Some Numerical Experiments for Warm Fog Clearing by Seeding with Hygroscopic Nuclei

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ABSTRACT

The effectiveness of warm fog clearing by seeding with hygroscopic nuclei was studied numerically. A parcel of air with a known distribution of fog droplets was assumed to be seeded with hygroscopic nuclei. While new drops were formed upon the seeding nuclei due to condensation, the fog droplets became smaller due to evaporation. The new distribution of drops and the corresponding visibility are given for 60 and 100 sec after seeding. The results indicate that seeding may be effective if the type of fog to be cleared is known as to droplet distribution and the seeding nuclei are chosen accordingly.

1. Introduction

Results of field trials indicate that seeding warm fog with hygroscopic nuclei may be effective in improving visibility. Seeding nuclei introduced into fog draw water from existing fog droplets. Large drops form on the seeding nuclei while fog droplets evaporate. The distribution of a given amount of water on fewer but larger drops improves visibility. If the drops formed on the seeding nuclei grow large enough to fall out, water is obviously removed from the fog, further improving visibility. In falling, the large drops also remove some small fog droplets by the process of coalescence (which is not considered in this paper).

The numerical experiments described in this paper show that the *degree* of visibility improvement with seeding depends on the fog droplet distribution and the appropriate selection of seeding nuclei. The seeding method is not in all cases equally effective. The droplet distribution of the fog to be cleared has to be studied before seeding is performed, and seeding nuclei have to be chosen to insure formation of new drops that will be large enough to fall out and thus also be effective from the point of view of coalescence.

Some recent laboratory and numerical experiments on fog seeding with hygroscopic nuclei are those of Jiusto *et al.* (1968) and Eadie (1969).

2. The mathematical model

a. Assumptions

Let us consider a parcel of air which has a volume of 1 cm^3 and contains a known distribution of fog droplets. We assume that these droplets were formed by water vapor condensation upon salt nuclei (NaCl) and that the amount of salt they include is known. The initial relative humidity of the air parcel is taken to be 100%. The air parcel does not move, and changes in its volume and pressure are neglected.

b. Insertion of seeding nuclei

With the insertion of a few hygroscopic nuclei into the air parcel, some amount of vapor is immediately absorbed by the seeding nuclei and the vapor pressure is reduced. The equilibrium between the vapor pressure over the fog droplets and the ambient vapor pressure is thus disrupted and the fog droplets begin to evaporate, releasing an additional amount of vapor which becomes available for condensation upon the seeding nuclei. Due to the process of evaporation and condensation the temperature of the system also changes.

c. Set of equations

The equation for the heat balance of the system[3] (see Byers, 1965, p. 8) is

$$-L d\rho_v - s\rho_w dT = (\rho_a+\rho_v)\left(c_p dT - R_m T\frac{dp_m}{p_m}\right). \quad (1)$$

By substituting $dp_m=0$, dividing by dt and rearranging, we obtain

$$\frac{dT}{dt} = -\frac{L(d\rho_v/dt)}{(\rho_a+\rho_v)c_p+s\rho_w}. \quad (2)$$

The radii of the fog droplets and of the drops growing upon the seeding nuclei change according to

$$\frac{dr_j}{dt} = (e-e_{sj}^*)\Bigg/\left[r_j\left(\frac{R_v T}{D}+\frac{L^2 e_{sj}^*}{R_v K T^2}\right)\right], \quad (3)$$

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[3] See Appendix for list of symbols.

(see Kornfeld, 1970). The index j with $1\leq j\leq m$ refers to the fog droplets, and the index j with $m<j\leq n$ refers to the drops growing upon the seeding nuclei.

The saturation vapor pressure over a drop is

$$e_s^*=e_s k R_c, \tag{4}$$

where, as shown in Murray (1967),

$$e_s(T)=6107.8\exp\left[\frac{17.2694(T-273.16)}{T-35.86}\right]. \tag{5}$$

The Kelvin corrector is

$$k=\exp\left(\frac{2\sigma}{TR_v r\rho_s}\right), \tag{6}$$

while the Raoult corrector is

$$R_c=\begin{cases}\dfrac{M_n}{M_n+i\alpha M_w}, & r\leq r_0\left(\dfrac{\rho_n}{\alpha}+1\right)^{\frac{1}{3}}\\[2ex] \dfrac{M_n(r^3-r_0^3)}{M_n(r^3-r_0^3)+ir_0^3\rho_n M_w}, & r>r_0\left(\dfrac{\rho_n}{\alpha}+1\right)^{\frac{1}{3}}\end{cases} \tag{7}$$

If the seeding nuclei are made of a different salt than the fog droplet nuclei, this fact has to be expressed in the Raoult corrector.

