2} \left\{ \left[(\alpha D + 1)^2 + \frac{\alpha^2 \epsilon^2}{4} + 1 \right] \sinh \frac{\alpha \epsilon}{2} - \alpha \epsilon (\alpha D + 1) \cosh \frac{\alpha \epsilon}{2} \right\}; \quad (24)$$

and, third, from (15), the *volume distribution equation*

$$v_f = \frac{\pi K \nu}{8 D_m^\beta} \int_{D-\epsilon/2}^{D+\epsilon/2} D^{3+\beta} e^{-\alpha D} dD, \quad (25)$$

which, integrated over the particular β range from -1 to $+1$, becomes

$$v_f = \left(\frac{\pi K \delta_\nu \nu}{8 D_m^\beta} \right) D^{3+\beta} e^{-\alpha D}, \quad d < D < D_m, \quad (26)$$

FIG. 11. Variation of the ratios δ_n/ϵ , δ_s/ϵ and δ_v/ϵ with α for three particular values of ϵ .

where

$$\delta_v = \frac{1}{\alpha(\alpha D)^{3+\beta}} \left(\sinh \frac{\alpha \epsilon}{2} \left\{ \rho \left[(\alpha D + 1)^2 + \left(\frac{\alpha \epsilon}{2} \right)^2 + 1 \right] \right. \right. \right. \\ \left. \left. + 2\sigma \left[(\alpha D)^3 + \frac{3}{4} \alpha^3 \epsilon^2 D \right] + 2\omega \left[(\alpha D)^4 + \frac{3}{2} \alpha^4 \epsilon^2 D^2 + \left(\frac{\alpha \epsilon}{2} \right)^4 \right] \right\} \right. \\ \left. - \alpha \epsilon \cosh \frac{\alpha \epsilon}{2} \left\{ \rho (\alpha D + 1) + \sigma \left[3(\alpha D)^2 + \left(\frac{\alpha \epsilon}{2} \right)^2 \right] \right. \right. \right. \\ \left. \left. + \omega \left[4(\alpha D)^3 + \alpha^3 \epsilon^2 D \right] \right\} \right). \quad (27)$$

The parameters δ_n , δ_s and δ_v may be regarded either as "parameters of convenience," which permit the basic distribution equations to remain relatively uncluttered, or as "effective bandwidths," which correspond approximately to the actual bandwidths of the integration. For example, the first diagram of Fig. 11 shows the ratio δ_n/ϵ , as determined from Eq. (22), vs α for three particular values of ϵ . The second and third diagrams show similar plots of the ratios δ_s/ϵ and δ_v/ϵ , which were computed from (24) and (27) for the observed limiting condition of $\alpha D \leq \alpha D_m \leq 10$ (see Table 4) and for $\nu = \bar{\nu} = 1.0$ and $\beta = \bar{\beta} = 0.0$ (corresponding to the average, empirically-established values of Table 3). It is seen that for relatively small values of ϵ , of the order of 0.1 mi or smaller, the ratios do not depart from unity by more than about 20% across the α range 2–23 (the observed range as shown in Table 4). This indicates that in the practical application of the distribution equations

TABLE 4. Equation values of population parameters.

Sample	Time (EST)	α (mi ⁻¹)	αD_m	χ	N_T (no. per 100 mi ²)	V_T (mi ³ per 100 mi ²)	D' (mi)	D_v' (mi)
10 Aug. 1957	0822	22.2	7.33	0.977	2185	0.57	0.090	0.135
	0906	9.15	6.40	0.954	1065	2.50	0.219	0.280
	0945	5.40	7.02	0.971	509	9.46	0.371	0.513
	1055	4.80	8.16	0.988	298	9.20	0.417	0.552
16 Aug. 1957	0957	12.3	10.4	0.998	824	1.12	0.163	0.244
	1030	8.75	6.13	0.943	954	2.37	0.228	0.287
	1138	9.08	5.90	0.933	853	1.85	0.220	0.392
Typical sky cover	08–09	14.6	7.30	0.976	751	0.54	0.137	0.206
	09–10	7.10	4.97	0.874	617	3.09	0.282	0.422
	10–11	3.80	4.94	0.870	265	8.43	0.526	0.764
	11–12	4.35	5.66	0.921	379	7.52	0.460	0.690
	12–13	3.30	5.28	0.897	256	12.9	0.606	0.870
	13–14	2.25	4.72	0.848	172	25.5	0.888	1.30
	14–15	2.10	5.04	0.879	92	21.0	0.951	1.43
	15–16	2.39	5.97	0.937	66	9.84	0.837	1.42
	16–17	4.00	6.60	0.960	68	2.89	0.500	0.824
Large sky cover	09–10	6.20	5.58	0.916	1058	8.80	0.323	0.484
	12–13	2.27	5.68	0.922	220	29.1	0.882	1.45
	15–16	2.39	5.97	0.937	107	15.3	0.837	1.13

