Directional Wave Spectra from a Swath Ship at Sea

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ABSTRACT
During the Surface Wave Dynamics Experiment (SWADE), the swath ship Frederick G. Creed was equipped with an array of wave staffs for the estimation of wave directional spectra. This paper reports on the first such estimates taken from a ship at sea. An algorithm for removing the effects of the ship motion, including those resulting from the Doppler shifting of observed frequencies, is presented along with some results from the SWADE experiment. A comparison with directional wave spectra taken from a nearby buoy shows the fidelity of the method.

1. Introduction
The accurate measurement of wave directional spectra is an important step toward understanding the dynamics of the evolution of the wave field. The Surface Wave Dynamics Experiment (SWADE), which took place off the Virginia coast between October 1990 and March 1991, was carried out with the goal of studying the temporal and spatial response of the wave field in open ocean conditions. This was to be achieved through the deployment of an array of buoys that would record wave and flux data continuously during the six-month period, and through the use of aircraft-based remote-sensing techniques during three intensive operating periods (IOPs) (see Weller et al. 1991).

High-resolution directional spectra critical to the study were to be obtained from a Spar buoy equipped with an array of capacitance wave gauges. Unfortunately, the Spar sank in late October 1990, just prior to the first IOP. As a replacement, the swath (small water plane area twin hull) ship Frederick G. Creed, operated by the Canadian Department of Fisheries and Oceans, was chartered and equipped to perform the high-resolution measurements. We report here the instrumentation and special analysis techniques employed to make the first in situ measurements of wave directional spectra from a ship at sea.

The Creed operated in the SWADE area during IOPs 2 (14–25 January 1991) and 3 (25 February–9 March 1991). Although the opportunity for six months of continuous high-resolution data collection was lost with the Spar, the mobility of the ship allowed for several innovations. For instance, it became possible to study the effects of currents and fetch on wave spectra. Also, cross-calibration of the various discus buoy and aircraft measurements was feasible.

The next section describes the Creed and the special equipment used on it during SWADE. Section 3 discusses the analysis required to obtain directional wave spectra from the ship-based measurements. In section 4 results are presented, and comparisons are made with spectra obtained from a neighboring buoy. Finally, the last section gives a summary of the conclusions.

2. Swath ship apparatus
The ship, 20 m long by 10 m wide, was designed to produce minimal flow disturbance at the water surface
and, because of its high design cruising speed, is much
better streamlined than typical ships (see Fig. 1). It is
thus an excellent air–sea interaction research platform.
Buoyancy is provided by two pontoons located below
the surface that are attached to the hull by two narrow
struts running the length of the ship. Engines located
in each pontoon allow for ship speeds of up to 13 m s⁻¹,
although for the purposes of SWADE speeds of 2–8
m s⁻¹ were typical.

The transformation of the swath ship into a mobile
research platform required the addition of a variety of
special equipment (see Fig. 1). Mounted on a bowsprit
2 m forward of the bow was the principal wave-sensing
apparatus that included an array of wave staffs em-
ployed to estimate the directional properties of the wave
field. The staffs were about 4 m ahead of the pontoons,
well out of the way of the ship’s wake. Two configu-
trations of wave staffs were employed during SWADE.
The initial configuration consisted of six capacitance
wave gauges arranged in a centered pentagon of 75-
cm radius. During the third IOP, this apparatus was
damaged beyond repair. It was replaced by a smaller
array of three staffs: two staffs across the bow 2 m apart
with a third 1 m ahead of the centerline of the first
two. For both arrays, the wave staffs were 4.5 m long
and constructed from 0.9-mm-diameter high-carbon
steel (piano) wire coated with Teflon. The wave staffs,
tensioned with strong springs, were held in place in a
support structure that intersected the surface down-
stream of the wave staffs. The forward motion of the
ship assured that any turbulence shed from the supports
was carried downstream, away from the gauges. This
was confirmed visually, from the foredeck of the ship.
Prior to the experiment various types of wires were
tested by dragging them through the water in the Can-
da Centre for Inland Waters (CCIW) towing tank.
The wire type chosen yielded the minimum error in
sensing the water level at speeds up to 4 m s⁻¹. The
arrays were designed so that wave staffs could be re-
placed at sea under relatively calm conditions. During
the cruise, wave-staff breakages due to either fatigue
(especially in extreme conditions) or dolphins (who
enjoyed swimming through the array) occurred at the
rate of about two staffs per day. Also mounted on the
bowsprit, about 2 m below the surface, was an acoustic
current meter. In addition to providing near-surface
turbulence data, the current meter yielded the mean
ship speed through the water. The current meter was
lost with the original array. The mean ship speed
through the water was then obtained from an acoustic
Doppler current profiler that produced 1-min current
averages along a vertical line in bins of 4 m.