The ambient vapor pressure is

$$e=\rho_v R_v T, \tag{8}$$

with

$$\rho_v=W-(4/3)\pi\rho\sum_{j=1}^{n}N_j(r_j^3-r_{0j}^3), \tag{9}$$

where the total water content of the system is

$$W=\rho_v^0+(4/3)\pi\rho\sum_{j=1}^{m}N_j(r_j^3-r_{0j}^3). \tag{10}$$

From Eq. (9) we obtain

$$\frac{d\rho_v}{dt}=-4\pi\rho\sum_{j=1}^{n}N_j r_j^2\frac{dr_j}{dt}. \tag{11}$$

TABLE 1. Three types of fog.

Type of fog	Range of fog droplet radii (μ)	Liquid water content (gm m^{-3})	Initial temperature (°C)	Initial visibility (m)
A	1–7	0.1	10	180.5
B	10–20	0.3	18	207.0
C	4–16	0.6	10	72.5

The liquid water content per unit volume of air is

$$\rho_w=(4/3)\pi\rho\sum_{j=1}^{n}N_j(r_j^3-r_{0j}^3). \tag{12}$$

d. Initial conditions

We choose an initial temperature T^0 and an initial pressure p_m. The pressure is assumed to remain unchanged during the process, and the relative humidity is assumed to have an initial value of 100%. With these conditions, the following parameters are defined by

$$e^0=e_s(T^0), \tag{13}$$

$$\rho_v^0=\frac{e^0}{R_v T^0}, \tag{14}$$

$$\rho_a=\frac{p_m-e^0}{R_a T^0}. \tag{15}$$

When the seeding nuclei are inserted, some amount of vapor immediately condenses upon them and reduces the vapor density and pressure. We assume that the seeding nuclei instantly become drops of radius r_j; as a result, the vapor density immediately reduces to

$$\rho_v=\rho_v^0-(4/3)\pi\rho\sum_{j=m+1}^{n}N_j(r_j^3-r_{0j}^3). \tag{16}$$

The reduced vapor pressure is given by

$$e=\rho_v R_v T^0. \tag{17}$$

3. Visibility

The effectiveness of seeding is measured by the improvement in visibility. We know that the fog droplets will get smaller. On the other hand new droplets will form upon the seeding nuclei. Using a derivation from MacCready (1969), the visibility in centimeters can be estimated from

$$V_j=3/\sum_{i=1}^{j}N_i\pi r_i^2, \tag{18}$$

where r_i is also in centimeters, and V_j is the visibility if we assume that drops with radii $\leq r_j$ remain in the air parcel and that those with radii $>r_j$ fall out.

4. Results

Computations were made for three types of fog. Type A fog had low liquid water content (LWC) and was composed of small droplets at an initial temperature $T^0=10$C. Type B fog had medium LWC and included large droplets at $T^0=18$C. Type C fog with high LWC was composed of medium size droplets at $T^0=10$C (see Table 1). The pressure in all cases was assumed to be $p_m=900$ mb.

The seeding material for fog of types A and B was sodium chloride. For type C fog, computations were made for seeding nuclei of NaCl, magnesium chloride ($MgCl_2$), and, to demonstrate the behavior of highly hygroscopic material, for seeding nuclei of beryllium fluoride (BeF_2) as well.

The results are given in Tables 2–7. Each table first shows the initial fog droplet distribution, including the size of the condensation nuclei upon which the fog droplets were formed. The size and number of seeding nuclei are given in the next two columns. For both fog droplets and seeding nuclei, the distributions are shown with their corresponding j indexes. The drop sizes for 60 and 100 sec after seeding are then shown, along with the corresponding visibility values.

During the process of water content redistribution the relocation of the drops due to their relative fall velocity was not considered; therefore, only illustrative results, shortly after seeding, were presented in this paper. The values of visibility computed omitting drops with radii $>r_j$ are the estimated visibility values after those drops have fallen out from the new droplet distributions.

TABLE 2. Initial distribution of type A fog droplets, distribution of NaCl seeding nuclei, and the final drop sizes with corresponding visibility values. Initial visibility is 180.5 m.

