Intercomparison of Single-Frequency Methods for Retrieving a Vertical Rain Profile from Airborne or Spaceborne Radar Data

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ABSTRACT

This paper briefly reviews several single-frequency rain profiling methods for an airborne or spaceborne radar. The authors describe the different methods from a unified point of view starting from the basic differential equation. This facilitates the comparisons between the methods and also provides a better understanding of the physical and mathematical basis of the methods. The application of several methods to airborne radar data taken during the Convective and Precipitation/Electrification Experiment is shown. Finally, the authors consider a hybrid method that provides a smooth transition between the Hitschfeld–Bordan method, which performs well at low attenuations, and the surface reference method, for which the relative error decreases with increasing path attenuation.

1. Introduction

The study of attenuation effects on radar returns goes back to the work of Hitschfeld and Bordan (Hitschfeld and Bordan 1954). Their paper shows that an unacceptably large error may result in the attenuation correction unless the radar constant is accurately determined and the parameters chosen in the model very closely represent the actual rain. Owing to these difficulties, the use of attenuating frequencies for meteorological radars has generally been avoided. A revival of interest in methods for attenuation correction has been promoted by the development of spaceborne weather radars. To achieve adequate spatial resolution with an antenna size of less than 2 m requires the use of relatively short attenuating wavelengths (i.e., less than 3 cm).

In this paper we formulate the problem in terms of estimating the true radar reflectivity factor $Z(r)$ from the apparent measured radar reflectivity factor $Z_m(r)$. Ambiguities associated with the conversion of $Z(r)$ into the rainfall rate $R(r)$ will be discussed later.

When there is attenuation, the radar equation becomes

$$P_r(r) = \frac{C|K|^2}{r^2} Z_m(r), \quad (1)$$

where

$$Z_m(r) = Z(r) \exp\left[-0.2 \ln 10 \int_0^r k(s)ds\right], \quad (2)$$

and

$$K = \frac{m^2 - 1}{m^2 + 2}. \quad (3)$$

In this paper, we use the following notation: $P_r(r)$ is the received power, r is the range from the radar, C is the radar constant, m is the complex index of refraction of the precipitating particles, and $k(r)$ is the specific attenuation (dB km^{-1}) (or attenuation coefficient). We will call the quantity $\int_0^r k(s)ds$ the attenuation. The attenuation to the surface, that is, $\int_0^{r_s} k(s)ds$ where r_s is the range to the surface, is called the "path-integrated attenuation" (PIA). The quantity $\exp[-0.2 \ln 10 \times \int_0^r k(s)ds]$ is called the "attenuation factor."

If C, m, and r are known, we can calculate $Z_m(r)$ from the measured $P_r(r)$. To calculate the rainfall profile $R(r)$, we require $Z(r)$ or $k(r)$. When the attenuation is negligible, we can equate $Z(r)$ to $Z_m(r)$ and the rainfall rate $R(r)$ can be estimated by using a Z–R relationship. In order to attain a reasonable beamwidth with a small antenna, however, the wavelength used in a spaceborne or airborne radar must be rather short and, therefore, the signal is attenuated by rain. In such

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a case we need to solve equation (2) for the unknown functions $Z(r)$ and $k(r)$ for a given function $Z_m(r)$.

This problem is obviously ill posed and cannot be solved uniquely unless some relation exists between $Z(r)$ and $k(r)$. One such solution, derived by Hitschfeld and Bordan (1954) with a power-law relation ($k = \alpha Z^{\beta}$), gives a reasonable estimate if the attenuation effect is small. When the attenuation is large, however, the Hitschfeld–Bordan solution can become unstable. To solve this instability problem, several methods have been proposed. In what follows, some of the representative methods (Fujita 1989; Hitschfeld and Bordan 1954; Kozu et al. 1991; Marzoug and Amayenc 1991; Meneghini 1978; Meneghini et al. 1983; Meneghini and Nakamura 1990) are reviewed and compared.

2. Surface reference methods

Surface reference methods estimate the PIA through rain from the decrease in the surface return. In particular, an estimate of the attenuation factor at $r = r_s$ is obtained from a ratio of the surface return power measured in rain to that measured in adjacent rain-free areas:

$$\hat{A}(r_s) = \frac{P_r(r_s;\,\text{rain})}{P_r(r_s;\,\text{no rain})}. \quad (4)$$

An estimate of the PIA, which follows directly from $\hat{A}(r_s)$, is then used to constrain the Hitschfeld–Bordan solution.

