Comments on "VHF Radar Measurements of In-Beam Incidence Angles and Associated Vertical-Beam Radial Velocity Corrections"

PETER T. MAY

Bureau of Meteorology Research Centre, Melbourne, Australia

17 January 1992

The paper by Larsen and Rottger (1991, hereafter referred to as LR) analyzes the apparent tilt angles of scattering layers in the troposphere using interferometric techniques applied to a mesosphere–stratosphere–troposphere (MST) radar employing the spaced antenna technique. The technique employs pairs of receiving antennas to measure the horizontal wind, while the vertical wind is measured by the line-of-site Doppler shift as in other Doppler radars. As shown by Rottger and Vincent (1978), these radars can also be used as interferometers to investigate the radar scattering. The angle of arrival of the signal can be calculated using the phase of the returned signals at different receiving antennas. The mean phase difference between the returned signals of the pair is measured from the cross-correlation function of the time series at zero lag. This phase difference is then converted to an angle of arrival using the spatial separation of the receiving arrays [tilt angle $\delta = \sin^{-1}(\lambda\Delta\phi/2\pi d)$ where $\Delta\phi$ is the phase difference between the signals at the two antennas, d is the antenna separation, and λ is the radar wavelength]. The interferometer technique and the implications of such measurements are important, because that are many profilers studying vertical wind motions that cannot correct for such tilts and thus may be ignoring an important source of error. Vertical velocity measurements are one of the primary research applications of 50-MHz wind profilers. However, the angle-of-arrival measurements presented in the paper are difficult to unambiguously interpret in terms of tilted layers. This is because the radar in question used spatially separated transmitting and receiving antennas. Therefore, this type of spaced antenna radar is a special case of a bistatic radar where the transmitting and receiving antenna dimensions are dramatically different. This has important implications for both the measurement of the tilt and also the "leakage" of the horizontal wind into the vertical beam.

The leakage of the horizontal wind component into the measured apparent vertical motions with such bistatic radars was discussed by Vincent et al. (1987). As was also shown by LR, this means that some correction must be made to the observed "vertical" motions in order to obtain useful vertical velocity fields with such radars. Following Vincent et al., suppose there exists a layer of scatterers at an altitude z (Fig. 1). We have the transmitting and receiving antennas located at positions $-x'$ and x', respectively. Then the pathlength P to a scatterer at position ζ is given by

$$P = [(x' + \zeta)^2 + z^2)]^{1/2} + [(x' - \zeta)^2 + z^2)]^{1/2}.$$

The phase ϕ of the scattered signal is $2\pi P/\lambda$. Differentiating the phase with time (assuming no vertical motions) to obtain the Doppler shift of the signal we obtain

$$\frac{d\phi}{dt} = \frac{2\pi}{\lambda}\{[(x' + \zeta)^2 + z^2)]^{-1/2}(x' + \zeta) - [(x' - \zeta)^2 + z^2]^{-1/2}(x' - \zeta)\}\frac{d\zeta}{dt},$$

but $d\zeta/dt$ is just the horizontal wind velocity. The backscattered signal will be the sum of the signals from all the scatterers weighted by the illumination of the antenna polar diagrams (centered over $-x'$ and x') and the "aspect sensitivity" of the scatterers. For an extreme case where one of the polar diagrams (e.g., the one at x') is very narrow compared with the other and the scatter is isotropic, the mean Doppler shift, $d\phi/dt$, will be given by $= (2\pi/\lambda)(4x'^2 + z^2)^{-1/2} \times (2x')d\zeta/dt$. On the other hand, if the scatter is specular (mirrorlike), then the scatter will be dominated by contributions from $x = 0$ and there will be no leakage. The data of Vincent et al. suggested that their observations were midway between these extremes.

Figure 2 shows curves for the leakage versus height for a radar similar to the one used in LR for isotropic scatter, where the angular dependence of the scatter is such that it falls off with angle as does the narrowest radar antenna and where the scatter falls off twice as fast. This is represented as an "apparent tilt angle."

Corresponding author address: Dr. Peter T. May, Bureau of Meteorology Research Centre, GPO Box 1289K, Melbourne, 3001, Australia.

