The Effect of Antenna Sidelobes on Multiple-Parameter Radar Measurements

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ABSTRACT

Multiple-parameter radar measurements involve the combination of several different radar measurements, such as reflectivity measurements at different frequencies or polarizations, into a single parameter. Prediction of the effects of antenna sidelobes on such measurements can be rather involved since each component measurement is subject independently to sidelobe effects. This paper develops a simple model that can be used to examine sidelobe contributions to multiple-parameter radar measurements and identifies the factors controlling the magnitude of the sidelobe contribution.

1. Introduction

All radar measurements can be classified as single-parameter or multiple-parameter. To date, most meteorological studies have employed single-parameter measurements. For example, the vast majority of radar observations involve only simple estimates of the equivalent reflectivity factor based on the amount of power returned to the radar. Although Doppler radars measure additional parameters, they are normally analyzed and recorded as separate data fields and would still be classed as single-parameter data. In a true multiple-parameter measurement, two or more separate measurements are combined into a *single* new parameter. Most frequently, these multiple-parameter measurements involve measurements at different frequencies or different polarizations.

Perhaps the simplest form of a multiple-parameter measurement (and the only one to be discussed in this note) is the ratio of the equivalent reflectivity factor measured at two different wavelengths or polarizations. If Z_α and Z_β are the two estimates of the reflectivity factor (mm^6 m^{-3}), then their combination can be termed ξ, where

$$\xi = Z_\alpha/Z_\beta. \quad (1)$$

If Z_α is the *S*-band (10 cm wavelength) reflectivity factor and Z_β is the attenuation corrected *X*-band (3 cm wavelength) reflectivity factor, then ξ can be used to detect the presence of large (i.e., non-Rayleigh) scatterers (Atlas and Ludlam, 1961). In this usage it has been termed the *hail signal* and has come to be identified with the symbol Y or Y'. Alternatively, if Z_α is the reflectivity factor estimated from measurements at horizontal linear polarization and Z_β is estimated at vertical polarization, then ξ is a measure of an effective mean shape of the scatterers. In this context, ξ is usually expressed in dB units and has been termed Z_{DR} (Seliga and Bringi, 1976). As a class, any multiple-parameter variable defined by (1) can be described as a differential reflectivity measurement, since the ratio (when expressed in dB units) can be obtained by simple subtraction of Z_β from Z_α, so long as the individual reflectivity factors are first converted to a decibel scale.

In recent papers, Rinehart and Tuttle (1982, 1984) have discussed some of the problems that mismatched antenna patterns can introduce in dual-wavelength processing. While several aspects of their analysis need to be viewed with caution (Jameson and Heymsfield, 1984), their work is useful in that it highlights some of the sensitivities of multiple-parameter radar measurements to sidelobes. This note reexamines the issue of sidelobe effects and attempts to place the discussion into a more general framework.

2. A simple model

If the full antenna pattern is known, the evaluation of the effects of the sidelobes on radar measurements is basically straightforward. Such an evaluation, however, quickly becomes so enmeshed in the interaction between the beam geometry and the three-dimensional reflectivity field under consideration that it is easy to lose sight of the main influences over the magnitude and even the direction of the sidelobe contributions. In this context, it is useful to introduce a simplification to permit analytic examination of sidelobe effects on multiple-parameter radar measurements. For ease in development, the model will be introduced using dual-wavelength nomenclature. The same formulation, however, can be directly applied to other differential measurements such as Z_{DR}.

In this model, the area illuminated by the radar is divided into two parts. The first part (area 1) includes the main lobe of the radar. This area, which will generally include some of the sidelobes as well, is assumed to be filled with a uniformly distributed array of scatterers having S- and X-band reflectivity factors Z_{s1} and Z_{x1}, respectively. These quantities can be combined to define an appropriate hail signal for the region, $Y_1 = Z_{s1}/Z_{x1}$. The remaining area (area 2) is also assumed to be filled with a uniformly distributed array of scatterers. These scatterers exhibit S- and X-band reflectivity factors of Z_{s2} and Z_{x2}. As was done for area 1, these reflectivity factors can be combined to define an appropriate hail signal for area 2, $Y_2 = Z_{s2}/Z_{x2}$.