If k is related to Z by $k = \alpha Z^{\beta}$ and β is constant in range, (2) can be rewritten as a differential equation in the following way:

$$\frac{du}{dr} + u\beta \frac{d}{dr} \ln Z_m + q\alpha = 0, \quad (5)$$

where $u = Z^{-\beta}$ and $q = 0.2\beta \ln 10$. Note that α can be a function of r. A general solution to this equation is

$$Z(r) = Z_m(r)[C_1 - qS(r)]^{-1/\beta}, \quad (6)$$

where C_1 is an arbitrary constant, and $S(r)$ is defined by

$$S(r) = \int_0^r \alpha(s) Z_m^{\beta}(s) ds. \quad (7)$$

If the initial condition

$$Z(r) = Z_m(r) \text{ at } r = 0 \quad (8)$$

is given, then C_1 becomes 1. This corresponds to the Hitschfeld–Bordan solution:

$$Z_{\text{HB}}(r) = Z_m(r)[1 - qS(r)]^{-1/\beta}. \quad (9)$$

Note that the differential equation (5) with this initial condition is equivalent to the integral equation (2) with $k = \alpha Z^{\beta}$.

If, instead, the final condition on $Z(r)$ is given at $r = r_s$, the following condition must be satisfied by C_1:

$$C_1 = \left[\frac{Z_m(r_s)}{Z(r_s)}\right]^{\beta} + qS(r_s). \quad (10)$$

Substituting this into (6), we get

$$Z_{\text{fv}}(r) = Z_m(r)\left\{\left[\frac{Z_m(r_s)}{Z(r_s)}\right]^{\beta} + q[S(r_s) - S(r)]\right\}^{-1/\beta}. \quad (11)$$

If we define A_s by

$$A_s \overset{\text{def}}{=} \exp\left(-0.2 \ln 10 \int_0^{r_s} k ds\right), \quad (12)$$

then

$$\frac{Z_m(r_s)}{Z(r_s)} = A_s, \quad (13)$$

and the solution can be written as

$$Z_{\text{fv}}(r) = Z_m(r)\{A_s^{\beta} + q[S(r_s) - S(r)]\}^{-1/\beta}. \quad (14)$$

We will call this method the "final value method" hereafter. This solution to the final value problem is equivalent to the solution given in terms of k by Marzoug and Amayenc (1991). In fact,

$$\begin{aligned} k_{\text{fv}}(r) &= \alpha Z_{\text{fv}}^{\beta} \\ &= \alpha Z_m(r)^{\beta}\{A_s^{\beta} + q[S(r_s) - S(r)]\}^{-1} \\ &= \alpha Z_m(r)^{\beta}\left[A_s^{\beta} + q\int_r^{r_s} \alpha Z_m^{\beta}(s) ds\right]^{-1}. \end{aligned} \quad (15)$$

If we define $w_0(r)$ by

$$w_0(r) = \frac{q\alpha Z_m(r)^{\beta}}{A_s^{\beta}}, \quad (16)$$

then (15) becomes

$$k_{\text{fv}}(r) = \frac{1}{q} w_0(r)\left[1 + \int_r^{r_s} w_0(s) ds\right]^{-1}. \quad (17)$$

This is identical to the form derived by Marzoug and Amayenc (1991).

It should be noted that A_s need not be obtained from a surface reference measurement $\hat{A}(r_s)$; for example, an estimate of A_s determined from a radiometer with a similar beamwidth at the same frequency as the radar would suffice.

The final value method uses the PIA as the single condition to choose the solution. Unlike the forward solution of the Hitschfeld and Bordan equation, this solution is stable. Since the solution is solved backward from the surface, it depends only on the measured $Z_m(r)$ between the point of interest and the surface. The values of $Z_m(r)$ above the range r do not affect the solution. This is an advantage of the final value method since it is not easy to model the k–Z relation appropriately at high altitudes where the phase and size distribution of precipitating particles are not known. Since the integration constant C_1 is adjusted to satisfy the surface condition, the solution does not satisfy the natural initial condition (8).

In order to satisfy both initial and PIA conditions, it is necessary to introduce an adjustable parameter. Different surface reference methods adjust different model parameters so that the total attenuation calculated from the Hitschfeld–Bordan solution with the adjusted parameters equals the PIA. Mathematical formulation reveals the differences and similarities among the different methods.

