Statistical Characteristics of Subtropical Jet-Stream Features in Terms of MRIR Observations from Nimbus II

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ABSTRACT

A sample of 60 individual cases of southwesterly jet-stream maxima located over the continental United States during the Nimbus II period is studied in relation to the MRIR characteristics surrounding the jet. The analysis has been conducted primarily from a statistical point-of-view applied to the equivalent black-body temperature (T_{BB}) patterns of channels 1, 2 and 4, and to hypsometric combinations of these T_{BB} values representing layer-mean temperatures. It was found that certain gross features of the fields of convergence and divergence associated with the moisture distribution in certain quadrants surrounding the jet are consistently indicated in these patterns. A stepwise, multiple regression analysis for specification of the maximum wind speeds gives predictive skill in several channels, particularly in the 5–30 μ channel, indicating that baroclinic effects can be discerned by MRIR observations applied to suitably stratified data samples.

1. Introduction

Several useful papers have appeared in the literature (Oliver *et al.*, 1964; Whitney *et al.*, 1966; Johnson, 1966) describing the feasibility of locating jet streams through characteristic cloud features observable in meteorological satellite photographs. The parallel objective of characterizing jet streams, and in particular jet wind speed maxima, in terms of digitized fields of emitted terrestrial radiation detected by the various sensors on meteorological satellites is an equally useful problem and is the objective of this paper.

Beran *et al.* (1968) have addressed themselves to the latter problem. They examined the continuity of four independently selected jet-stream case sequences observed over North America during the lifetime of the Nimbus II Medium Resolution Infrared Radiometer (MRIR) data period (15 May–28 July 1966). They concluded that a general pattern of radiometric values, particularly in the 6.4–6.9 μ spectral region, may “in general be associated with jet streams and upper tropospheric cyclones in both the extratropical and subtropical regions.”

In this paper the approach is somewhat more quantitative in that statistical procedures have been employed on a random sample of 60 jet maxima in which the direction of flow was southwesterly. Through the use of a grid system, radiometric values at 13 objectively selected points symmetrically located about each jet maximum were obtained (see Fig. 1). In order to reduce the variation in as many physical variables as possible,

case stratification to southwesterly subtropical jets occurring over the United States was set forth as a requirement. While this represents a limitation upon the generality of the approach, insight into the more general problem of jet analysis through radiometric parameters results. The statistical approach adopted here was designed to achieve two general objectives:

- 1) The determination of spatial consistencies in the medium resolution infrared radiation (MRIR) fields relative to jet stream maxima.
- 2) The assessment of the usefulness of stepwise regression analysis for selecting MRIR variables that

FIG. 1. The jet-maximum case of 1200 GMT 5 June 1966. The grid overlay used for MRIR map sampling is shown within the dashed boundary. The sampling points are labeled 1, 2, ..., 13, and are denoted by circles containing a cross. The line 5–9 of the overlay is aligned with the jet maximum wind direction of the radiometric base map (1:10,000,000). The MRIR analysis shown is that for channel 1.

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FIG. 2. The NMC maximum wind analysis for 1200 GMT 5 June 1966. The symbol SR indicates that the run terminated before the target reached the level of the maximum wind. The symbol S indicates areas of maximum winds with speeds <20 kt.

allow the determination of the jet maximum wind speed ($V_{\max}$) and the lateral wind shear (ΔV) measured normal to the jet maximum wind direction.

2. Data sources

a. Maximum wind analysis data

As has already been noted, a decision was made, for purposes of stratification, to limit this analysis to subtropical jet streams occurring over or near the continental United States in which the direction of flow was southwesterly. For further selection of cases in which the jet maximum could be located and its speed determined, the maximum wind analyses (MWA) prepared by the National Meteorological Center (NMC) during the *Nimbus II* MRIR period were used. A good example of such a chart is shown in Fig. 2. No jet maximum was chosen in any instance that was within 5° longitude of either the western or eastern edges of the area illustrated in Fig. 2.

It was found useful to make systematic reference to the concurrent NMC wind shear analysis (WSA) in combination with the MWA chart for the purpose of establishing the final location and resultant magnitude of $V_{\max}$. As an example, the jet wind maximum in Fig. 2 was designated to be 125 kt in magnitude and located midway between Tucson and El Paso on the subtropical

jet axis shown on the MWA where there was a definitive local maximum of mean vertical shear in excess of 6 kt $(1000 \text{ ft})^{-1}$. If further question remained as to the location of the jet wind speed maximum, reference was made additionally to the jet maximum locations indicated on the concurrent 200-mb analysis also prepared by NMC. Using these jet-maximum analysis procedures for each case, an estimate of the expected rms errors in location, direction and speed are 1° latitude, 10° azimuth and 10 kt, respectively. While the speed error estimate is open to some question, the location and direction errors are not likely to be greater than postulated. The care taken in the use of the three separate NMC analyses mentioned above would tend to confirm this statement.

A further useful tool was available as a check on the location of each subtropical jet axis. This additional data source consisted of the pictorial multi-spectral documentation prepared in the *Nimbus II* Medium Resolution Infrared Pictorial Data Catalog, Vols. I and II (1967). For many of the jet cases studied there was found to be a long plume of cirrus and/or water vapor rich atmosphere equatorward of the jet axis and parallel to the jet. This feature was particularly noticeable in the photo-imagery of the radiances observed in channel 1 ($6.4\text{--}6.9 \mu$; see Fig. 3). The cirrus plume in Fig. 3 is

FIG. 3. The MRIR multispectral pictorial display based upon data orbit 286. Note especially the area near 30N, close to 1850 GMT 5 June 1966.

recognizable to a lesser extent in channel 4 ($5-30 \mu$), and in channel 2 ($10-11 \mu$). The patterns in the water vapor channel (as exemplified in Fig. 3) bear a remarkable resemblance to the jet-stream cirrus patterns reported by Kadlec (1964) in his studies of jet-stream cloudiness from jet aircraft. A consistent cross check of all the NMC data sources noted above with the MRIR pictorial data was maintained throughout the study, introducing a fourth analysis procedure for determining southwesterly jet locations. However, while this latter tool was a valuable adjunct, only the digitized grid-print values of equivalent blackbody temperature (T_{BB}) were used in the statistical analyses to be described.