	Fog droplets			Seeding nuclei		Drop sizes		Visibility	
	Radius		Number	Radius	Number	After 60 sec	After 100 sec	After 60 sec	After 100 sec
Index j	r_{0j} (μ)	r_j (μ)	N_j (cm^{-3})	r_{0j} (μ)	N_j (cm^{-3})	r_j (μ)	r_j (μ)	V_j (m)	V_j (m)
1	0.1	1.0	2			0.344	0.326		
2	0.2	2.0	18			0.700	0.663		
3	0.3	3.0	75			1.056	0.999		
4	0.4	4.0	120			1.413	1.336		
5	0.5	5.0	75			1.770	1.673		
6	0.8	6.0	18			2.844	2.683		
7	1.0	7.0	2			3.566	3.358	1293.8	1448.7
8				6	1	20.827	20.674	814.9	878.8
9				8	1	25.675	26.400	521.6	535.4
10				10	1	29.896	31.381	350.5	345.0

TABLE 3. Initial distribution of type B fog droplets, distribution of NaCl seeding nuclei, and the final drop sizes with corresponding visibility values. Initial visibility is 207.0 m.

	Fog droplets			Seeding nuclei		Drop sizes		Visibility	
	Radius		Number	Radius	Number	After 60 sec	After 100 sec	After 60 sec	After 100 sec
Index j	r_{0j} (μ)	r_j (μ)	N_j (cm^{-3})	r_{0j} (μ)	N_j (cm^{-3})	r_j (μ)	r_j (μ)	V_j (m)	V_j (m)
1	0.36	10.0	1			1.752	1.648		
2	0.45	12.5	5			3.992	2.069		
3	0.54	15.0	8			9.109	2.508		
4	0.63	17.5	5			12.810	8.132		
5	0.72	20.0	1			16.057	12.610	523.3	1692.9
6				6	1	22.846	24.569	406.9	817.8
7				8	1	27.751	30.133	306.4	460.0
8				10	1	32.081	35.048	230.3	289.0

TABLE 4. Initial distribution of type C fog droplets, distribution of NaCl seeding nuclei and the final drop sizes with corresponding visibility values. Initial visibility is 72.5 m.

	Fog droplets			Seeding nuclei		Drop sizes		Visibility	
	Radius		Number	Radius	Number	After 60 sec	After 100 sec	After 60 sec	After 100 sec
Index j	r_{0j} (μ)	r_j (μ)	N_j (cm^{-3})	r_{0j} (μ)	N_j (cm^{-3})	r_j (μ)	r_j (μ)	V_j (m)	V_j (m)
1	0.4	4.0	1			2.548	2.483		
2	0.6	6.0	8			3.914	3.782		
3	0.8	8.0	30			5.839	5.229		
4	1.0	10.0	48			8.130	7.203		
5	1.2	12.0	30			10.409	9.484		
6	1.4	14.0	8			12.628	11.785		
7	1.6	16.0	1			14.797	14.042	105.3	128.4
8				6	1	22.964	25.193	99.5	118.3
9				8	1	27.472	30.261	92.2	106.2
10				10	1	31.465	34.772	84.2	93.6

TABLE 5. Initial distribution of type C fog droplets, distribution of $MgCl_2$ seeding nuclei, and the final drop sizes with corresponding visibility values. Initial visibility is 72.5 m.

Index j	Fog droplets Radius r_{0j} (μ)	Radius r_j (μ)	Number N_j (cm^{-3})	Seeding nuclei Radius r_{0j} (μ)	Number N_j (cm^{-3})	Drop sizes After 60 sec r_j (μ)	After 100 sec r_j (μ)	Visibility After 60 sec V_j (m)	After 100 sec V_j (m)
1	0.4	4.0	1			2.555	2.491		
2	0.6	6.0	8			3.923	3.793		
3	0.8	8.0	30			5.833	5.242		
4	1.0	10.0	48			8.118	7.211		
5	1.2	12.0	30			10.397	9.487		
6	1.4	14.0	8			12.616	11.786		
7	1.6	16.0	1			14.786	14.042	105.5	128.2
8				6	1	22.934	25.136	99.7	118.1
9				8	1	27.485	30.226	92.4	106.1
10				10	1	31.557	34.781	84.3	93.6

TABLE 6. Initial distribution of type C fog droplets, distribution of BeF_2 seeding nuclei, and the final drop sizes with corresponding visibility values. Initial visibility is 72.5 m.