As all velocity and displacement measurements were
made with the ship in motion, it was crucial that this
motion be recorded. To achieve this, a motion package
consisting of a pair of angular accelerometers and a
gimbaled gyroscope (each measuring pitch and roll)
and two triplets of linear accelerometers (each mea-

![Diagram of Swath ship Frederick G. Creed, showing SWADE equipment and coordinate system.](image)

**Fig. 1.** Swath ship Frederick G. Creed, showing SWADE equipment and coordinate system.
surging heave, surge, and sway) was installed at the bow, just aft of the bowsprit. Yaw was measured with a two-channel fluxgate magnetometer and a magnetic compass. Duplicate sensing of the ship motion allowed for the possibility of equipment failure. The ship's position and heading were obtained using a Global Positioning System (GPS) receiver and two magnetic compasses (digicourse and fluxgate). A mast mounted on the foredeck just ahead of the cabin held a variety of meteorological sensors, including anemometers, wet- and dry-bulb thermometers, and a hygrometer (see Katsaros et al. 1993).

All equipment was calibrated after completion of the experiment either at CCIW (anemometers, motion package, wave array, and compasses) or at the University of Washington (all meteorological apparatus). The compasses and K-Gill anemometer were also calibrated against the ship's gyrocompass and GPS receiver during several special data runs. During all the runs, each of the signals was filtered through one-pole resistor–capacitor (R-C) antialiasing filters with 5-Hz cutoff (3 dB) and sampled at 20 Hz prior to being stored on optical disks. (An inverse filter transfer function was applied to the data during the analysis.) Data quality was monitored during the runs, and adjustments and repairs were carried out as necessary.

Limitations on the material strength of the wave wires and on the fidelity of their measurements required the cruising speed to be limited to 4 m s⁻¹ when the wave staff array was deployed. Hence, two types of data runs were made: low-speed runs (typically 2–3 m s⁻¹) into the wind with the array deployed, and high-speed flux runs with the array out of the water. During both types of run, the ship speed and heading were kept constant. Here we are concerned only with the low-speed runs. A total of some 29 h of data suitable for the estimation of wave directional spectra was collected. The limited length of the wave wires restricted, to some degree, the wave amplitudes that could be measured from the ship. Although the position of the array in the water could be adjusted somewhat by using control surfaces on the pontoons to change the pitch angle of the ship, the practical operating limit for the collection of useful wave data was a significant height of less than 2.8 m.

3. Analysis

The rendering of the collected data into directional wave spectra involves two principal stages: the calculation of the surface displacement time series and application of a maximum likelihood technique to determine the spectral estimates from the time series. The procedure for carrying out the first stage follows that used for correcting flux measurements from aircraft, and has been documented for flux measurements from a ship at sea (e.g., Fujitani 1985; Katsaros et al. 1993).

The true surface displacement can be written as the sum of three components: the wave height as measured by a wave staff, \( z_m \); the displacement of the wave staff due to the translational motion of the ship, \( z_t \); and the displacement of the wave staff due to relative rotational motion between it and the point at which the translational motion is measured, \( z_r \). We describe below how each of these three terms is determined.

In calculating the surface displacement time series, one must first account for the fact that, although both the linear accelerometers and wave staffs are mounted in the moving reference frame of the ship (SR), one wants the final surface displacements in an earth-referenced (ER) system. The conversion from the moving SR system to the fixed ER one is carried out at each instant in time using the pitch, roll, and yaw (\( \theta \), \( \phi \), and \( \psi \), respectively) signals from the gyrocompass and magnetometer. For a gimballed gyrocompass, these signals are Euler angles and one can then, following Goldstein (1950), derive a transformation matrix \( T_{SE} \) between ship and earth coordinates—this is given in both Fujitani (1985) and Anctil et al. (1994). If \( a_S \) represents the translational accelerations (surge, sway, heave) measured by strapped-down accelerometers on the ship (i.e., SR), then the ER accelerations \( a_E \) are given by

\[
 a_E = T_{SE} a_S + g, \]

where \( g \) is the acceleration due to gravity, \((0, 0, -g)\). Here, \( a_E \) as given above represents the full three dimensional acceleration vector, and the three components of ship displacement are readily determined from \( a_E \) by double integration. For the present purposes, we are concerned only with the vertical component. Then

\[
z_{te} = \int \int [a_S \cdot (-\sin \theta, \cos \theta \sin \phi, \cos \theta \cos \phi)] \cdot dt \cdot dt
\]

represents the earth-referenced vertical displacement due to the translational motion of the ship. The above integrations are carried out in the frequency domain, employing a high-pass filter at 0.04 Hz. Similarly, the earth-referenced measured wave height is given by

\[
z_{me} = z_m \cos \theta \cos \phi.
\]

This expression differs from that above for ship displacement in that, while all three components of linear ship acceleration were recorded, the wave staffs only record displacement in the SR vertical direction.