Since the rawinsonde reports after smoothing are subject to randomly distributed errors (Reiter, 1963; Klein, 1968), the exact rms speed error will not, in general, obscure coherent T_{BB} pattern-analyses relative to the jet, provided the indicated quality control of the rms errors of location and azimuth are maintained.

In addition to the maximum wind speed, the lateral wind shear ΔV was measured across each jet maximum transverse to the jet wind direction. This parameter was defined as

$$\Delta V = (V_{\max} - V_{5n}) + (V_{\max} - V_{5s}), \quad (1)$$

where V_{5n} is the speed indicated on the MWA at a point

TABLE 1. Total sample of jet maxima which fulfilled the selection criteria (see text). The group of 30 cases in the left half (sub-sample 1) was used as the dependent sample for the regression statistics of Section 4. Sub-sample 2 was used for the independent verification.

Date (1966)	Map time (GMT)	Location of jet maxima (N lat./W long.)	Wind direction/ speed (kt)	Wind shear [kt (10° lat.) ⁻¹]	Date (1966)	Map time (GMT)	Location of jet maxima (N lat./W long.)	Wind direction/ speed (kt)	Wind shear [kt (10° lat.) ⁻¹]
<i>Sub-sample 1</i>					<i>Sub-sample 2</i>				
20 May	0000	30/103	260/120	130	20 May	1200	34/091	270/105	40
21 May	0000	30/103	260/110	150	21 May	1200	30/105	260/80	75
22 May	1200	33/107	270/110	100	22 May	0000	30/109	260/110	70
24 May	0000	32/110	260/120	145	25 May	1200	32/114	250/105	140
27 May	0000	27/101	270/105	80	28 May	0000	26/097	270/80	80
30 May	1200	36/116	210/110	160	31 May	1200	32/116	230/135	130
1 June	1200	32/115	220/140	140	1 June	0000	34/112	260/120	100
2 June	1200	33/114	240/120	100	2 June	0000	32/112	250/110	70
5 June	1200	30/109	260/130	145	5 June	1200	26/080	250/80	100
6 June	0000	39/104	260/105	120	7 June	0000	38/111	260/105	105
10 June	1200	39/101	240/80	75	11 June	1200	31/103	230/95	115
12 June	1200	33/100	240/80	80	13 June	0000	32/100	240/95	140
14 June	0000	33/098	260/80	35	15 June	0000	33/091	260/80	40
16 June	0000	31/100	260/80	75	17 June	0000	31/096	260/85	100
18 June	0000	32/116	240/90	90	19 June	0000	33/111	250/85	125
27 June	0000	36/113	220/80	100	26 June	0000	35/103	260/60	50
28 June	1200	32/116	210/75	95	28 June	1200	28/080	240/60	70
29 June	0000	39/109	230/80	100	30 June	0000	30/081	230/80	100
29 June	1200	39/111	240/75	75	30 June	1200	30/080	230/80	110
30 June	1200	35/114	210/95	110	2 July	1200	37/116	230/130	115
2 July	1200	31/081	230/80	100	3 July	0000	42/110	230/120	140
3 July	0000	31/083	240/80	120	3 July	1200	38/115	220/105	85
4 July	0000	43/112	230/105	90	4 July	1200	39/115	220/105	130
5 July	0000	43/108	230/100	140	6 July	0000	37/116	240/80	95
7 July	0000	42/107	260/105	130	8 July	1200	36/116	220/90	100
9 July	1200	39/115	210/140	170	15 July	0000	36/117	220/85	100
15 July	1200	37/120	220/100	85	16 July	0000	39/120	210/110	90
16 July	1200	40/118	210/105	120	21 July	1200	45/111	230/105	120
22 July	0000	47/102	240/95	110	23 July	0000	46/103	260/110	95
25 July	1200	43/109	250/105	90	26 July	0000	46/116	240/130	155

5° latitude along the positive transverse axis shown in Fig. 1, and V_{56} is an analogous measurement taken at a point 5° latitude transverse to the jet axis in the opposite direction. The shear ΔV is subject to considerably greater uncertainty than $V_{\max}$ since wind speed estimates at three points, rather than at a single point, are involved.

b. Radiometric data

The radiometric data used in this study were obtained in the form of multi-resolution Mercator grid print maps having 1.25° of longitude per mesh interval. The data for these maps came from the Nimbus II meteorological radiation tapes on file at the Goddard Space Flight Center of the National Aeronautics and Space Administration. Orbits were selected for each jet case that would provide *composited* radiometric data coverage at each of the 13 grid points indicated in Fig. 1. Using this criterion, a final selection of jet stream cases was made from the relatively large number of jets which could be selected from the MWA, as already described. A substantial number had to be rejected because the Nimbus II Data Catalog (1966) indicated a lack of MRIR coverage over the grid shown in Fig. 1. As a result, a great deal of scrutiny was involved in acquiring 60 cases where radiometric fields provided "synoptic" coverage for the jets selected up to this point. The 60 cases finally selected are listed in Table 1.