the simplifying assumption

$$\delta_n \cong \delta_s \cong \delta_v \cong \epsilon \quad (28)$$

may be employed, providing that reasonable equation accuracy is acceptable and that the bandwidths chosen are not excessively large. For example, if ϵ , in practice, is chosen to be something smaller than $D_m/5$, the approximation (28) will always hold to within 15% accuracy, except in the single instance noted below.⁴

It is also desirable, for purposes of practical application, to express the constant K of the distribution and totals equations in terms of population parameters that are conventionally observed, or are observable, albeit with difficulty. These parameters are 1) the total sky cover S_T , 2) the coefficient of the exponential number density variation α , and 3) the maximum cloud diameter D_m . We obtain this desired expression for K by writing (10) as

$$K = \frac{2\alpha^3 S_T}{\pi\chi}. \quad (29)$$

As demonstrated in Section 3, a representative cumulus population occupies a surface area of several hundred square miles, an area that is two orders of magnitude larger than any of the conventional measures of area, such as the square mile or the square kilometer. Moreover, individual clouds of the populations frequently have base areas considerable in excess of either of these unit measures. Hence, to avoid the inconvenience of having to deal with cloud numbers in units such as 10^{-2} clouds mi^{-2} (if we work strictly with number density), it is preferable, in the author's opinion, to specify and work with a larger surface area of reference, such as 100 mi^2 , or 100 km^2 . We are then able to state, in better conformance with the facts of the actual composition of the populations, that two clouds were physically present over 100 mi^2 of surface area (or that 10, or 100, or some integral number of clouds were present).

If A_r is this reference surface area larger than unity, then we may specify that

$$N = n_f A_r, \quad (30)$$

$$N_T = n_t A_r, \quad (31)$$

$$V = v_f A_r, \quad (32)$$

$$V_T = v_t A_r, \quad (33)$$

where the quantities on the left are the number, total number, volume and total volume of the clouds over this reference area. Such specifications are not required

⁴ The ratios δ_s/ϵ and δ_v/ϵ can depart considerably from unity, particularly for small α values, in the first bandwidth interval of the integration, which extends from $D=0$ to $D=\epsilon$. The ratio values in this first interval increase without limit as $\epsilon \rightarrow 0$. This causes no serious difficulties in the distribution equations, however, when (28) is used in simplification, because the sky cover and cloud volume amounts contained in this first interval are negligibly small compared to the population totals of these parameters.

in the case of the sky cover or total sky cover, these being area ratio quantities whose values are independent of A_r .

When (29) is substituted into (8), (17), (21), (23) and (26), to replace the K 's in these equations, and when the assumption of (28) and the new definitions of (30)–(33) are also employed, the three distribution equations may be written in their final practical form as

$$N = \left(\frac{2A_r \epsilon \alpha^3 S_T}{\pi\chi} \right) e^{-\alpha D}, \quad d < D < D_m, \quad (34)$$

$$S = \left(\frac{\epsilon \alpha^3 S_T}{2\chi} \right) D^2 e^{-\alpha D}, \quad d < D < D_m, \quad (35)$$

$$V = \left(\frac{A_r \epsilon \nu \alpha^3 S_T}{4\chi D_m^\beta} \right) D^{3+\beta} e^{-\alpha D}, \quad d < D < D_m, \quad (36)$$

and the total number and total volume equations as

$$N_T = \left(\frac{2A_r \alpha^3 S_T}{\pi\chi} \right) [e^{-\alpha d} - e^{-\alpha D_m}], \quad (37)$$

$$V_T = \left(\frac{A_r S_T}{4\chi D_m^\beta \alpha^{1+\beta}} \right) \times \{ \rho\chi - e^{-\alpha D_m} [\sigma(\alpha D_m)^3 + \omega(\alpha D_m)^4] \}. \quad (38)$$