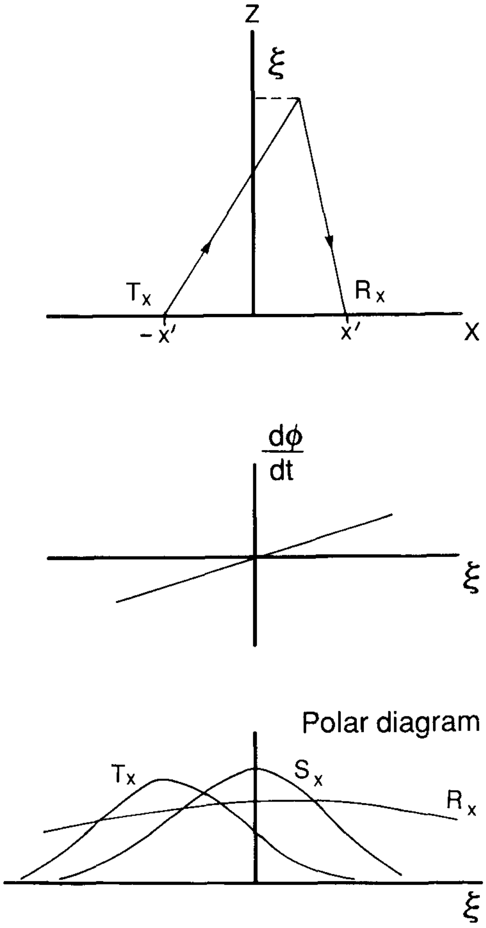

FIG. 1. The scattering geometry of a bistatic radar together with the amount of "leakage" of the horizontal velocity into the Doppler shift as a function of the position of the scatterers. The leakage is weighted by the polar diagram of the antennas (T_x, R_x) varying the illumination of the scatterers and the angular dependence of the scatter S_x. For isotropic scatter, S_x is constant, while for pure specular scatter, it is a delta function at $\xi = 0$ (after Vincent et al. 1987).

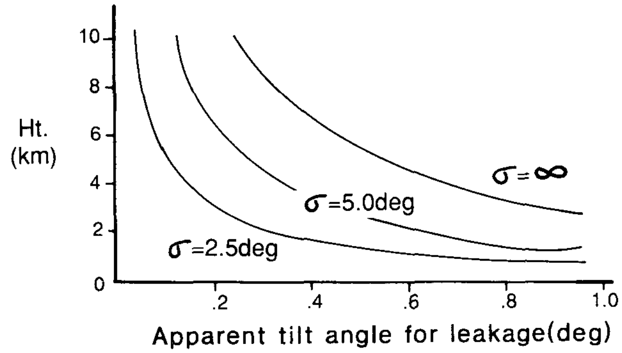

FIG. 2. Leakage versus height for a bistatic radar similar to the SOUSY radar in Germany in its spaced antenna mode. The transmitting beamwidth is 5°, the receiving array beamwidth is 15°, and the antenna separation is 60 m. The curves are for isotropic scatter ($\sigma = \infty$) and scatter where the aspect sensitivity is equivalent to the transmitting antenna beamwidth ($\sigma = 5°$) and half the beamwidth ($\sigma = 2.5°$).

Note that this is similar in magnitude and falls off with altitude in a similar manner to the data presented by LR (e.g., their Fig. 6).

The separation is also important for the phase at zero lag of the cross-correlation function of the received signals at pairs of antennas. It is this quantity that is used to infer tilt angles with LR's interferometer analysis. In order to demonstrate this, consider the case of a mirrorlike reflector. Then the pathlength P between the transmitting and a receiving antenna i separated by a distance $2x_i$ for scatter from an altitude z is equal to

$$P = 2(z^2 + x_i^2)^{1/2},$$

as is shown in Fig. 3. Now consider the pathlength difference, PLD, between two receiving antennas separated by $2x_1$ and $2x_2$ from the transmitting antenna, respectively. To a good approximation, this is given by

$$\mathrm{PLD} = \frac{x_1^2 - x_2^2}{z},$$

since $z \gg x_i$. This relative path difference leads to mean phase differences between the signals at the two antennas similar to those observed by LR. PLD corresponds to a phase difference $\Delta\phi = 2\pi/\lambda$ PLD, where λ is the radar wavelength. The tilt angle δ becomes $\delta = \sin^{-1}(\mathrm{PLD}/d)$, where d is the separation between the two receiving antennas as in LR. The phase difference will about double for isotropic scatter where the mean scattering point is from almost directly above the transmitting antenna (since its polar diagram is much narrower than the receiving antenna's).