Let us introduce a correction factor defined by

$$\epsilon \stackrel{\text{def}}{=} \frac{1 - A_s^\beta}{qS(r_s)}. \quad (18)$$

If the model parameters α and β accurately represent the k–Z relationship and if there is no error in the radar calibration or the PIA estimate, ϵ becomes unity. This is the case when the total attenuation is equal to the attenuation calculated from the retrieved $Z(r)$ with an assumed k–Z relationship. Note that the converse of the above statement is not necessarily true: $\epsilon = 1$ does not necessarily imply that the model parameters are free of errors.

The α adjustment method (Meneghini et al. 1983; Meneghini and Nakamura 1990) adjusts the coefficient α by a factor of ϵ and makes the two attenuations equal. The corrected $Z(r)$ is given by

$$\begin{aligned} Z_\alpha(r) &= Z_m(r)[1 - \epsilon qS(r)]^{-1/\beta} \\ &= Z_m(r)\{A_s^\beta + \epsilon q[S(r_s) - S(r)]\}^{-1/\beta}. \quad (19) \end{aligned}$$

The solution is expressed in two different forms here in order for the easy comparison with the Hitschfeld–Bordan solution (9) and with the final value solution (14), respectively. The equivalence of these two forms can be easily verified by using the relation (18).

The radar constant adjustment method (Meneghini et al. 1983; Meneghini and Nakamura 1990) adjusts the radar constant C and makes the two attenuations equal. The corrected $Z(r)$ is given by

$$\begin{aligned} Z_C(r) &= \epsilon^{1/\beta} Z_m(r)[1 - \epsilon qS(r)]^{-1/\beta} \\ &= Z_m(r)\left\{\frac{A_s^\beta}{\epsilon} + q[S(r_s) - S(r)]\right\}^{-1/\beta}. \quad (20) \end{aligned}$$

It is easy to see from (9), (14), (19), and (20) that $Z_C(r) = \epsilon^{1/\beta} Z_\alpha(r)$ and that the following inequalities hold:

$$\begin{aligned} &\text{if } \epsilon > 1, \quad Z_C(r) > Z_{\text{fv}}(r) > Z_\alpha(r) > Z_{\text{HB}}(r), \\ &\text{if } \epsilon = 1, \quad Z_C(r) = Z_{\text{fv}}(r) = Z_\alpha(r) = Z_{\text{HB}}(r), \\ &\text{if } \epsilon < 1, \quad Z_C(r) < Z_{\text{fv}}(r) < Z_\alpha(r) < Z_{\text{HB}}(r). \quad (21) \end{aligned}$$

It may be worthwhile to mention a few special cases here. As indicated in (21), if the correction factor ϵ is unity, that is, if no correction is necessary, the solutions are identical. It is also observed from (14) and (19) that at the surface $S(r) = S(r_s)$, and hence, $Z_{\text{fv}}(r_s) = Z_\alpha(r_s)$. On the other hand, if the total path attenuation is large [$A_s \ll 1$ and $qS(r_s) \approx 1$], then $Z_{\text{fv}}(r) \approx Z_C(r)$ near the rain top where attenuation is not yet significant [$S(r) \approx 0$].

In contrast to the above methods that assume independent choices for k–Z and Z–R laws, Kozu et al. (1991) has proposed a method in which the number density N_0 of the exponential raindrop size distribution is adjusted to satisfy the PIA condition. When N_0 is adjusted, the coefficients in k–Z and Z–R relations change accordingly. If the unadjusted coefficients in the k–Z relation used in the α or C adjustment method are the same as the coefficients used in this model at the initial N_0, and if the k–R relation is almost linear—for example, for light rain at Ka band—then this method gives a rain profile similar to the one given by the C adjustment method. (See the appendix.)

Fujita (1989) takes the ratios of adjacent range bin data, thereby eliminating the radar constant C from the equation. The Z–R and k–R relations are used to express Z and k in terms of R. Finally, the integrated rainfall rate over the path is introduced as an additional constraint so that the set of simultaneous equations becomes solvable. It is possible to use the PIA condition as the constraint instead of the path-integrated rainfall rate (PIRR). In this case, the solution obtained from Fujita's method is identical to the solution given by the C adjustment method (except in the finite-binning effect) because both methods adjust the radar constant C, either explicitly or implicitly, to find the solution to (2) with the PIA constraint. Fujita's method, however, requires a nonlinear least-square fit routine and is extremely slow.