Finally, the displacement due to the relative rotational motion is found by integrating the rotational velocity vector \( \Omega \times T_{SE} L_S \), where \( \Omega \) represents the angular velocity of the ship as measured by the gyrocompass and \( L_S \) the SR displacement between the accelerometers and water surface at each wave staff. Again, we are concerned only with vertical displacement, and this is given by
\[ z_{RE} = \int \left[ L_{E2}(-\dot{\theta} \sin \phi + \dot{\phi} \cos \theta \cos \psi) - L_{E1}(\dot{\theta} \cos \phi + \dot{\phi} \cos \theta \sin \psi) \right] dt, \]  

where a dot represents time differentiation, \( L_E = T_{SE} L_S \), and the numerical subscripts represent components of the vector. The two expressions in parentheses are, respectively, \( \Omega_1 \) and \( \Omega_2 \). We note that \( L_S \) is different for each wave staff in the array. The true surface elevation at each staff is then given by the sum of \( z_{mE} \), \( z_{fE} \), and \( z_{RE} \). In Fig. 2, an example of a recorded wave-staff signal, \( z_{mE} \), and fully corrected sea surface elevation is given. The relative contributions of the three expressions, \( z_{mE} \), \( z_{fE} \), and \( z_{RE} \), to a typical one-dimensional surface elevation spectrum are shown in Fig. 3.

The maximum-likelihood (ML) method, developed by Capon (1969), is employed to obtain the directional estimates from the surface elevation measurements. Consider an array of \( N \) wave staffs, each located at position \( \bar{x}_n \) in the array. The wave height time series measured at each staff are corrected for ship motion as described above and the \( N \times N \) cross-power spectral densities (CPSD) among the various signals are calculated. The ML estimate of the directional energy distribution at a given frequency \( \omega_m \) and wave direction \( \Theta_i \) is then given by

\[ S(\omega_m, \Theta_i) = \kappa \left| H(\omega_m, \Theta_i) \right|^2 \text{C}^{-1}(\omega_m) \times \left| H(\omega_m, \Theta_i) \right|^{-1} J(\omega_m, \Theta_i), \]

where \( \text{C} \) represents the \( N \times N \) CPSD matrix at frequency \( \omega_m \), and \( H \) the \( N \times 1 \) array of complex phase lags between the \( N \) staffs and the center of the array for waves of frequency \( \omega_m \) and wavenumber \( k \) coming from direction \( \Theta_i \), with \( H_n = \exp(ik \cdot \bar{x}_n) \). For convenience, we employ a directional vector with resolution or spacing \( d\Theta \) centered around the ship’s bow.

Here, \( \kappa \) and the asterisk represent the vector operations transpose and conjugate, respectively, and \( J(\omega_m, \Theta_i) \) the Jacobian described below. Here, \( \kappa \) is a normalizing factor defined so that the integrated total energy is equal to the average energy of the \( N \) one-dimensional spectra.

For a stationary array, \( k \) is easily and uniquely determined from the frequency \( \omega_m \) through the dispersion relation—it is independent of the propagation direction of the waves. When one considers a moving array, however, this is not the case. Although the wave-staff signals are corrected for ship accelerations and angular motion, the resulting spectra are Doppler shifted in frequency due to the forward velocity of the ship. Furthermore, the amount of Doppler shifting depends on the relative direction of travel of the ship and wave component. Due to the motion of the ship and therefore of the array, waves with an actual or intrinsic frequency \( \omega \) will be observed at an apparent frequency \( n \). The relationship between the two frequencies is given by

\[ n = \omega + kU \cos \alpha = (gk)^{1/2} + kU \cos \alpha, \]

where \( U \) is the ship speed and \( \alpha \) the angle between the ship direction and wave component. We take \( \alpha = 0 \) to correspond to waves traveling against the ship. The deep water dispersion relation has been assumed in writing the last equation. The function \( J(\omega_m, \Theta_i) \) introduced above is the Jacobian of the transform between apparent and intrinsic frequency:

\[ J(\omega_m, \Theta_i) = 1 + 2 \left( \frac{k}{\omega_m} \right) U \cos \Theta_i. \]