There are two other equations which also are useful to the description of the cumulus populations. These are the equations specifying the diameters of the sky cover and volume modes. From (9), on differentiation with respect to D , and setting $ds/dD=0$, it is readily shown that a maximum of sky cover occurs at the particular modal diameter

$$D' = 2/\alpha, \quad (39)$$

and, similarly from (15), that a maximum of cloudy volume occurs at

$$D_v' = \frac{3+\beta}{\alpha}. \quad (40)$$

Eqs. (34)–(40), together with (12), describe an empirical cumulus population model of exponential type. The equations may be evaluated for any assumed or measured values of S_T , α , D_m , ν and β , whereupon they may then be employed to provide first-order estimates of the number, sky cover and volume distributions of the clouds, also of the number and volume totals.

The equations of this model were applied to the 19 Florida population samples discussed earlier. They were evaluated using the observed values of d , D_m and S_T , of Table 2, and the empirically deduced values of ν and β , of Table 3. The values of α were established from the normalized number density data by the least-squares determination (nonlinear) of the exponential curves of

best fit. The data of the first two size classes of the cumuli (clouds < 0.1 mi diameter) were deliberately excluded from these determinations, since it was recognized that the sampled numbers of these clouds listed by the analyst were highly uncertain. The α values determined by this least-squares method are listed in Table 4. Also listed are the values of αD_m and χ , which entered the computations, and those of N_T , V_T , D' and D_v' , as established from (37)–(40).

The distribution equations (34), (35) and (36) were evaluated for each of the samples. Plots of the equations are shown in Figs. 5a–8a, superimposed over the histograms. The bandwidth ϵ was taken to be 0.05 mi; the reference area A_r was assumed to be 100 mi², values which corresponded to the normalized values of the histogram plots.

With reference to the first three of these figures, and to Tables 2 and 4, it is seen that the distribution equations were reasonably descriptive of the Florida samples of the morning hours. The general characteristics of the histograms of sky cover and cloudy volume, their modal peaks and distribution trends, were replicated quite satisfactorily by the equations, and the volume totals, with one exception, corresponded to within 20%. The number density totals evidenced considerable departure, however. The N_T values computed from the equation were much smaller than those of the direct sampling. These differences stemmed directly from the fact that the number distribution equation failed to account adequately for the observed numbers of the clouds of the very smallest size, which were clouds that were deliberately ignored in the method of determining α .

Although it was recognized in the beginning that the equations of the exponential model would fail to describe the Florida populations of the afternoon, also those of large total sky cover, it was nevertheless of interest to assess the quantitative amounts of the departures. Figs. 7a and 8a show that the number distribution equation underestimated the observed numbers of the clouds of the smallest and largest size, and overestimated the numbers of intermediate size. The consequence of the underestimate of the smallest clouds is strikingly apparent in the comparative N_T values of Tables 2 and 4. The equation values are smaller than the observed by as much as a factor of 10. The consequence in the case of the largest clouds is most obvious in the comparative volume distribution trends of the equation and histograms, the cloud volume being the parameter most sensitive to number density errors at large D . The distribution of cloud volume prescribed by the equation is seen to differ radically from the observed, the equation failing completely to describe the volume mode at $D = D_m$. Somewhat surprisingly, however, perhaps by chance, the volume totals for the afternoon populations, computed from (38), are only some 5–30% smaller than those determined analytically.

TABLE 5. Observed and equation values of population parameters for Blackmer-Serebreny typical samples.

	Population type		
	Fair weather cumuli C _{L1}	Cumulus-cumulus congestus C _{L2}	Cumulus-cumulonimbus C _{L3}
No. of samples analyzed	4	4	3
Sites of samples	Mississippi, Texas, New Mexico and California	Florida, Mississippi and Texas	Florida, Alabama, New Mexico and Arizona
Range of occurrence times of cumuli (local times)	1210–1510	1220–1530	1140–1530
Observed parameters			
D_m (mi)	2.2	4.1	7.1
N_T (no. per 100 mi ²)	140	74	56
Equation parameters			
S_T (assumed)	0.3	0.3	0.3
α (mi ⁻¹)	3.73	1.88	1.10
αD_m	8.20	7.70	7.80
χ	0.984	0.983	0.984
D' (mi)	0.54	1.06	1.82
N_T (no. per 100 mi ²)	186	57	21