With these assumptions, the value of Z actually measured by the S-band system can be expressed as

$$Z_{sm} = \frac{Z_{s1} \iint_{\text{area 1}} I_s(\theta, \phi) d\theta d\phi + Z_{s2} \iint_{\text{area 2}} I_s(\theta, \phi) d\theta d\phi}{\iint I_s(\theta, \phi) d\theta d\phi}, \tag{2}$$

where $I_s(\theta, \phi)$ is the two-way radiation pattern of the antenna expressed in terms of azimuth (θ) and elevation (ϕ) relative to the beam axis (Donaldson, 1964). In actual practice, the evaluation of the normalizing integral in the denominator of (2) is limited to the main lobe. Since the first integral in the numerator will be dominated by the main lobe contributions, (2) can be simplified to

$$Z_{sm} = Z_{s1} + f_s Z_{s2}, \tag{3}$$

where

$$f_s = \iint_{\text{area 2}} I_s(\theta, \phi) d\theta d\phi \Big/ \iint_{\substack{\text{main}\\ \text{lobe}}} I_s(\theta, \phi) d\theta d\phi.$$

The term f_s is simply a measure of the relative sidelobe intensity in area 2.

The corresponding expressions for the measured X-band reflectivity factor and dual-wavelength hail signal are

$$Z_{xm} = Z_{x1} + f_x Z_{x2}, \tag{4}$$

$$Y_m = Y_1 + \frac{f_s R_s (R_f Y_2 - Y_1)}{R_f R_y + f_s R_s}, \tag{5}$$

where

$$f_x = \iint_{\text{area 2}} I_x(\theta, \phi) d\theta d\phi \Big/ \iint_{\substack{\text{main}\\ \text{lobe}}} I_x(\theta, \phi) d\theta d\phi,$$

$R_s = Z_{s2}/Z_{s1}$, $R_y = Y_2/Y_1$ and $R_f = f_s/f_x$. In (3), (4) and (5), the second term on the right represents the sidelobe contribution to the measurement. While the sidelobe contributions to the individual reflectivity measurements are relatively simple, the effect on the hail signal is considerably more complex. This complexity can be reduced if the sidelobe contribution is reexpressed in terms of a multiplicative factor

$$Y_m = Y_1 \left(\frac{R_f R_y (1 + f_s R_s)}{R_f R_y + f_s R_s} \right), \tag{6}$$

which simultaneously facilitates presentation of the sidelobe contributions in dB units. This approach effectively reduces the sidelobe effect to a simple function of the terms $f_s R_s$ and $R_f R_y$. The variables f_s and R_f represent a measure of the sidelobe intensity and mismatch, while R_s and R_y are merely the ratios of the area 1 and area 2 reflectivities and hail signals. Figure 1 uses (6) to illustrate the possible magnitude of the sidelobe contributions affecting the dual-wavelength hail signal. This figure is completely general and can be used for any configuration of the 2-area model and for any appropriate radar system. In examining Fig. 1, it is interesting to note that the sidelobe contribution to the S-band reflectivity factor can be expressed as

$$Z_{sm} = Z_{s1}(1 + f_s R_s). \tag{7}$$

This means that the x-axis can be directly labeled in terms of the sidelobe contribution (in dB) to the S-band reflectivity (see scale at the top of the figure). While there are conditions under which the sidelobe effects can grow quite large, sidelobe contributions are not always a problem. The shaded region of Fig. 1, for example, indicates the conditions for which the sidelobe contributions are less than 1 dB and may generally be ignored.