Since the MWA charts are drawn at 0000 and 1200 GMT, daily "synoptic" radiometric coverage consisted of composited MRIR non-void fields occurring within 5 or 7 hr on either side in time of the jet maximum case being considered. If both the "before" and "after" MRIR fields had a data gap over the gridded area around a jet maximum, the case was rejected. In general, for the cases selected (Table 1), it was possible to obtain non-void composited MRIR data over the grid consisting of two orbital swaths both in the periods prior to and after the jet wind analysis under consideration. The composited MRIR radiation fields (represented on the multi-resolution Mercator maps by equivalent blackbody temperatures) were analyzed in considerable detail for each case selected so that the data could be carefully interpolated at each of the 13 grid points lying on an acetate overlay (within the dashed boundary in Fig. 1), the tangential axis of which was aligned with the jet wind direction.

The radiation data compositing procedure described above resulted in smoothed radiometric fields *nearly synoptic* with 0000 and 1200 GMT wind analyses. As a result, a nighttime orbit was always involved, and the solar reflectance channel was not used at all. The 14–16 μ data were not used because relatively small populations composited at individual points on the grid print maps were not large enough to overcome the noise in the basic data. This situation could have been

TABLE 2. Sixty-case sample means of equivalent blackbody temperature $\bar{T}_{BB}(j)$ of the 13 grid points in channels 1, 4 and 2, and the corresponding pressure-equivalents $\bar{P}_s(j)$ in the *U. S. Standard Atmosphere*. Standard deviations $\sigma_T(j)$ for each grid point-mean temperature and its pressure-equivalent $\sigma_{P_s}(j)$ are also given. The sample means of $V_{\max}$ and ΔV are included in the bottom section of the table.

Channel	Grid point j	$\bar{T}_{BB}(j)$, (°K)	$\sigma_T(j)$ (°K)	$\bar{P}_s(j)$, (mb)	$\sigma_{P_s}(j)$ (mb)
1	1	236.767	5.685	360.912	45.568
	2	239.333	5.939	381.956	49.834
	3	238.317	6.320	373.505	52.073
	4	238.400	5.603	374.189	46.230
	5	244.550	5.688	427.799	52.316
	6	243.217	5.624	415.676	50.542
	7	240.900	5.993	395.282	51.692
	8	239.683	5.315	384.897	44.883
	9	238.050	4.806	371.312	39.420
	10	240.667	4.576	393.271	39.321
	11	239.767	4.626	385.604	39.119
	12	238.617	3.445	375.985	28.535
	13	236.933	3.635	362.345	29.227
4	1	262.967	12.004	624.566	149.870
	2	265.916	9.869	664.463	129.643
	3	264.483	12.051	645.783	154.713
	4	265.517	9.596	653.164	124.135
	5	268.767	7.280	702.695	100.063
	6	269.017	6.726	706.133	92.828
	7	267.166	11.108	680.985	148.868
	8	265.267	9.667	655.909	125.691
	9	262.017	9.864	614.764	121.686
	10	267.483	7.391	685.232	99.564
	11	266.050	9.358	666.148	123.204
	12	265.383	7.921	657.414	103.162
	13	262.250	8.280	617.644	102.541
2	1	282.067	16.671	905.762	248.288
	2	284.583	13.379	949.044	212.310
	3	284.350	16.706	944.971	257.688
	4	285.017	12.747	956.677	204.589
	5	285.500	11.519	965.227	187.954
	6	287.250	10.089	995.591	170.649
	7	286.300	14.797	979.527	238.525
	8	285.067	12.879	957.560	206.662
	9	279.983	13.767	871.132	202.895
	10	285.450	10.777	964.340	176.687
	11	285.050	12.163	957.260	196.166
	12	284.817	11.383	953.154	184.001
	13	279.616	12.010	865.151	178.355
Predictand		Mean	Standard deviation		
$V_{\max}$		98.667 kt	19.064 kt		
ΔV		104.083 kt (10° lat.) ⁻¹	29.950 kt (10° lat.) ⁻¹		

partially remedied by the water vapor cloud contamination removal procedures suggested by Warnecke and McCulloch (1968) or by Woolf and Smith (1970). However, the basic hypothesis invoked here involved inferring tropospheric baroclinicity as it could be detected by channels whose maximum response occurred in the troposphere. This limits the consideration to channels 1, 4 and 2.

The equivalent blackbody temperatures [$T_{BB}(j)$; $j=1,2,\dots,13$] for each of these three channels at the thirteen points shown in Fig. 1 were determined and encoded onto IBM cards in a consistent format amenable to further analysis by computer. The statistical features of the $T_{BB}(j)$ fields at the 13 points will now be described.

3. Pattern analysis of mean MRIR equivalent blackbody temperature fields

a. Analysis of variance of the sample means of equivalent blackbody temperature T_{BB}

As an initial step, the 60-case sample means $\bar{T}_{BB}(j)$ were computed for each grid point j illustrated in Fig. 1 and for each of channels 1, 4 and 2. The pressure-equivalent $\bar{P}_s(j)$ was also computed by determining the pressure level corresponding to the observed $T_{BB}(j)$ at each grid point. This was done by using tables of temperature vs pressure for the *U. S. Standard Atmosphere* (COESA, 1962). Table 2 lists the grid-point means of the distributions, $\bar{T}_{BB}(j)$ and $\bar{P}_s(j)$, as well as their standard deviation, $\sigma_T(j)$ and $\sigma_{P_s}(j)$, respectively, for each grid point. Inasmuch as the latter standard deviation extends over a considerable atmospheric depth, the implied vertical variability in the equivalent pressure level is noteworthy. Since P_s is functionally related to the temperature in these equivalences, the major thrust of the analysis of variance will be placed upon the fields of $\bar{T}_{BB}(j)$. However, one cannot fail to observe that $\sigma_{P_s}(j)$ in channel 2 is of such large magnitude at every grid point as to intersect the ground level unless it receives its major contribution from levels where $P_s(j)$ is consistently smaller than $\bar{P}_s(j)$. This suggests the conclusion that the sample of $P_s(j)$ for channel 2 is representative of a distribution strongly skewed away from near-surface levels toward cloud-top levels whenever middle or high clouds exist within the analysis grid.