All waves traveling against the ship \((-\pi/2 < \alpha < \pi/2)\) are uniquely mapped onto an expanded Doppler-shifted frequency range, with wavenumbers given by
where \( u = U \cos \alpha \). However, as pointed out by Kats and Spevak (1981), the mapping between apparent frequency and wavenumber is not one to one over the full range of frequencies for waves traveling in the same direction as the ship ("following waves"). Hence, a portion of the wavenumber field cannot be unscrambled (see Fig. 4). The effects of this were minimized by pointing the ship into the wind (±20°), so that the observed wind sea was propagating against the ship and therefore shifted (uniquely) up in frequency. It is then assumed that short waves \( (k > g/4u^2) \) are unlikely to be traveling against the wind, and the low wavenumber \( (0 < k < g/4u^2) \) branch of the curve is used. At ship speeds of 4 m s\(^{-1}\) the inversion process is unique for following waves of periods 5.12 s and longer. Wave data were gathered at ship's speeds of 4 m s\(^{-1}\) and less. We note that for shallower depths, the depth-dependent dispersion relation is used, and the solution for wavenumber is carried out numerically.

Finally, although the wave-staff signals are corrected for the ship motion so that the true surface elevations are determined, the horizontal orientation of the array (i.e., heading of the ship) may also vary during a run. Typically, this variation is between 2° and 5° rms, so that a 5°–10° angular resolution is the best possible. Any large deviations or systematic changes in the ship heading during a run were noted and the runs flagged.

4. Results

In Fig. 5 we produce a typical directional spectrum (1840 UTC 22 January 1991) plotted as a polar contour plot. North and east appear, respectively, at the top and right-hand side of the plot, following standard meteorological convention. Grid lines are 30° apart, with frequencies every 0.1 Hz to a maximum of 0.4 Hz. The wave spectrum is shown in the direction of propagation with equally spaced contour lines normalized to the energy maximum. The data were collected using the six staff array, and the ML analysis carried out with an angular resolution \( d\theta \) of 10°. The average wind vector appears on the plot as an arrow pointing in the direction of the wind, with 0.1 Hz equivalent to a wind speed of 10 m s\(^{-1}\). The wind data are from a K-Gill anemometer mounted on the ship, with full accounting for the ship-induced motions—see Katsaros et al. (1993). The significant wave height \( H_s \) given in the figure caption is defined as four times the rms wave height.

These data were taken on the continental shelf some 50 km southeast of the Maryland coast at the end of a storm. About 6 h prior to the data collection, the wind and waves measured at a National Data Buoy Center (NDBC) SWADE discus buoy a further 50 km offshore peaked at about 14 m s\(^{-1}\) and 3.5-m significant wave height, respectively. Throughout the storm, the wind was offshore, varying from north-northwest to north. We note that the 1.47-m significant wave height recorded by the Creed reflects the fetch-limited conditions of the measurement site. The swath ship spectrum clearly shows an active wind sea with the higher-frequency components in the direction of the wind and the lower-frequency components following the coastline, at some 60° to the wind.

In Fig. 6, the spectrum multiplied by the fourth power of frequency \( f \) is plotted. This presentation emphasizes the high frequency or wind sea components that achieve an equilibrium spectral slope of \( f^{-4} \) (see

\[
k = \frac{g + 2un - (g^2 + 4ung)\sqrt{1}}{2u^2},
\]

(7)
Donelan et al. (1985) and allows one to study the response of the wind sea (at 0.2–0.4 Hz) to the wind forcing. One-dimensional spectra, both as measured (Doppler shifted) and as determined from the directional analysis, integrating over direction, appear on log–log axes in Fig. 7. The Doppler shift of the higher frequencies is clearly evident.

In Fig. 8, a spectrum from early 27 February 1991 is shown. During this time the Creed was situated over the shelf break about 4 km from Discus-North, one of the NDBC SWADE discus buoys. At the time of the measurements, the wind was blowing from the north-northwest, but as seen in Fig. 9, for most of the previous two days, the wind had been from the northeast direction. The directional spectrum reflects this changing wind with older (lower frequency) sea from the east and a prominent peak in the wind direction.

NDBC buoys are designed to measure directional wave spectra using heave, pitch, and roll data, with averaged CPDPS values transmitted hourly to satellite receivers. Although the signals are routinely analyzed by NDBC assuming \( \cos^2 \theta \) directional spreading (Longuet-Higgins et al. 1963), we use, for comparison purposes, an extension of the above ML technique for
5. Conclusions

We have demonstrated a means of obtaining in situ directional wave spectra from a moving ship in the ocean. As these measurements can be made simultaneously with others in the atmospheric and marine boundary layers, a small fast ship of this type provides a versatile platform for air–sea interaction studies.

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