Vertical Velocity Variances and Reynold Stresses at Brookhaven

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ABSTRACT

The response characteristics obtained through wind tunnel tests of the Brookhaven National Laboratory bivane are presented and used in correcting the measured values of $(\overline{w'^2}/u^{*2})$, the variance of the vertical wind velocity fluctuations normalized by the square of the friction velocity. In neutral through unstable atmospheres the corrected values appear to be independent of thermal stability and height of observation, except for extreme instability at heights >46 m. The average value based on measurements at heights of 23, 46 and 92 m is 1.48 with a standard deviation of 0.38.

1. Introduction

In recent papers by Busch *et al.* (1968) and Frizzola *et al.* (1967), several years' accumulation of turbulence data from Brookhaven (BNL) was summarized. The observational procedure and data reduction have previously been described by Brown (1959) and Singer (1964). The purpose of this paper is to present the results of wind tunnel tests of the Brookhaven annular bivane. The energy transfer functions describing the instrument response and the numerical filter employed in the data reduction process have been used to obtain corrected values of the normalized variance of the vertical wind velocity component.

According to the Monin-Obukhov similarity hypothesis, the nondimensional variance $\overline{w'^2}/u^{*2}$ (w' being the vertical velocity component and u^* the friction velocity) should be a universal function of the stability parameter z/L, where z is the height of observation and L the Monin-Obukhov-Lettau (MOL) stability length (Lumley and Panofsky, 1964). Busch and Panofsky (1968) report $\overline{w'^2}/u^{*2}$ to be constant and about 1.7 for stable, neutral and unstable air up to $|z/L|=0.5$. The present paper is a study of the variation of the nondimensional variance with height and thermal stability at Brookhaven in neutral through unstable air.

Due to lack of heat flux measurements the MOL lengths could not be calculated. Instead, another stability parameter z/L' was used, where L' is defined by $L'=LK_H/K_M$, K_H and K_M being the turbulent diffusivities for heat and momentum, respectively. If the KEYPS interpolation formula for the wind profile is accepted, z/L' may be related to the gradient Richardson number Ri through

$$z/L'=\mathrm{Ri}(1-\gamma'\mathrm{Ri})^{-1/4}$$

(Lumley and Panofsky, *loc. cit.*). For all 58 sets of turbulence measurements reported, z/L' was computed for $z=23$ m using $\gamma'=18$ and Ri($z=23$ m) estimated from profile measurements. The range of values encountered was $-0.74\leq(z/L')_{23}\leq 0$. Maximum and minimum mean wind speeds were 17 and 2.7 m sec^{-1}, respectively.

2. Experimental procedure

The bivane shown in Fig. 1 was mounted in the working section of a wind tunnel, the vane displaced by a

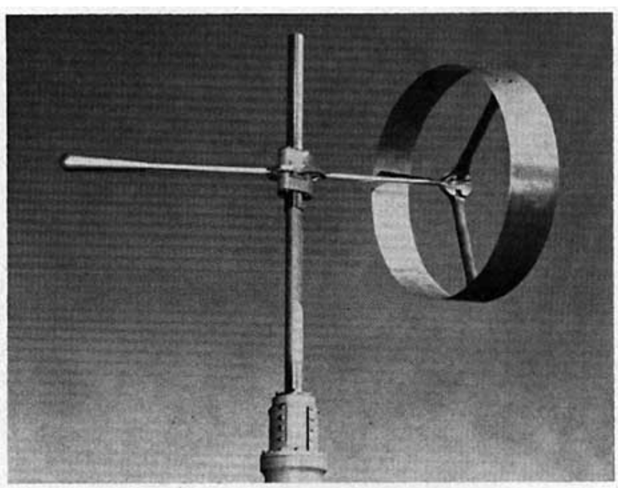

FIG. 1. Brookhaven annular bivane: outer diameter of fin, 0.41 m; distance from counterbalance to fin, 0.82 m; weight of fin, 404 gm. See Mazzarella (1952).