The values of $\bar{T}_{BB}(j)$ and their standard deviations have been plotted for each of the thirteen points of channels 1, 4 and 2 in Fig. 4, $\bar{T}_{BB}$ isotherms having been drawn at 2K intervals. The $\sigma_T(j)$ values are also plotted and are enclosed in parentheses. The dashed lines drawn across each grid-panel denote the median values of the standard deviation. The areas delineated by the introduction of the dashed lines afford a visual aid in separating relatively high vs low variabilities in $\bar{T}_{BB}(j)$.

It should be observed that in this study a special set of MRIR channel-radiances have been isolated that are associated with cloud and/or water vapor configurations distributed relative to jet maxima. As such, these data cannot be considered in any *a priori* sense as being restricted to the radiances to be expected from cloudless model atmospheres (for example, see Möller and Raschke, 1964). Therefore, when considering information from the three channels 1, 4 and 2, one has a greater expectancy of observing nonredundant data in specifying MRIR patterns relative to subtropical jet maxima, and their attendant cloud features.

A more detailed examination of Fig. 4 involves analysis of the patterns of $\bar{T}_{BB}(j)$ values over each grid array. This examination reveals a warm tongue of $\bar{T}_{BB}$ oriented generally parallel to, but slightly north of, the mean jet wind axis in each case (Figs. 4a-c). The central value of the warm tongue appears in the mean to have a

FIG. 4. Mean equivalent blackbody temperatures for each of the three channels, (a) channel 1, (b) channel 4, (c) channel 2, plotted at grid points 1, $\dots$, 13, after averaging over the sample set of 60 southwesterly subtropical jets. Isotherms are drawn as solid lines at 2K intervals. Standard deviation σ_T for each grid are displayed below each point by the number within parentheses. The dashed line(s) in each panel represents the median value of the σ_T distribution.

westward tilt with increasing height. This arrangement of warm centers together with the location of cold pools near the southernmost point of the grid is consistent with statistical-physical considerations that are relevant to the southwesterly jet sample which has been collected for this study. A discussion of these considerations, as well as the cold pool to the north, will be given in Section 5. In the remainder of this section the statistical significance of the channel patterns of $\bar{T}_{BB}(j)$ is considered. The discussion below is modeled after Crow *et al.* (1960, pp. 119–126), and involves inferences concerning one factor analysis of variance, and a Type I problem.

The Type I problem considered here assumes as the null hypothesis that the spatial patterns displayed in Fig. 4 are not significantly different from the grand sample means. The three 13-point mean spatial² arrays of $T_{BB}(j, \cdot)$ will be examined and the sum-of-squares between (SSB) and the sum-of-squares within (SSW) treatments determined. The treatments discussed here are those of case-averaging at each of the 13 points over the analysis grid. The symbols SSB and SSW are defined as:

$$SSB = 60 \sum_{j=1}^{13} [T_{BB}(j, \cdot) - \bar{T}_{BB}]^2, \quad (2)$$

$$SSW = \sum_{j=1}^{13} \sum_{k=1}^{60} [T_{BB}(j, k) - T_{BB}(j, \cdot)]^2, \quad (3)$$

where $T_{BB}(j, \cdot)$ is the treatment mean at grid point j ($j=1, 2, \dots, 13$) and $T_{BB}(j, k)$ the individual equivalent blackbody temperature occurring in each case at the 13 grid points ($j=1, 2, \dots, 13$; $k=1, 2, \dots, 60$). The values of $T_{BB}(j, \cdot)$ are given in Table 2 and the values of $\bar{T}_{BB}(\cdot, \cdot)$ in each channel are computed from $T_{BB}(j, \cdot)$ by averaging. The results developed for SSB and SSW are given in Table 3 along with the mean sum of squares (MSB, MSW), the degrees of freedom (DF), and the F ratio, which can be expressed as

$$F = MSB/MSW. \quad (4)$$

In each channel, the null hypothesis is tested for possible rejection using as the critical F level ($F_{crit.} = 2.18$) the 99% level with 12 and 767 degrees of freedom. It is clear that the critical F level is exceeded in both channels 1 and 4, but not in channel 2. Consequently, the null hypothesis $T_{BB}(j, \cdot) = \bar{T}_{BB}(\cdot, \cdot)$ is to be rejected in the grid arrays for channels 1 and 4, but should not be rejected in the case of channel 2. The statistically significant patterns in channels 1 and 4 do indeed have some physical interpretation, particularly

TABLE 3. Analysis of spatial variance for channels 1, 4 and 2, between and within grid-point means for the 13-point grid array of Fig. 1. See text for meaning of symbols.

Channel	SSB	SSW	DF (SSB)	DF (SSW)	MSB	MSW	F ratio
1	3717.817	17,961.834	12	767	309.818	23.418	13.230
4	3657.576	68,715.955	12	767	304.798	89.591	3.402
2	3792.204	132,268.22	12	767	316.017	172.449	1.833

² Note the change of notation for the 60-case sample mean from $\bar{T}_{BB}(j)$ to $T_{BB}(j, \cdot)$ in accordance with the notation of Crow *et al.* (1960).