3. Iteration method

The iteration method (Meneghini 1978) solves (2) directly by iteration assuming a particular k–Z relationship. One of the advantages of this method relative to the Hitschfeld–Bordan method is the fact that any functional form can be chosen for the k–Z relationship. In particular, unlike the surface reference technique and the Hitschfeld–Bordan method where it is necessary to assume that the power β is constant, this assumption is not necessary in the iteration scheme.

If the same power law k–Z relationship with constant β is used, the iteration method converges to the Hitschfeld–Bordan estimate, provided, of course, that the Hitschfeld–Bordan method itself converges. If the measured $Z_m(r)$ is used as the first approximation of $Z(r)$ in the iteration, the nth approximation to the solution is an increasing function of n for a fixed r. However, the convergence is not uniform. The iteration

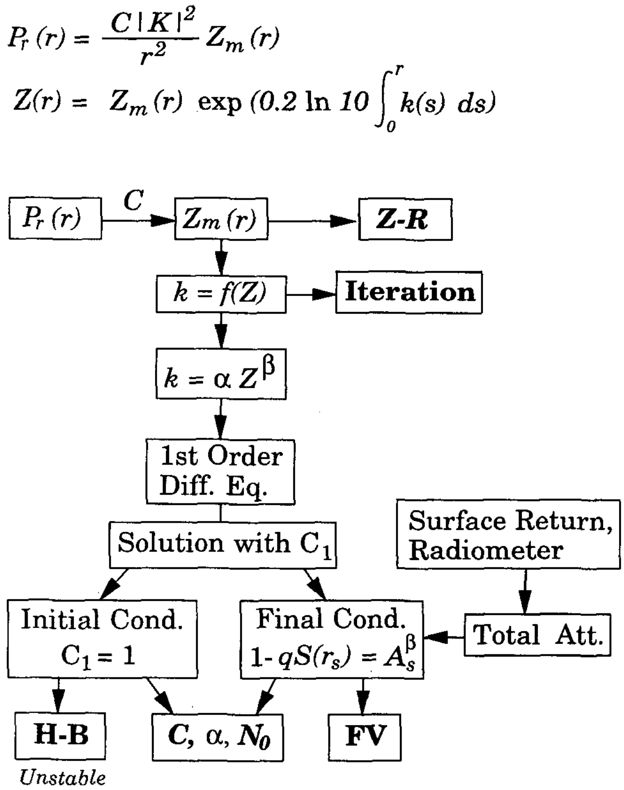

FIG. 1. Relation among different rain profile retrieving methods. The abbreviations H–B and FV stand for the Hitschfeld–Bordan and the final value methods, respectively. Other symbols are defined in the text.

converges to the Hitschfeld–Bordan solution in moving outward from the near to the far ranges. In other words, if the Hitschfeld–Bordan estimate is the true estimate, the iteration solution that is stopped at a finite order will underestimate at far ranges even if the estimate at close ranges is approximately correct. Even if the Hitschfeld–Bordan solution diverges, we can obtain a nondivergent estimate by stopping the iteration at a certain order. However, since the iteration does not converge throughout the entire range, this estimate is no more reliable than the Hitschfeld–Bordan estimate.

In practice, when the Hitschfeld–Bordan method converges, the second- or third-order iteration generally gives rather good estimates if the attenuation is small. Since we do not know any rain model that uses a k–Z relationship other than a power law with a constant power and since the iteration method gives exactly the same solution as the Hitschfeld–Bordan solution in the case of a power-law k–Z relationship, we will not consider this method any further.