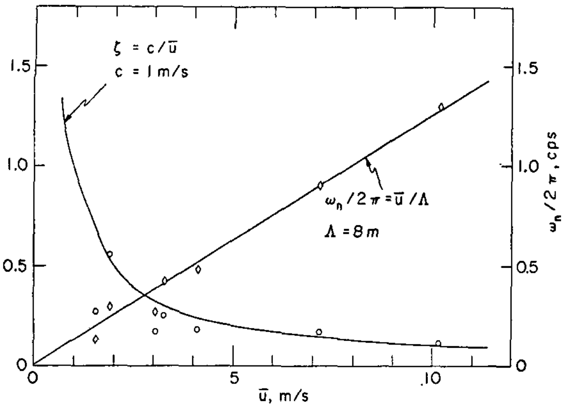

FIG. 2. Observed natural angular frequency ω_n (diamonds) and damping ratio ζ (circles) as a function of wind speed.

small angle either horizontally or vertically, and then released. The response to this type of step input was recorded for wind speeds varying from 3–10 m sec^{-1} with three replications for each wind speed and each direction of displacement. The initial displacement α_0, the first overshoot α_1, as well as the period T_f of the oscillations were inferred from the trace of a fast recorder.

3. Vane response

If the vane is assumed to behave as a damped second-order system (MacCready and Jex, 1964), the ratio of successive displacements on either side of the equilibrium position is given by

$$\frac{\delta}{2}=\ln\left(\frac{\alpha_n}{\alpha_{n+1}}\right)=\ln\left(\frac{\alpha_0}{\alpha_1}\right)=\frac{\pi\zeta}{(1-\zeta^2)^{\frac{1}{2}}}, \tag{1}$$

where $\zeta=\omega_d/(2\omega_n)$ is the damping ratio, ω_d being the damping coefficient for the system and ω_n its natural or eigen frequency. The constant δ is often called the logarithmic decrement.

The ratio of the frequency of the damped oscillations to that of the undamped can be written as

$$\frac{\omega_f}{\omega_n}=(1-\zeta^2)^{\frac{1}{2}}. \tag{2}$$

Through (1) and (2) the parameters ζ and ω_n can be derived from measurements of α_0/α_1 and $\omega_f=2\pi/T_f$. The results obtained for the BNL bivane are shown in Fig. 2.

The low values of ζ characteristic of many early vanes used in turbulence investigations mean a pronounced resonance peak in the transfer functions. For the BNL vane, ζ was found to be inversely proportional to the wind speed, which is unfortunate because it means an increasing resonance peak and thus a decreasing resolution (in terms of wavelength) with increasing wind speed. Since ω_n was found to be directly proportional to the wind speed, as was expected, it is a curious consequence of the said inverse proportionality that the damping coefficient ω_d is constant and independent of the wind speed. This indicates that mechanical friction rather than aerodynamic damping is responsible for the damping of the vane (Wieringa, 1967).

4. Correction of variances

According to Busch *et al.* (1968) the spectrum of the vertical velocity component may be represented by

$$\frac{nS_w(n)}{u^{*2}}=\frac{Af/f_m}{(1+1.5f/f_m)^{5/3}}, \tag{3}$$

where A is constant and $f=n/(z\bar{u})$ is the reduced frequency, n being the frequency (cps), z the height of observation, $\bar{u}$ the mean wind speed, and f_m the reduced frequency at which the logarithmic spectrum, $nS_w(n)$, has its maximum.

By logarithmic integration of (3) we obtain

$$\frac{\overline{w'^2}}{u^{*2}}=\frac{A}{1.5}\int_0^\infty(1+y)^{-5/3}dy=A. \tag{4}$$

The energy transfer function for the vane may be written as

$$T_1(\omega)=\{[1-(\omega/\omega_n)^2]^2+4\zeta^2(\omega/\omega_n)^2\}^{-1}. \tag{5}$$

If we let $\Lambda=2\pi\bar{u}/\omega_n$ denote the distance constant, i.e., the wavelength corresponding to the frequency of resonance of the undamped system, (5) is transformed into

$$T_1(\omega)=\{[1-(c_2y)^2]^2+4\zeta^2(c_2y)^2\}^{-1}, \tag{6}$$

with

$$c_2=\Lambda f_m/(1.5\,z) \quad\text{and}\quad y=1.5f/f_m.$$

Since f_m appears to be constant in neutral and unstable atmospheres, c_2 is only variable with height for a given vane.