FIG. 5. Simple correlations between pairs of channel temperatures T_{BB} in channels 1, 2 and 4, at each of the 13 grid points surrounding the jet maximum. The sample size is 60. The solid curved line denotes the median value of the correlation coefficient in each panel.

in the case of channel 1 where the F ratio was quite large. More discussion of this point will be given in Section 5.

b. Intra-channel correlations between equivalent blackbody temperature values

Considerable discussion has been devoted to the analysis of mean patterns of T_{BB} in the three channels. It is also of interest to examine intra-channel simple correlations in order to derive further diagnostic clues which may help to characterize the MRIR radiance patterns that are observed near subtropical jet maxima. Our major effort here is directed toward deducing some of the statistical-physical implications suggested by the simple correlation coefficients of Fig. 5, and their lateral variations.

Figs. 5a–c depict in sequence the panels $r_j(T_1, T_2)$, $r_j(T_1, T_4)$ and $r_j(T_2, T_4)$, where T is again the equivalent blackbody temperature, but the subscript BB has been dropped and is replaced henceforth by numbers corresponding to the MRIR channel number. The subscript j refers to the point on the analysis grid. Note that all correlation coefficients displayed are positive and are of increasing magnitude from panels 5a–c, respectively.

In Fig. 5a, values of $r_j(T_1, T_2)$ are generally of the order of 0.6 or higher except near the left corner of the grid. Here low correlations evidently indicate points where the warm tongue of Fig. 4a undergoes considerable variability depending upon the actual dynamical properties of the jets being sampled. In general, the overall pattern of $r_j(T_1, T_4)$ values bears a close resemblance to $r_j(T_1, T_2)$ but has a higher scale of values. The highest value which occurs in the $r_j(T_2, T_4)$ panel is 0.977 at point 1. The 0.95 isopleth extends nearly parallel to the transverse axis, but is inclined slightly toward the northeast across the axis. There are, however, two areas of relatively low $r_j(T_2, T_4)$ values at points $j=6, 8$ along the axis tangent to the jet.

The relatively small spatial variation in r_j across panel 5c makes even qualitative conclusions difficult. In panels 5a and 5b, however, in view of the fact that both channels involved are sensitive in some degree to water vapor, it appears that the more moist and/or cloudy conditions would be compatible with the resulting increase in intra-channel correlations occurring in the lower right-hand portion of the analysis grid. The lowest correlations and driest or most variable conditions tend to persist in the upper left-hand portions of Figs. 5a and 5b.

Since one of our objectives was to be able to discern baroclinic effects by regression methods, a new radiometric “thickness” variable ΔZ was defined in the layer T_1 – T_2 for each point of the grid and for all 60 cases. This new variable ΔZ was modeled using the hypsometric formula applied with mean radiometric temperatures, and extending between the pressure-equivalent levels in each case. Thus ΔZ , as modeled, assumes the familiar form

$$\Delta Z = \frac{R_d}{g} \left[\left(\frac{T_1 + T_4}{2} \right) \ln \left(\frac{P_4}{P_1} \right) + \left(\frac{T_2 + T_4}{2} \right) \ln \left(\frac{P_2}{P_4} \right) \right]. \quad (5)$$

Here T_1 , T_2 and T_4 are the equivalent blackbody temperatures from each channel grid point and case, and P_1 , P_2 and P_4 are the corresponding pressure-equivalent levels.

If one assumes the usual equation in a polytropic atmosphere as a means of determining pressure-equivalent levels, P_1 , P_2 and P_4 , it is only necessary to use the U. S. Standard Atmosphere [COESA, 1962, Eq. (I.2.10–4)], with the standard lapse rate of $0.0065\text{K (gpm)}^{-1}$, to derive the relationships

$$\left(\frac{P_4}{P_1} \right) = \left(\frac{T_4}{T_1} \right)^{5.25588}, \quad \left(\frac{P_2}{P_4} \right) = \left(\frac{T_2}{T_4} \right)^{5.25588}.$$

FIG. 6. Multiple correlation coefficients R_k for $V_{\max}$ with the k th step prediction equation, shown using channel 1, 4 and 2 grid-point temperatures (T_{BB}) as separate predictors, and plotted vs k . Tabular inset gives the order of entry by the stepwise regression method from $k=1, \dots, 6$ of the overlay grid-point variables selected.

The resulting expression for ΔZ is

$$\Delta Z = C \left[(T_1 + T_4) \log_{10} \left(\frac{T_4}{T_1} \right) + (T_2 + T_4) \log_{10} \left(\frac{T_2}{T_4} \right) \right] = CT_M, \quad (6)$$

where C of (6) is defined by

$$C = \frac{(2.30259)(5.25588)}{2g} R_d,$$

and has the value $177.1089 \text{ gpm } (^{\circ}\text{K})^{-1}$ in the Standard Atmosphere. Since C is a constant throughout the regression analysis, its value will be suppressed from this point on. All of the temperature sums, quotients, logarithms, etc., can be introduced as preliminary steps in the ensuing stepwise regression program. The mean temperature within the brackets in (6) may be regarded as the sum of an upper and lower layer-mean temperature over the grid for each sampling. The expressions for the upper and lower layer are, respectively:

$$T_U = (T_1 + T_4) \log \left(\frac{T_4}{T_1} \right), \quad (7a)$$

$$T_L = (T_2 + T_4) \log \left(\frac{T_2}{T_4} \right). \quad (7b)$$

One may view the application of the stepwise multiple regression procedure to the new parameter ΔZ as a means of estimating the contribution of the spatial variations of T_m to the specification of the jet wind speed and the wind shear. A further question to be answered is whether T_U or T_L may be equally or more effective in the specification problem to be pursued next.