The mean phase difference that can be expected has been calculated numerically assuming that the separation of the receiving antennas from the transmitting antenna was 50 and 70 m (i.e., $d = 20$ m). These values are similar to the SOUSY radar used by LR. Three curves are plotted as in Fig. 2. The results for specular

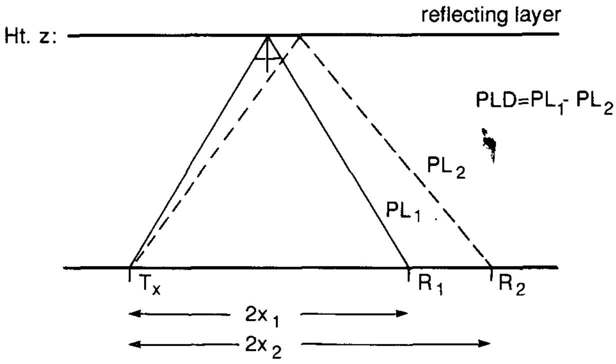

FIG. 3. Geometry for the interferometer measurements for a specular reflector.

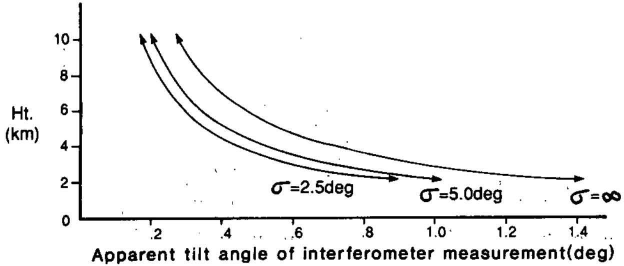

FIG. 4. Measured tilt angle for interferometer measurements versus height for similar scattering characteristics to Fig. 2. The beamwidths of the antennas are the same as for Fig. 2, but the two receiving arrays are separated from the transmitting array by 50 and 70 m (i.e., $d = 20$ m).

reflections are very close to the curve for the 2.5° scattering dependence; this is to be expected since the scattering angles are significantly less than the narrowest radar beamwidth. The apparent tilt angles have a similar magnitude to the tilts revealed by LR and similar vertical behavior in that they decrease with altitude. However, the detailed behavior of the phase difference is different from the leakage. Here the phase difference occurs for mirror reflectors where there is no leakage of the horizontal wind. The leakage is related to the mean separation of the receiving antennas from the transmitting antenna, while the phase difference is related to difference in the separation of the receiving antennas from the transmitting antenna. Note that if there was no true tilt of the scatterers, then Figs. 2 and 4 could be used to remove the effect of horizontal winds from the measurements. However, if the scattering layers had a real tilt in addition to the displacement effect, no simple solution is possible. These results imply that the success of using the interferometer tilt angle to correct the vertical motions may be somewhat fortuitous in this case.

These results may explain much of the tilt observed by LR in the east–west plane, but not in the north–south plane. We also note that in meteorological terms the tilt angles averaged over several days observed by LR are quite large: of the order of 5/1000. This slope is similar in magnitude to that expected of frontal surfaces. However, the persistence of the tilts over several days is puzzling.

As noted, interferometer measurements to measure the tilts of scattering layers have important implications for profilers using the Doppler technique. However, this technique should be applied to radars using collocated transmitting and receiving arrays in order to unambiguously interpret the angle of arrival measurements in terms of tilted scattering layers. An example of an existing radar system that can do this kind of measurement is the MU radar in Japan.

REFERENCES

Larsen, M. F., and J. Rottger, 1991: VHF radar measurements of in-beam incidence angles and associated vertical-beam radial velocity corrections. *J. Atmos. Oceanic Technol.*, **8**, 477–490.

Rottger, J., and R. A. Vincent, 1978: VHF radar studies of tropospheric velocities and irregularities using spaced antenna techniques. *Geophys. Res. Lett.*, **5**, 917–920.

Vincent, R. A., P. T. May, W. K. Hocking, W. G. Elford, B. H. Candy, and B. H. Briggs, 1987: First results with the Adelaide VHF radar: Spaced antenna studies of tropospheric winds. *J. Atmos. Terr. Phys.*, **49**, 353–366.