The data obtained at Brookhaven were recorded every 0.6 sec. Before computation of statistics, the data were block-averaged over 10 readings, which corresponds to an averaging time $\Delta t=6$ sec. The transfer function for this type of numerical filter can be closely approximated by

$$T_2(\omega)=\left[\frac{\sin(\Delta t\omega/2)}{\Delta t\omega/2}\right]^2.$$

Using the notation introduced above together with $\zeta=c/\bar{u}$, we arrive at

$$T_2(\omega)=\left[\frac{\sin(\pi\Delta tcc_2y/\Lambda\zeta)}{(\pi\Delta tcc_2y/\Lambda\zeta)}\right]^2, \tag{7}$$

where for the BNL vane $\Lambda\approx8$ m and $c\approx1$ m sec^{-1}.

The measured variances can then be written as

$$\frac{\sigma_w^2}{u^{*2}}=\frac{A}{1.5}\int_0^\infty (1+y)^{-5/3}T_1(\omega)T_2(\omega)dy,$$

and the true variances as

$$\frac{\overline{w'^2}}{u^{*2}}=\frac{1}{K}\left(\frac{\sigma_w}{u^*}\right)^2,$$

where

$$K=\frac{2}{3}\int_0^\infty (1+y)^{-5/3}T_1(\omega)T_2(\omega)dy. \tag{8}$$

For the measurements referred to, the heights of observation were 23, 46 and 92 m, corresponding to values of c_2 equal to 0.070, 0.035 and 0.018, respectively, if $f_m=0.3$ (Busch and Panofsky, 1968; Busch *et al.* 1968). The results of numerical integrations of (8) using (6) and (7) are shown in Figs. 3 and 4. In Fig. 3 the correction factor K is plotted against the damping ratio, while it is plotted as a function of wind speed in Fig. 4. Also plotted in Fig. 3 is the correction factor for vane response only, i.e., for $T_2=1$ (no block-averaging).

In the range of values for ζ (or $\bar{u}$) generally encountered in the atmosphere, the overshoot of the vane will tend to counteract the effect of the high-frequency cutoff inherent in any wind vane. It is seen that this counteraction, while growing more pronounced as the damping ratio decreases, is more than compensated by the rapid high-frequency cutoff due to the block-averaging, the effect of which becomes increasingly severe. The net effect is a systematic underestimation of the variances.

Note also that the greater the height the less the correction will be, since, as the ratio Λ/z decreases, the two filters will allow a greater percentage of the total energy to pass for a given value of the damping ratio.

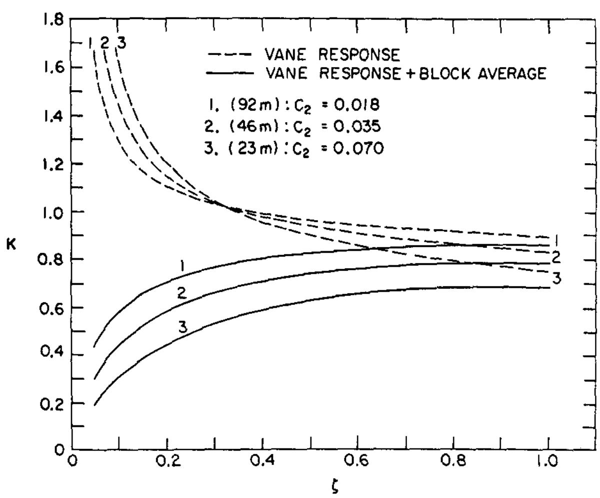

FIG. 3. Correction factors for velocity variances as functions of the damping ratio and height of observation.