The general statistical approach to be used here, while pooling together a collection of jets in various stages of their development, has the effect of approximating the steady-state features of the jet.

4. The regression analysis and verification

a. Preliminary examination of the stepwise regression analysis

A preliminary stepwise regression analysis, based upon the procedure of Miller (1962), was performed to specify $V_{\max}$ and ΔV from the three individual channel variable sets. In this initial step, the full sample population of 60 cases were used. The multiple correlation between $V_{\max}$ and the k th T_{BB} grid point selection derived using the Miller technique is shown for the three channels in Fig. 6. After the 10th step, any further increase in successive R_k was negligibly small. However, the process of adding redundant predictors causes the standard error of estimate of $V_{\max}$ to approach its standard deviation. This is characteristic of the over-specification process described by Lorenz (1956). The first six predictors selected in the three stepwise re-

FIG. 7. Multiple correlation coefficients R_k for $V_{\max}$ with the k th step prediction equation, shown using the grid-point values of T_U , T_L and T_M of Eqs. (6) and (7) as separate predictors, and plotted vs k . Tabular inset gives the order of entry from $k=1$, ..., 6 of the grid-point variable selected in each regression.

gressions are enumerated, one set for each channel, on the right side of Fig. 6. As it turns out, the use of prediction equations beyond the step $k=6$ is not of interest since it is found that the minimum shrinkage in R (and, therefore of unexplained variance R^2) occurs either at step 4 or 5 in the case of $V_{\max}$. Based upon the 60-case stepwise regressions, the multiple correlations at the sixth step³ for $R_6(V, \tilde{V})$ and for $R_6(\Delta V, \Delta \tilde{V})$ are listed below in Table 4 as well as the standard errors of estimate. The graph of $R(\Delta V, \Delta \tilde{V})$ is not presented here.

The standard errors in estimating V are to be considered in relationship to the standard deviation of V , namely, $\sigma_V=19.0642$ kt. However, the fractional explained variance is given (Crow *et al.*, 1960) as

$[1 - (\text{S.E.}/\sigma_V)^2]$ and has the resultant expression

$$\text{Fractional explained variance} = \left(\frac{n-1}{n-k-1} \right) R_k^2, \quad (8)$$

where n is the sample size, k the number of predictors selected, and R_k the multiple correlation coefficient determined at step k . The values of $R_6(\Delta V, \Delta \tilde{V})$ listed in Table 4 next to $R_6(V, \tilde{V})$ were not considered high enough to be of possible physical significance.

A similar preliminary procedure was applied to the three standard atmosphere mean-temperatures, T_M , T_U and T_L , defined in Eqs. (6), (7a) and (7b), respectively. The 60-case grid samples of T_M , T_U and T_L are now treated as the analogs of T_1 , T_4 and T_2 in determining the stepwise entry of grid point temperatures into the

TABLE 4. Multiple correlation coefficient (R) and standard error of estimate (S.E.) at step 6 in the stepwise multiple regression procedure for V and ΔV in channels 1, 4 and 2.

Channel no.	$R_6(V, \tilde{V})$	S.E. ₆ (kt)	$R_6(\Delta V, \Delta \tilde{V})$	S.E. ₆ [kt (10° lat.) ⁻¹]
1	0.6778	14.7893	0.4406	28.3679
4	0.7002	14.3660	0.6203	24.7865
2	0.7085	14.1958	0.6060	25.1364

³ Henceforth, the subscript "max" will be deleted from $V_{\max}$ and it will be understood that $V=V_{\max}$. The superscript *tilde* is used to denote the statistical estimate of the variable V or ΔV .

TABLE 5. Multiple correlation levels (R) and standard error of estimates (S.E.) at step 6 in the stepwise multiple regression procedure for V and ΔV for radiometrically-defined mean temperatures T_U , T_M , T_L of Eqs. (6) and (7).

Layer-mean variable	V		ΔV	
	R_6	S.E. ₆ (kt)	R_6	S.E. ₆ [kt (10° lat.) ⁻¹]
T_U , upper	0.7098	14.1686	0.6151	24.9142
T_M , middle	0.7269	13.8130	0.5577	26.2293
T_L , lower	0.7020	14.4020	0.6014	25.2459

TABLE 6. Comparisons of the multiple correlation coefficients and related standard errors of estimate (kt) associated with prediction equations involving k radiometric temperature predictors. Results (a) are based upon the *dependent* sample, while those of (b) are based upon the identical prediction equations applied to the *independent* sample; S_4 , S_5 , S_6 are based upon Eq. (9)

					Predictors						
					Channel 1	Channel 4	Channel 2	Upper column	Entire column	Lower column	
Number of variables in prediction equation	(a)	Dependent sample	4	R_4 S.E.	0.6235 15.9069	0.7116 14.2954	0.6621 15.2489	0.6966 14.5579	0.6989 14.5517	0.7142 14.2424	
			5	R_5 S.E.	0.7176 14.4176	0.7352 14.0769	0.6882 15.0673	0.7276 14.2461	0.7095 14.6343	0.7301 14.1899	
			6	R_6 S.E.	0.7267 14.5724	0.7598 13.7918	0.7180 14.7661	0.7427 14.2040	0.7181 14.7650	0.7389 14.2927	
	(b)	Independent sample	4	R_4 S.E.	0.5146 16.9847	0.6679 14.7423	0.5986 15.8677	0.4585 17.6038	0.6406 15.2098	0.6300 15.3834	
			5	R_5 S.E.	0.6051 15.7709	0.6484 15.0803	0.5889 16.0099	0.5449 16.6099	0.6554 14.9607	0.6158 15.6078	
			6	R_6 S.E.	0.5639 16.3582	0.6460 15.1206	0.4798 17.3800	0.4798 17.3793	0.6467 15.1086	0.5997 15.8513	
	(c)	Skill	Percent shrinkage	4	S_4	31.88	11.90	18.26	56.68	15.99	22.19
				5	S_5	28.90	22.22	26.78	43.91	14.67	28.86
				6	S_6	39.79	27.71	55.34	58.27	18.90	34.13

specification equations for V and ΔV . The increase of the multiple correlation coefficient for V with increasing step number k in the resulting regression equations is shown in Fig. 7 as it pertains to the selection of predictors from the T_U , T_M and T_L grid-point arrays, respectively. The sixth-step values of R_k depicted in Fig. 7, with the various column-mean, grid-point temperatures as predictors, are given in Table 5. The results for ΔV are included again, but the standard errors are too large to suggest any significance.