9. Comparisons with the typical data of Blackmer and Serebreny

Blackmer and Serebreny (1962) have presented tabulated size-distribution data for typical cumulus populations of the C_{L1}, C_{L2} and C_{L3} types of international definition. It is of interest to compare their results with those found herein, utilizing the distribution equations to facilitate the discussion. Blackmer and Serebreny obtained their data from measurements performed on 11 population samples of cumulus clouds that were photographed during two U-2 photo-reconnaissance flights that extended across the southern United States from Florida to California. One flight was made on 28 July 1960, between the local times 1030 EST (leaving Florida) and 1330 PST (arriving California). The other was made on 5 August 1960, in the same direction, between the times 1335 EST and 1540 PST. The approximate times and locations of the samples are indicated in Table 5.

The Blackmer-Serebreny histograms of cloud number density and sky cover for the three population types are shown in Fig. 12. These have been converted into diameter and number density units compatible with the ones of the present study. The population sky cover, a parameter that they left variable for specification, was assumed to be 0.3. Their histograms herein have been normalized to a common class width of 0.2 mi, which was the approximate width of their narrowest counting class employed. Several assumptions were made in the conversion of their data but none is regarded as serious enough to alter the originally reported results in any material way. It should be noted that their C_{L3} population contained clouds that were much larger than any considered in the Florida study.

The maximum diameters and observed total numbers of the clouds of the Blackmer-Serebreny populations are given in Table 5. Also given are the least-squares

FIG. 12. Number density and sky cover histograms, and corresponding equation plots, for the typical population samples of Blackmer and Serebreny (1962). It should be noted that the N and S values plotted here are four times larger than those of the Florida histograms, because of the different normalized class widths employed.

values of α , and the values of αD_m , χ , D' and N_T , which were determined for the samples using the methods and equations previously discussed. The values of N_T were computed assuming that the minimum diameter of the cumuli sampled by Blackmer and Serebreny was 0.1 mi, which their report would suggest. The distribution equations (34) and (35), for N and S , were evaluated and plotted for each of their populations. The plots are shown in Fig. 12.

The variation of cloud number density with cloud size in the Blackmer-Serebreny populations was similar to that of the Florida populations. The histograms evidenced the same general exponential decreases of N with D , also the same trend of the least-squares α values in going from populations containing relatively small cumuli to those containing larger cumuli. The D_m values of their C_{L1} and C_{L2} populations were much larger, though, for comparable α and S_T , than the ones for the Florida samples. Their values correspond more nearly to the G_m (maximum group size) values determined herein. They did not discuss groups specifically, nor did

they tell how the problems posed by these groups were handled in the analyses.

The maximum sky cover in the Blackmer-Serebreny C_{L1} population is seen to occur in their first counting class, which extends to 0.7 mi diameter at its upper end. The equation plot for this population would suggest that the actual maximum within the class might have occurred at a cloud diameter of approximately 0.5 mi. A maximum must have occurred somewhere within their first class, at some diameter exceeding zero. This is prerequisite in view of the near exponential decrease of N within the population.

The sky cover maximum in their C_{L2} population occurred in the second counting class, which had a mid-diameter of approximately 0.85 mi. The predicted equation maximum for the population occurred at 1.06 mi (ref. Table 5), which is in reasonable conformance with the observations considering the rather large width of their counting classes.

It is interesting to note that the number density trend in the C_{L3} population of Blackmer and Serebreny de-

parted from the exponential in the same manner as did the Florida afternoon populations containing the largest clouds. The same tendency is apparent in the histograms, in which there is a relative deficiency of cumuli of an intermediate size.

All in all, considering the possible differences of definition and the different analytical methods employed, it would seem reasonable to conclude that the Blackmer-Serebreny study findings complement those herein. This is suggestive that the size distribution characteristics of cumuli over uniform terrain are relatively similar irrespective of the specific geographical site of the observations. Other photo-reconnaissance data that the author has examined, showing cumuli over Massachusetts, Ohio, New Mexico, Arizona and Brazil, would also appear to support this suggestion.

10. Applications and predictions of the population model

The equations of the exponential population model developed herein have been applied to several cloud physics problems that are worthy of brief mention.

The equations have been utilized by Kauth and Penquite (1967) to estimate the probabilities of obtaining clear lines of sight through cumulus populations. They assumed, however, to simplify the mathematics, that the cumuli were of ellipsoidal shape, rather than paraboloidal.