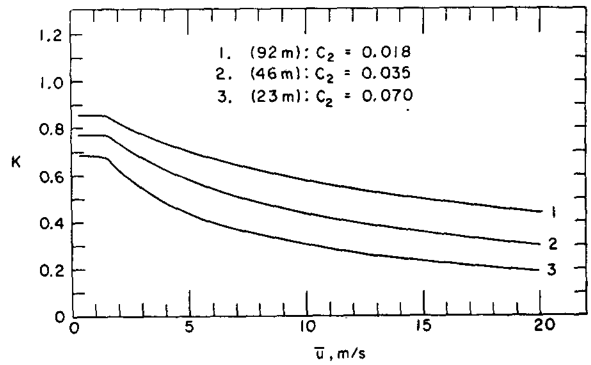

FIG. 4. Correction factors for velocity variances as functions of wind speed and height of observation.

5. Correction of friction velocities

So far we have assumed that the filtering has a negligible effect on the cross covariance between the longitudinal and the vertical velocity fluctuation $u^{*2}=-\overline{u'w'}$. This is not necessarily the case, although the error is expected to be minor because of the rapid decrease of the cospectrum at high frequencies.

It is readily shown that if the stochastic signals $u'(t)$ and $w'(t)$ are passed through linear filters, the complex cross-covariance spectrum of the outputs can be written as

$$\varphi_{uw}(\omega)=T_u^*(\omega)T_w(\omega)[Co_{uw}(\omega)+iQ_{uw}(\omega)], \tag{9}$$

where Co_{uw} is the cospectrum and Q_{uw} the quadrature spectrum of the input signals, and T_u and T_w are the Fourier transforms of the unit-step response functions pertaining to the filters working on $u'(t)$ and $w'(t)$, respectively. The asterisk denotes the complex conjugate.

At Brookhaven the longitudinal velocity components were measured by means of Aerovane propellers, which to a good approximation function as first-order linear filters with transfer functions of the form

$$T_3(\omega)=\frac{1-i\omega T_0}{1+(\omega T_0)^2}.$$

The distance constant $\Lambda_0=T_0\bar{u}\approx 7.5$ m (Mazzarella, 1954).

The vertical velocity components were measured by means of BNL bivanes, which have transfer functions of the form

$$T_4(\omega)=\frac{1-(\omega/\omega_n)^2-i2\zeta(\omega/\omega_n)}{[1-(\omega/\omega_n)^2]^2+4\zeta^2(\omega/\omega_n)^2}.$$

The transfer functions appearing in (9) may then be written as

$$\left.\begin{aligned}T_u^*(\omega)&=T_2^{1/2}(\omega)T_3^*(\omega)\\ T_w(\omega)&=T_2^{1/2}(\omega)T_4(\omega)\end{aligned}\right\}.$$

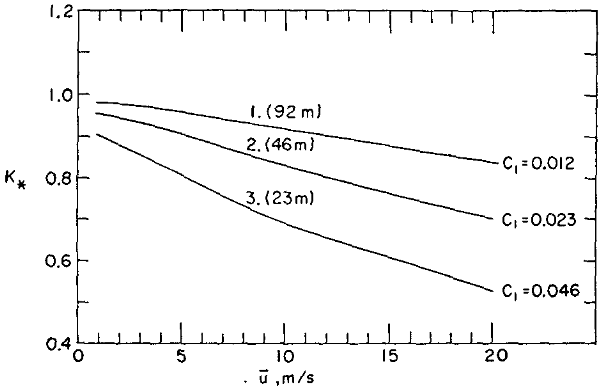

FIG. 5. Correction factors for Reynolds stresses as functions of wind speed and height of observation.

Considering only the real part of φ_{uw} in (9), after some manipulation we obtain

$$\mathrm{Re}[\varphi_{uw}]=\left[\frac{\sin(\Delta t\omega/2)}{\Delta t\omega/2}\right]^2\frac{1}{1+(\omega T_0)^2}$$
$$\times\frac{1}{[1-(\omega/\omega_n)^2]^2+4\zeta^2(\omega/\omega_n)^2}$$
$$\times\{[1-(\omega/\omega_n)^2+2\zeta T_0\omega_n(\omega/\omega_n)^2]Co_{uw}(\omega)$$
$$-\omega T_0[1-(\omega/\omega_n)^2-2(\zeta/T_0\omega_n)]Q_{uw}(\omega)\},\quad (10)$$