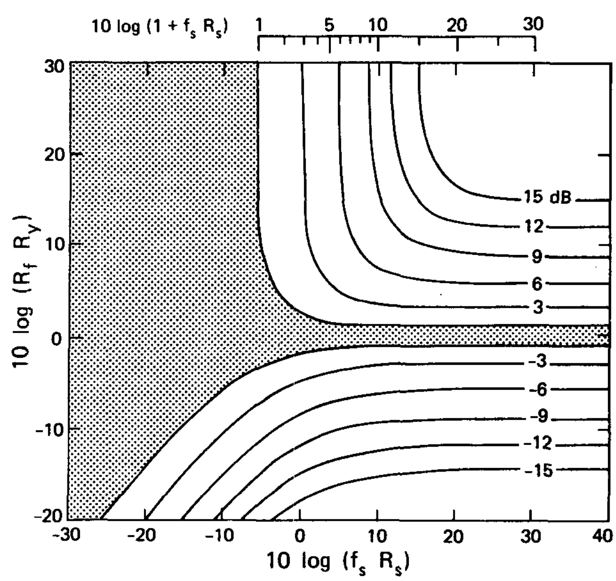

FIG. 1. Sidelobe contributions (dB) to the dual-wavelength hail signal in a two-area storm model as a function of sidelobe strength (f_s) and mismatch (R_f) and the differences in reflectivity factor (R_s) and hail signal (R_y) between areas 1 and 2. The shaded region indicates the range of conditions for which the absolute value of the sidelobe contribution is less than 1 dB.

To illustrate the use of both this figure and the 2-area model, assume that there is an intense shaft of precipitation beginning slightly beyond the edge of the main lobe and having a reflectivity factor 40 dB higher than the surrounding area. This precipitation shaft will comprise area 2, while an adjacent region of less intense precipitation will be considered area 1 ($R_s = 10^4$). Furthermore, assume that the precipitation shaft (area 2) is largely composed of hail, producing a strong dual-wavelength hail signal of 12.0 dB ($Y_2 = 16$), while the surrounding areas are hail free ($Y_1 = 1$, $R_y = 16$). If the full S- and X-band antenna patterns are known, it is easy to evaluate the relative sidelobe intensities in area 2 (f_s and f_x) for any location of the beam axis relative to the boundary of the hail shaft. In this example, we will simply assume that $f_s = 10^{-4}$ and that the sidelobes for the X-band antenna are twice as strong as those for the S-band antenna ($R_f = 0.5$). For this set of conditions, the sidelobe contributions would result in an overestimate of the S-band reflectivity factor in area 1 by 3.0 dB, while the hail signal would be overestimated by 2.5 dB. Equations (6) and (7), however, are appropriate only if both areas 1 and 2 contain detectable concentrations of scatterers. If area 1 were free of scatterers, then $Y_m = R_f Y_2$ and $Z_{sm} = f_s Z_{s2}$. Using the conditions specified in the preceding example ($R_f = 0.5$ and $Y_2 = 16$), the power returned from the sidelobes would then produce a measured hail signal of 9.0 dB. This is, of course, precisely the asymptotic value reached at the extreme right-hand side of Fig. 1, if we assume that $Y_1 = 1$, since $Y_m/Y_1 \rightarrow R_f R_y$ as $f_s R_s \rightarrow \infty$.

Even though this model was introduced using dual-wavelength nomenclature, it applies to Z_{DR} as well. If Z_v and Z_h represent the reflectivity factors at vertical and horizontal polarization, respectively, we can define a new variable $\zeta = Z_h/Z_v$. In this case, $Z_{DR} = 10 \log\zeta$ and (6) can be rewritten as

$$\zeta_m = \zeta_1\left(\frac{R_f R_\zeta(1 + f_h R_h)}{R_f R_\zeta + f_h R_h}\right), \qquad (8)$$

where $R_h = Z_{h2}/Z_{h1}$, $R_\zeta = \zeta_2/\zeta_1$, $R_f = f_h/f_v$, and f_v and f_h are the integrated sidelobe intensities at vertical and horizontal polarization. The Z_{DR} measurements, however, have a smaller natural range of values than hail signal measurements and require greater precision. Accordingly, Fig. 1 is of limited use in evaluating sidelobe effects on Z_{DR} measurements. Figure 2 gives an expanded view of the portion of Fig. 1 that is relevant for Z_{DR} measurements. As in Fig. 1, the shaded region indicates the conditions under which sidelobe effects can be safely ignored. Fortunately, this encompasses most of the conditions expected in the interior of real storms. The most serious potential for sidelobe-induced errors would be at the edges or top of severe storms, where sharp discontinuities and large gradients may exist. These are, of course, exactly the regions where sidelobe contributions have long been recognized to be a problem.