Index j	Fog droplets Radius r_{0j} (μ)	Radius r_j (μ)	Number N_j (cm^{-3})	Seeding nuclei Radius r_{0j} (μ)	Number N_j (cm^{-3})	Drop sizes After 60 sec r_j (μ)	After 100 sec r_j (μ)	Visibility After 60 sec V_j (m)	After 100 sec V_j (m)
1	0.4	4.0	1			2.262	2.095		
2	0.6	6.0	8			3.436	3.183		
3	0.8	8.0	30			4.872	4.305		
4	1.0	10.0	48			7.075	5.682		
5	1.2	12.0	30			9.487	7.780		
6	1.4	14.0	8			11.836	10.230		
7	1.6	16.0	1			14.109	12.673	132.0	190.8
8				6	1	25.525	27.835	121.1	165.2
9				8	1	30.633	33.556	108.2	138.3
10				10	1	35.222	38.679	94.9	113.7

TABLE 7. Initial distribution of type C fog droplets, distribution of NaCl seeding nuclei, and the final drop sizes with corresponding visibility values. Initial visibility is 72.5 m.

Index j	Fog droplets Radius r_{0j} (μ)	Radius r_j (μ)	Number N_j (cm^{-3})	Seeding nuclei Radius r_{0j} (μ)	Number N_j (cm^{-3})	Drop sizes After 60 sec r_j (μ)	After 100 sec r_j (μ)	Visibility After 60 sec V_j (m)	After 100 sec V_j (m)
1	0.4	4	1			1.930	1.664		
2	0.6	6	8			2.926	2.515		
3	0.8	8	30			3.959	3.375		
4	1.0	10	48			5.547	4.255		
5	1.2	12	30			8.034	5.260		
6	1.4	14	8			10.602	7.115		
7	1.6	16	1			13.053	9.808	190.0	367.9
8				6	2	22.260	23.626	158.7	257.3
9				8	2	26.878	28.941	128.0	177.3
10				10	2	30.945	33.617	101.8	124.9

Table 2 shows the results for type A fog. The drops formed upon the seeding nuclei after 100 sec are much larger than the initial fog droplets. After they fall out the visibility will improve immensely from an initial value of 180.5 to 1448.7 m. After fallout of drops with radii $r \geqslant 25\,\mu$, the droplet distributions formed after 60 and 100 sec will result in visibilities of 814.9 and 878.8 m, respectively.

Table 3 shows results for type B fog. If we assume that all the drops formed upon the seeding nuclei remain in the air parcel, then the visibility improves only slightly from an initial value of 207.0 to 289.0 m after 100 sec. However, after the fallout of drops with radii $>25\,\mu$ from the drop distribution formed after 100 sec, the visibility will be 817.8 m, a significant improvement.

For type C fog, which has the poorest initial visibility, seeding does not seem to be as effective as in the other two cases. Table 4 shows that seeding type C fog with NaCl improves visibility only slightly, even after all the drops formed upon the seeding nuclei fall out. Seeding with $MgCl_2$ gives similar results, as shown in Table 5.

If the seeding nuclei for type C fog are made of a more hygroscopic material, like BeF_2, the drops formed upon them grow much faster in the beginning. Very soon, however, they become quite dilute, and the dissolved material has almost no effect on the saturation vapor pressure over the drops. Table 6 shows that BeF_2 improves visibility only slightly more than NaCl.

Seeding type C fog with a larger amount of material seems to be more effective. Table 7 shows the results for seeding with a double amount of NaCl. Comparison of Tables 7 and 4, however, shows that drops formed upon the seeding nuclei are smaller when more seeding nuclei are used. This suggests that some caution may be necessary; using too many seeding nuclei may result in formation of drops not large enough to fall out.

The set of equations was solved by the Runge-Kutta method, with control on the integration step dt to insure the desired accuracy. The computations were performed on the NCAR Control Data Corporation 6600 computer. The computation for one case took about one-half minute of computer time.

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APPENDIX

List of Symbols

- c_p specific heat of dry air at constant pressure
- D diffusion coefficient of water vapor in air
- e ambient vapor pressure
- e_s saturation vapor pressure over flat surface of pure water
- e^*_s saturation vapor pressure over a drop
- i van't Hoff factor: $i=2$ for NaCl; $i=3$ for $MgCl_2$ and BeF_2
- K heat conduction coefficient of air
- k Kelvin corrector
- L heat released by condensation of one gram of vapor
- M_n molecular weight of salt
- M_w molecular weight of water
- N number of drops cm^{-3}
- p_m pressure of moist air
- R_a gas constant for dry air
- R_c Raoult corrector
- R_m gas constant for moist air
- R_v gas constant for water vapor
- r radius of a drop
- r_0 radius of nucleus
- s specific heat of water
- T temperature (°K)
- t time
- V visibility
- W total amount of water in air parcel
- α solubility of salt in water
- ρ density of water
- ρ_a density of dry air
- ρ_n density of salt
- ρ_s density of drop
- ρ_v density of vapor in air parcel
- ρ_w liquid water content per unit volume of air
- σ surface tension

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