where

$$u_m{}^{*2}=\int_0^\infty \mathrm{Re}[\varphi_{uw}(\omega)]d\omega$$

is the measured friction velocity squared. The cospectrum is defined so that

$$u^{*2}=\int_0^\infty Co_{uw}(\omega)d\omega.$$

From (10) it is seen that the quadrature spectrum contributes to the measured cross covariance. This is due to the difference in the phase shifts introduced by the two filters. Correction of the measured cross covariances therefore requires knowledge about the quadrature spectra which is not available. Notwithstanding this difficulty, it is assumed that the part of the correction involving the quadrature spectrum is negligible. Arguments may be produced indicating that this assumption leads to errors in u^{*2} of less than 10%. These arguments, however, await more careful experimental support.

Integration of (10) yields

$$K_*=\frac{u_m{}^{*2}}{u^{*2}}=\frac{5}{3}\int_0^\infty\left\{\frac{1}{(1+y)^{8/3}}\left[\frac{\sin(\pi\Delta t c c_1 y/\Lambda\zeta)}{\pi\Delta t c c_1 y/\Lambda\zeta}\right]^2\right.$$
$$\left.\times\left[\frac{1}{1+(c_1\eta y)^2}\right]\frac{1-(c_1y)^2[1-2\eta\zeta]}{[1-(c_1y)^2]^2+4\zeta^2(c_1y)^2}\right\}dy,\quad (11)$$

where we have used (Panofsky and Mares, 1968)

$$\frac{nCo_{uw}(n)}{u^{*2}}=\frac{f/f_c}{(1+0.6f/f_c)^{8/3}},$$

and the notation

$$\left.\begin{array}{l}\eta=T_0\omega_n=2\pi\Lambda_0/\Lambda\\ c_1=f_c\Lambda/(0.6z)\end{array}\right\}.$$

For $f_c=0.08$, as recommended by Panofsky and Mares, we obtain values of c_1 equal to 0.046, 0.023 and 0.012, corresponding to heights of observation of 23, 46 and 92 m, respectively.

The results of numerical integrations of (11) are shown graphically in Fig. 5. The systematic underestimation of the friction velocity appears to be quite significant in case of low heights or high winds.

6. Results

For 58 sets of runs taken under neutral and unstable conditions and classified according to the Brookhaven gustiness classification (Singer and Smith, 1953), the normalized variances of the vertical velocity component were corrected and averaged for each stability class and height.

The group-averaged wind speeds and modified Monin-Obukhov-Lettau stability parameters z/L' are summarized in Table 1. The nondimensional variances are displayed in Table 2. Apparently they vary slightly but systematically with height and stability. Except for the upper height and extreme instability, however, it is disputable whether this variation can be said to be

TABLE 1. Group-averaged wind speeds $\bar{u}$ (m sec^{-1}) and stability parameters z/L'.

Height (m)	Gustiness class* C $\bar{u}$	C $-z/L'$	B$_1$ $\bar{u}$	B$_1$ $-z/L'$	B$_2$ $\bar{u}$	B$_2$ $-z/L'$
23	8.68	0.003	5.44	0.063	4.00	0.36
46	11.05	—	6.99	—	4.55	—
92	13.74	—	8.38	—	5.23	—

* C(9 runs): Neutral or nearly neutral atmosphere.
B$_1$(35 runs): Moderately unstable atmosphere.
B$_2$(14 runs): Very unstable atmosphere.

TABLE 2. Average* values of $\overline{w'^2}/u^{*2}$

Height (m)	Gustiness class C	B$_1$	B$_2$
23	1.43 (0.14)	1.39 (0.32)	1.44 (0.32)
46	1.38 (0.14)	1.44 (0.42)	1.53 (0.48)
92	1.52 (0.20)	1.63 (0.46)	2.50 (1.18)

* Figures in parentheses are standard deviations.

significant. The average value, excepting data representing B_2 cases at 92 m, is 1.48 with a standard deviation of 0.38.

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