Comparison of this tabular summary with that given in Table 4 tends to indicate a somewhat higher level of predictability for V using column-mean variables T_U , T_M and T_L than is provided by the variables T_1 , T_4 and T_2 . This conclusion is tentative and an experiment has been designed to examine this question more objectively. Since the preceding tests have employed all available sample data, and no independent sample exists for verification, the presentation of specific stepwise regression equations has been reserved for the experiment to follow.

b. The prediction-verification experiment

The full data sample of $N=60$ cases was divided randomly into two equal sub-samples of 30 cases each. This was done by selecting consecutive odd and even numbered cases in the data sequence (see Table 1) as sub-samples 1 and 2, respectively. It should be recalled that the original synoptic situations were selected as objectively as possible in space and time. Therefore, the breakdown into the subdivisions just described has

not reduced the objectivity employed in the initial sampling.

The stepwise multiple regression analysis was performed utilizing sub-sample 1 as the dependent data sample to develop regression equations (after Miller, 1962) from each of the six families of grid arrays designated by the notation T_1 , T_2 , T_4 , T_U , T_M , T_L . It was observed that the prediction equations at step 3 were never as effective in providing explained variance either on the dependent sample nor on the independent data sample (sub-sample 2) as with $k=4$. At the other extreme, the selection of seven grid-point predictors led to increased standard errors of estimate as compared with the six-predictor case. Our tests were therefore confined to consideration of cases involving 4, 5 and 6 predictor equations, each testing the six possible correlations of V first with the equivalent blackbody temperatures from each of channels, 1, 4 and 2, and then with the three column-mean temperature variables T_U , T_M and T_L . The resulting prediction results obtained from the dependent and independent samples are summarized in Table 6.

A criterion for ascertaining the success of a combination of k predictors, termed here the "skill shrinkage" S , was developed. This criterion S simply computes the *fractional shrinkage* in explained variance between the dependent and independent tests, the highest skill being associated with the smallest S . The fractional shrinkage of explained variance at a particular step k is expressed in the form

$$S = \frac{R_k^2 - \hat{R}_k^2}{R_k^2}, \quad (9)$$

so that the common factor $(n-1)/(n-k-1)$ may be divided out. Here for the same prediction equation R_k and $\bar{R}_k$ refer to the multiple correlation coefficient of the dependent and independent samples, respectively. Reference to Table 6 reveals that the channel 4 prediction equation with four predictors gives the best skill as defined by Eq. 9. The prediction equation is

$$\tilde{V}_4^{(4)} = -408.40308 + 0.78643T_4(1) + 0.70356T_4(2) + 1.07342T_4(12) - 0.65427T_4(13). \quad (10)$$

While Eq. (10) specifies only the jet maximum speed, and not direction, the effect of baroclinicity may be inferred, in light of Fig. 4b, by the existence of as large a difference as possible between $T_4(2)$ and $T_4(13)$. The numbers in parentheses are the points of the analysis grid selected by the stepwise regression procedure to be predictors. The estimation of $\tilde{V}$ by (10) is related to V of the independent sample by the best-fit regression

$$V = 0.77716\tilde{V}_4^{(4)} + 21.52382, \quad (11)$$

which is equivalent to the more detailed form

$$V = -295.87072 + 0.61118T_4(1) + 0.54678T_4(2) + 0.98342T_4(12) - 0.50847T_4(13).$$

In (10) and (11) the *superscript* (4) indicates the number of predictors in both equations. Eq. (11) is subject to a coefficient of determination $R(V, \tilde{V}) = 0.6679$ and a standard error of estimate on the independent data of 14.7423 kt.

The next best prediction equation, based upon the criterion of (9), turns out to be the column-mean temperature T_M grid array at step 5, in particular. Thus,

$$\tilde{V}_M^{(5)} = 52.34735 + 0.64494T_M(1) + 0.85614T_M(2) + 0.30653T_M(4) - 1.33155T_M(5) + 0.59781T_M(10), \quad (12)$$

where $\tilde{V}_M^{(5)}$ exhibits just slightly more shrinkage upon application to the independent data-sample. The best-fit relationship between $\tilde{V}_M^{(5)}$ and V is given by

$$V = 0.59970\tilde{V}_M^{(5)} + 37.7000. \quad (13)$$

The shrinkage between the dependent and independent sample trials using Eq. (12) is 14.67% and only 11.90% using (10). While all the other independent tests exhibiting minimum shrinkage are statistically significant using the F ratio with 5 and 24 degrees of freedom at the 95% confidence level, $F_{crit.} = 2.57$, they do not equal the verification performance of the equation pairs just discussed. For example, in the case of channel 1 prediction-verification at step 5, the shrinkage is 28.90%, whereas for channel 2 at step 4, the corresponding shrinkage is 18.26%.