4. Discussion

Figure 1 summarizes the various methods reviewed in this paper. A simple Z–R method, in general, gives negatively biased estimates when the attenuation is not negligible. [The method in which the measured $Z_m(r)$ is used to calculate the rainfall rate R instead of $Z(r)$ in a Z–R relationship is called the "Z–R method" in this paper.] The Hitschfeld–Bordan method tries to compensate for this attenuation effect. The compensation factor estimated from the measured $Z_m(r)$ profile is given by $[1 - qS(r)]^{-1/\beta}$. [See (9).] If, for example, the two-way attenuation to range r is 20 dB, this factor must be 100 or $qS(r) = 0.99$ if $\beta = 1$. If $qS(r)$ is overestimated by 1%, then the compensation factor becomes 10 000 (or an estimated attenuation of 40 dB) and the retrieved rainfall rate becomes highly biased. If on the other hand $qS(r)$ is underestimated by 1%, the same factor becomes 50; that is, the estimated attenuation is 17 dB, which is equivalent to a −3-dB bias. This example shows that the solution is very sensitive to the estimate of the integrated attenuation. The surface reference method forces this estimate to the independently obtained integrated attenuation at the farthest point of the measurement, thereby avoiding these instabilities.

Figure 2 shows an example where all correction methods give almost identical profiles. In this and the following figures, the corrections are applied to the Ka-band (35 GHz) airborne radar data that were taken during the Convective and Precipitation/Electrification Experiment in July 1991 in the vicinity of Florida (Iguchi et al. 1992). The radar has two channels, X band (10 GHz) and Ka band (34.5 GHz), and their beams are matched. The data shown were all taken over the ocean and the total attenuations needed for the surface reference methods are calculated from the decrease of the surface return in the rain area from that in the rain-free area. A fixed Z–R relationship is used to convert the retrieved Z profiles into the R profiles. Also shown

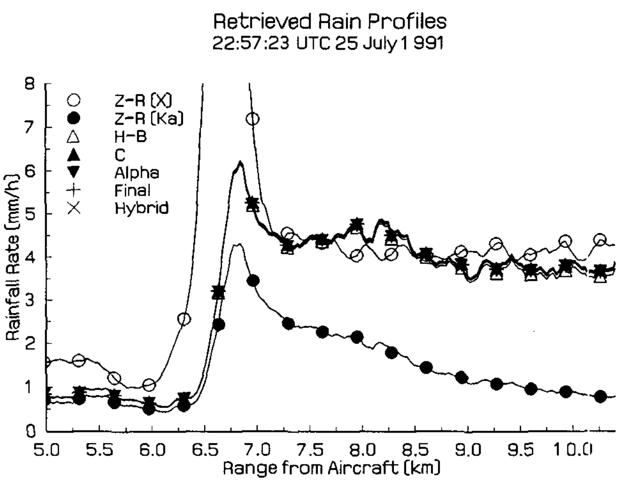

FIG. 2. Retrieved vertical rain profiles by various methods. The altitude of the airplane was 10.7 km during the measurement. In this example, $\epsilon = 1.01$ and all methods give nearly identical profiles. The hybrid method is described in the discussion section.

in the figure is the rainfall rate calculated with a Z–R relationship from the X-band (10 GHz) radar data. For both frequencies, a power law is assumed in the Z–R relationship. The constants used in $Z = aR^b$ are $a = 200$ and $b = 1.6$ for X band and $a = 330$ and $b = 1.30$ for Ka band. The initial parameters in the k–Z relationship are $\alpha = 0.0020$ and $\beta = 0.808$. The same parameters are used for the entire range even though they may be applicable only to the liquid phase precipitation. As a result, the apparent rainfall rate at the bright band (6.5–7.0 km from the aircraft) turns out rather high, especially in X band. Such high Z factors observed are considered to be caused by the partially melted snow aggregates. Because the dimensions of such aggregates are rather large and comparable to the wavelength of the Ka-band radar, the enhancement at Ka band is not as prominent as at X band.

Since $Z_C(r) = \epsilon^{1/\beta}Z_\alpha(r)$, the closeness of ϵ to unity implies $Z_C(r) \approx Z_{\rm fv}(r) \approx Z_\alpha(r)$; that is, all surface reference methods give approximately the same rain profiles. However, the fact that $\epsilon \approx 1$ does not imply that the correction to the Hitschfeld–Bordan solution is small when the attenuation is significant and $qS(r_s) \approx 1$. In other words, the correction is essential even if $\epsilon \approx 1$. Such an example is shown in Fig. 3 where ϵ, in this particular case, is 0.97. In this example, because of the large attenuation, the Ka-band data beyond the range of 9.5 km are masked by the system noise and convey no meteorological information. On the other hand, a large deviation of ϵ from 1 implies either that the initial choice of parameters (C, α, and β) are inaccurate or that the PIA estimate is in error. Figure 4 shows such an example in which $\epsilon = 1.44$. In order to simulate a large error in the surface attenuation estimate, data collected during a banking flight were used here. Because of the off-nadir incidence angle, the apparent surface return power decreased substantially so that the attenuation was overestimated.