In another line-of-sight problem, involving the ability of maintaining visual contact between two or more aircraft flying at cumulus levels, it is important to know how the "slice sky cover" of a cloud population (defined for a horizontal plane that slices through the cumuli at an altitude z above the average cloud base level) is variable with z from this base level to the altitude of the highest cloud tops. The equations of the exponential model, when applied to this problem, and neglecting the details of the derivation, predict that the slice sky cover S_z will decrease with z in the manner

$$S_z = S_T [1 - f_1(z) - f_2(z)], \quad (41)$$

where

$$f_1(z) = \frac{1}{\chi} \left[1 - e^{-\mu \left(\frac{\mu^2}{2} + \mu + 1 \right)} \right], \quad (42)$$

$$f_2(z) = \left(\frac{\mu^{1+\beta}}{2\chi} \right) \{ e^{-\mu [\lambda \mu^2 + \eta \mu + \tau]} - e^{-\alpha D_m [\lambda (\alpha D_m)^2 + \eta \alpha D_m + \tau]} \}, \quad (43)$$

$$\mu = \alpha \left(\frac{z D_m^\beta}{\nu} \right)^{1/(1+\beta)}, \quad (44)$$

$$\lambda = \begin{cases} \frac{-\beta(1-\beta)}{2}, & \text{for } \beta < 0 \\ 0, & \text{for } \beta \geq 0 \end{cases}, \quad (45)$$

$$\eta = 1 - \beta, \quad (46)$$

$$\tau = \Gamma(2 - \beta), \quad (47)$$

and where the other parameters are as defined previously. This is an extremely useful equation which was utilized not only for the line-of-sight problem mentioned but for other applications as well, one of which is cited below. Although seemingly unwieldy as written, the equation actually simplifies very quickly once the characteristic constants of the populations are designated.

The time rate of release of latent heat within the Florida morning populations was estimated with the aid of Eq. (41). The point values of liquid water content (LWC) within the cumuli were assumed to be everywhere equal to one-third the theoretical moist adiabatic values.⁵

Then, assuming that the slicing plane was of unit thickness Δz , the population total amounts of the cloud LWC were determined for each of the morning hours by vertical summation. Typical values of α , D_m , S_T , ν and β were utilized that were established from the sum and total of all available Florida data (including considerable not discussed herein). The time changes in the hourly total amounts of the LWC were computed and the corresponding amounts of the released latent heat energy were assessed. These latent energy values, when averaged per unit surface area, were found to increase from about 0.4×10^4 ergs $\text{cm}^{-2} \text{sec}^{-1}$, across the 0900-1000 EST interval of the convection, to about 3.0×10^4 , across the 1100-1200 interval. The hourly values of the solar (plus sky) radiation incident at the ground surface were also computed, using Fritz's (1949) curves and correcting for the Florida observed air mass, precipitable water and cloud albedo. These radiant energy values, for the mid-times of the intervals cited previously, were determined to be 6.0×10^5 and 7.5×10^5 ergs $\text{cm}^{-2} \text{sec}^{-1}$, respectively. Thus, the latent energy released aloft was found to be of the order of 1-5% of the incident solar energy, the percentage increasing with time. This implies, since the upward heat flux from the surface during convection is, on an energy comparison basis, about one-third the solar incident radiation [one-half is the maximum; see, e.g., Priestly (1959)] that the latent energy available for secondary cloud development aloft was some 3-15% of the basic driving energy of the convection, associated with the upward heat flux.

11. Concluding remarks

It should be emphasized, in conclusion, that the characteristic population parameters S_T , α , D_m , ν and β , that were identified and used herein, were obtained for population samples that occupied several hundred square

⁵ These LWC values correspond reasonably well to averages of the measurements reported by Durbin (1956), Ackerman (1959), Zaitsev (1950) and Chuvaev and Kryukova (1954).

miles or more of surface area. None of these parameters can be measured or estimated satisfactorily from surface observational data. A ground observer is only able to see some 2–5 mi² of the cloud base plane, even assuming that he looks out to 60° from the zenith (at which point, if not before, he begins looking intolerably at the sides of the cumuli, rather than at their bases). The first of the parameters, the population sky cover, might perhaps be estimated by averaging the separate sky cover reports of many ground observers. But none of the other parameters can be successfully determined from surface data. Photo-reconnaissance or high-resolution satellite data are mandatory.

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