The fact that channel 1 did not give better results than channel 4 is due primarily to the smaller range of the T_1 variables over the channel 1 grid as compared to

that of channel 4. Since the ratios of the extreme ranges observed in channels 1 and 4 in any particular case seldom exceeded 0.4, errors in recorded T_{BB} values in channel 1 act as a larger noise source. Similar remarks apply to the comparison of verifications for channel 1 relative to channel 2.

It appears significant that channel 4 and the column-mean "channel" represented by the variable T_M resemble one another in the stability of their verifications. This result lends support to the earlier hypothesis that channel 4 approximates a *tropospheric* column-mean temperature in defining areas of strong vertical wind shear.

With regard to the specifications of ΔV by the various channels, and the subsequent verification, too much noise was compounded into the shears to permit useful verification. The most promising result was again afforded by channel 4 predictors at step 4 where the shrinkage was only 35.44%. With all the *other* channels the prediction-verification process led essentially to standard errors in ΔV equal to the standard deviation of the independent sample, indicating no apparent predictive skill.

5. Some statistical-physical conclusions regarding the southwesterly jet during the Nimbus II period

We have found that the mean radiometric isotherm patterns at the levels of channels 1 and 4 have a consistent warm tongue lying somewhat north and west of the jet maximum. The westward tilt of the center of the warm tongue bears some resemblance to the westward tilt of a shortwave ridge in a baroclinic atmosphere. The warm tongue pattern suggests an area of descending motion to the west of the subtropical jet maximum, with a consistent tilt in the vertical from the tropopause to levels near the ground.

The coldest radiometric temperatures occur toward the extreme right transverse side of the jet, indicating vertically ascending air and a tendency for a moisture-rich atmosphere to be indicated in all channel grid-arrays in this location. Generally high T_{BB} correlations between channels exist along a curved line extending close to grid points 1, 2, 7 and 12, and show a tendency for cloud and/or moisture-rich air to envelop the area in the neighborhood of the line passing through these points. The areas just described are essentially those described by Reiter (1963, p. 136) as being close to the centers of divergence associated with the southwest and northeast quadrants relative to the "advective jet model."

The vertical tilt of the maximum T_{BB} centers (Fig. 4) and the persistence of these centers from case to case indicates the strong likelihood of a region of upper tropospheric convergence to the left and west of the jet. The mean T_{BB} values presented in Fig. 4 also tend to confirm the presence of jet stream cirrus and/or water-

vapor rich atmosphere extending not only in the SW and NE quadrants, *but continuing well into* the SE quadrant, generally paralleling the jet axis. This latter phenomenon is not entirely consistent with the Reiter *advective* model which would predict convergence in this sector, but is in agreement with time-lapse movies made from ATS-1 Lagrangian-type photographic sequences of individual subtropical jets made by Fujita,⁴ who has ascribed the prevalence of cirrus in the SE quadrant as being due to advection of thick cirrus, formed initially in the SW quadrant, and displaced into the SE quadrant, as long as the jet-level winds south of the core maintain a strength nearly equal to those of the jet axis. Visual reconnaissance by Kadlec (1964) indicates that wide areas of cirrus persist in the SE quadrant of the subtropical jet provided the jet stream "exhibits pronounced anticyclonic curvature," and if also the wind speeds remain in excess of 40 kt. Both sets of qualitative conditions just put forth, especially those of Fujita, indicate that while horizontal convergence of some magnitude exists in the SE quadrant, the rate at which a reversal to significant downdraft motions and ensuing cloud dissipation occurs is slow.

In most cases a polar front jet axis crossed the northern portion of the grid. The cool radiometric temperatures at the northernmost point (point 13) indicate the likelihood of an interaction between the subtropical jet and a polar jet to the north. This interaction appears to be indicative of moist updraft tropospheric motions in this intra-jet region, much like those found in the divergent zone on the right side of the subtropical jet.

While the analysis grid is large enough to allow only a semi-quantitative discussion, enough evidence is present to make inferences from the mean MRIR measurements regarding the location of the major convergent and divergent areas, as well as areas of most highly developed clouds, around the jet maximum. These inferences appear to be in agreement with recent observations of subtropical jets, where ample separation has been maintained between the polar front and southwesterly subtropical jets [Beran *et al.*, 1968; Reiter and Whitney, 1969; Kadlec, 1964; Fujita (*loc. cit.*)]. It must be remembered that this sample comes from summer situations where subtropical jet stream features are not as pronounced as in the winter, and separations between polar front and subtropical jet streams are greater.

6. Conclusions

The objective here was to characterize southwesterly subtropical jet streams over North America during the Nimbus II (summer) period as observed in different spectral regions afforded by the MRIR capability. In particular, the differing responses attributable to the

6.3 μ water vapor vibration-rotation band (sensed by channel 1), and the combined effects of this band, the water vapor rotational band ($\lambda > 20 \mu$), the 15 μ band of CO₂, and the water vapor window (all encompassed within channel 4) have been studied. Consistent patterns near jet maxima were observed in both these channels. This was evidenced by properties of the mean T_{BB} patterns, which displayed features that could be associated with areas of convergence and divergence observed in other recent jet stream studies. The 5–30 μ channel provided the best overall statistical results in specifying the wind speed in the jet maximum and the shear across the jet.

In general, the results were encouraging enough, particularly in the specification of wind speed, to conclude that efforts along this line need to be continued using properly stratified wintertime jet stream cases. Presumably the 20–23 μ channel available on the Nimbus III MRIR would lend itself to a continued effort of this sort in that it would provide information from lower layers in the troposphere that was only obtainable indirectly from the 5–30 μ channel. If possible, the CO₂ channel (15 μ) should be included in further studies in order to deduce possible linkage between stratospheric and tropospheric flow patterns.

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