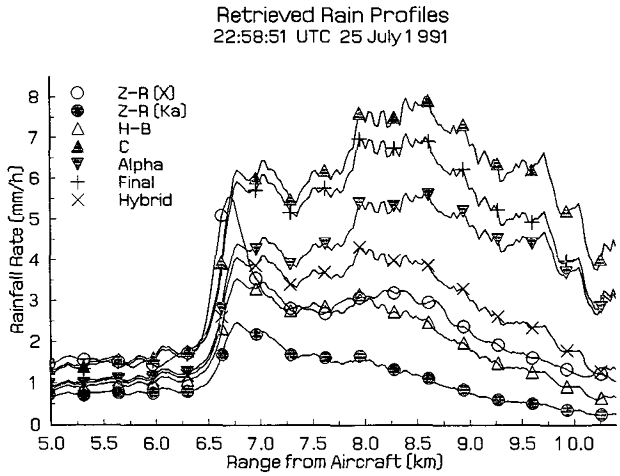

FIG. 4. Retrieved vertical rain profiles by various methods. The airplane was flying at the altitude of 10.7 km but slightly banking (14°) during the measurement. In this example, the correction factor ϵ is not close to 1 ($\epsilon = 1.44$). As a result, all methods give different profiles.

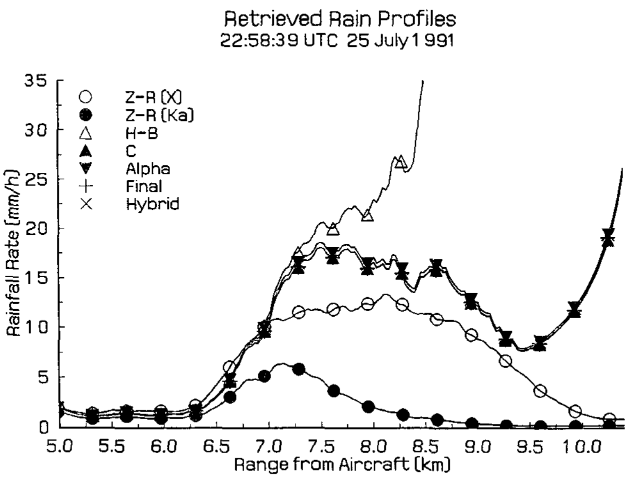

FIG. 3. Retrieved vertical rain profiles by various methods. The altitude of the airplane was 10.7 km during the measurement. In this example, the correction factor ϵ is very close to 1 ($\epsilon = 0.97$). As a result, all surface reference methods give essentially the same profile but the Hitschfeld–Bordan estimate diverges.

The answer to the question as to which method is the most appropriate depends on the situation. The C adjustment method is appropriate if the calibration error is the most likely source of discrepancy between the measured attenuation factor A_s and the calculated attenuation factor $[1 - qS(r_s)]^{1/\beta}$. If the radar is stable, the radar constant C should not change substantially in a short period of time. Therefore, the radar constant C should not be adjusted scene by scene. If the estimate of PIA is in error, none of the surface reference methods can retrieve a correct profile, because they all use the PIA as the basis for correcting the parameters. However, if the PIA is biased by the same magnitude as in the measured Z, and if there is no other error in the model, the final value method will retrieve the correct profile. Such a situation may happen if the radar constant C is in error and if the PIA is determined relative to a model of σ^0 that is independent of the radar calibration error. On the other hand, if the PIA is determined relative to the surface return measured in the rain-free area adjacent to the rain, then the bias in C cancels out in the estimate of PIA, and remains only in the measured Z. In this case, the C adjustment method may be more appropriate.