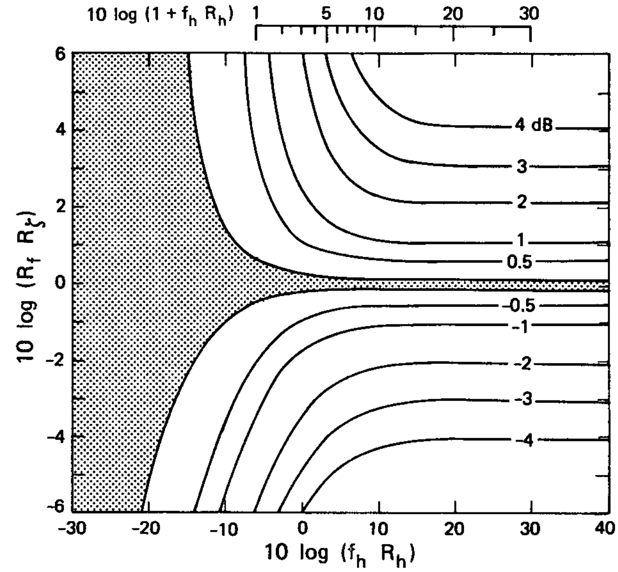

FIG. 2. Sidelobe contributions (dB) to Z_{DR} data in a two-area storm model as a function of sidelobe strength (f_h) and mismatch (R_f) and the differences in reflectivity factor (R_h) and $Z_{DR}(R_\zeta)$ between areas 1 and 2. The shaded region indicates the range of conditions for which the absolute value of the sidelobe contribution is less than 0.1 dB.

Figures 1 and 2 show that if the hail signal (or Z_{DR}) is assumed to be identical in areas 1 and 2 ($R_y = 1$) and the sidelobes are well matched ($R_f = 1$), then the sidelobe contributions will not bias the multiple-parameter measurement. Conversely, any mismatch in the sidelobes under these conditions will show up as a sidelobe-induced perturbation in the signal, even though the field was actually uniform. This was the case ($R_y = 1$, $Y_1 = Y_2 = 1$) examined by Rinehart and Tuttle (1982, 1984). In the more general case where the hail signal is not uniform ($R_y \neq 1$), however, matching the sidelobes at S- and X-bands will not eliminate the sidelobe contribution (see Fig. 1). Similarly, in dual-polarization studies using a single antenna, the sidelobes at vertical and horizontal polarization should be quite similar, but sidelobe effects can still contaminate the Z_{DR} data (Fig. 2).

3. Discussion

Antenna sidelobes can be troublesome in single-parameter radar measurements, but are conceptually simple. Multiple-parameter measurements, on the other hand, are more involved. At least two different measurements are needed and each measurement has the potential for inconsistencies in the magnitude or location of the sidelobes. Furthermore, the essence of the multiple-parameter measurement is the difference between the reflectivity fields at different wavelengths or polarizations. This means that the sidelobe contribution to each measurement used in the multiple-parameter analysis will be different, even if the beam patterns are identical.

Concern over the possible effects of mismatched sidelobes led Rinehart and Tuttle (1982, 1984) to suggest that specially designed antennae having matched sidelobes may be necessary for dual-wavelength analyses. While it is apparent that sidelobe effects cannot be eliminated by matching the antenna patterns, the goal of having perfectly matched sidelobes is well rooted in the fundamental assumption of multiple-parameter analysis: that the measurements to be combined are obtained while looking at the same volume of scatterers. Indeed, it is clearly desirable that the main lobes be closely matched for almost all multiple-parameter studies. Precise matching of the sidelobes as well, however, may not gain a great deal since sidelobe mismatch, or lack of mismatch, is only one aspect of the sidelobe problem. To the extent that a significant design effort would be required to match both main and sidelobes, generally it will be better to attempt to simply minimize the magnitude of the sidelobes irrespective of how well they match.

While reduction in sidelobe intensities should be the goal of new antenna systems, there may be some situations in which intentional sidelobe mismatch may be an advantage. For example, if the dual-wavelength hail signal is to be used to try to resolve the boundary of a hail shaft, sidelobe mismatch (larger sidelobes at X-band than at S-band) or even some mismatch in the main lobe (X-band broader than S-band) may improve the measurement. Intentional use of such mismatched beams, however, would require careful consideration of possible advantages and disadvantages before implementation.

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