NOTES AND CORRESPONDENCE

An Alternate Method for Eigenvector Computations

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An alternate method for eigenvector computations with possible computational advantages is herein described. For brevity, the notation used is identical to that used in a previous paper (Kutzbach, 1967) in which the alternate method could have been applied. In the cited paper, a diagonal eigenvalue matrix $\mathbf{L}$ and associated eigenvector matrix $\mathbf{E}$ were computed from an M by M symmetric matrix $N^{-1} \mathbf{F}\mathbf{F}'$ where $\mathbf{F}$ represents an M by N data matrix and the prime denotes the matrix transpose. This will be referred to as the direct method. Alternatively, and with a suitable transformation, identical results (in the form of eigenvalues and eigenvectors) could have been obtained starting with the N by N symmetric matrix $N^{-1} \mathbf{F}'\mathbf{F}$. The required transformation is $\mathbf{E} = \mathbf{F}\mathbf{E}_a(\mathbf{L}_a N)^{-1}$ where $\mathbf{L}_a$ and $\mathbf{E}_a$ are, respectively, the diagonal eigenvalue matrix and associated eigenvector matrix obtained from $N^{-1} \mathbf{F}'\mathbf{F}$. The subscript a has been used to identify matrices obtained via the alternate method. Attention is restricted to positive eigenvalues (for which $\mathbf{L}_a = \mathbf{L}$) and their associated eigenvectors.

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If M is appreciably greater than N , it is computationally convenient to obtain $\mathbf{L}_a$ and $\mathbf{E}_a$, and to then use the transformation to solve for $\mathbf{E}$. In this case the alternate method allows savings of computer time as well as permitting the calculation of larger eigenvectors than would likely be possible if the direct method were used, since computing time and computer space requirements increase rapidly with the size of the symmetric matrix which is to be diagonalized.

We have subsequently learned that this result has been described elsewhere but apparently not in the meteorological literature; see Gower (1966) for a brief discussion or Law and Fariss (1968) for a more detailed derivation.

REFERENCES

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Comments on "Spatial Variation of Net Radiation, Albedo and Surface Temperature of Forests"

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Federer (1968) has demonstrated that spatial variation need not be of major concern in sampling net radiation over uniform plant canopies. During his discussion

of radiative exchange, however, he estimated albedo and net infrared radiation through an erroneous interpretation of the linear regression used to relate net