If the error in the surface reference is the only source of error, the α adjustment method is least sensitive to such an error among the three surface reference methods. In this case the Hitschfeld–Bordan estimate can be regarded as the true profile, and the inequalities in (21) show that the Z_α estimate is closest to the $Z_{\rm HB}$ estimate. The example shown in Fig. 5 was taken while the airplane was banking so that the apparent surface

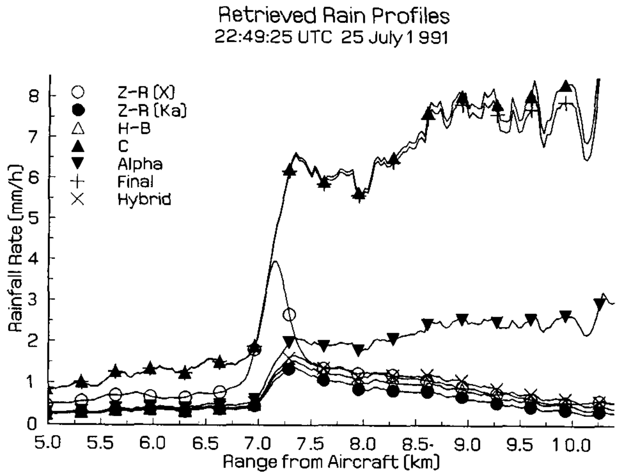

FIG. 5. Retrieved vertical rain profiles by various methods. The airplane was flying at the altitude of 10.7 km with the bank angle of 23° during the measurement. As a result, the path-integrated attenuation is significantly overestimated and all surface reference methods give higher rainfall rates than the Z–R estimate as derived from the X-band data. Since the rainfall rate is very small in this example, the Hitschfeld–Bordan estimate and the X-band Z–R estimate are considered to be closer to the true profile than the estimates produced by surface reference methods. The correction factor ϵ in this example is 3.4.

echo decreased substantially in spite of the very light rain above the surface. In the surface reference methods without the correction of incidence angle effect, this decrease was totally attributed to the attenuation by rain. As a result, they all overestimate the rainfall rates. In this case, $S(r_s) = \int_0^{r_s} \alpha Z_m^\beta ds$ is close to 1 and its major contributor is large Z_m due to the surface clutter. As a result, $S(r)$ remains small in comparison with $S(r_s)$ for the range shown in the figure. In this example, A_s^β is also small. Because ϵ is rather large (=3.4), the denominator of (19) is much larger than that of (14). This is the reason that the α adjustment solution is much smaller than the final value solution. However, when the range goes into the clutter region, which is not shown in the figure, $S(r)$ suddenly increases and eventually becomes equal to $S(r_s)$ at the surface; that is, $Z_\alpha(r_s) = Z_{fv}(r_s)$, as stated previously.

In the discussion above, we adjusted either the radar constant C or the coefficient α. Both, however, can be adjusted. In this case, if the original α and C are denoted by α_0 and C_0, respectively, the PIA constraint gives the relation that the new parameters α and C must satisfy

$$\alpha C^{-\beta} = \epsilon\alpha_0 C_0^{-\beta}. \tag{22}$$

This relation defines a simple curve on the α–C plane. Any point on this curve will give a $Z(r)$ profile that is consistent with both the measured reflectivity profile and the surface attenuation. The α adjustment method and the C adjustment method are special cases of this more general solution in which either C or α is fixed to the given value and not adjusted. Because there is no a priori criterion to choose the correct point on the curve, a further constraint is needed to select a particular point or to limit the range of choices. For example, a good calibration will restrict the magnitude by which C can be adjusted.

To relate the measured radar return power to the rainfall rate, we need to chose two relationships from Z–R, k–Z, and k–R relationships. Note that any of these relationships is automatically defined by the other two. Because the measured power is most closely related to Z, we formulate the different methods in terms of Z in this paper. However, we can also formulate the same relationships in terms of k if preferred. Once we assume these relationships among Z, k, and R, the results are equivalent, and hence, the retrieved rainfall profile will be the same regardless of which relationship is used to calculate R. Sometimes, the use of k–R relationship is preferred to the Z–R relationship because of its robustness to the variations in the drop size distribution. However, in so far as we start from the radar return power, the choice of k as a fundamental variable does not provide any improvement to the Z formulation.

It may be worthwhile to note here that the α adjustment method and C adjustment method give identical k profiles (Marzoug and Amayenc 1993). This can be seen easily by substituting (19) and (20) into the relations $k_\alpha(r) = \epsilon\alpha Z_\alpha(r)^\beta$ and $k_C(r) = \alpha Z_C(r)^\beta$, respectively. Note that we need to use the adjusted α (i.e., $\epsilon\alpha$) in the α adjustment case. As a result, if we use a constant k–R relationship, we get identical rainfall profiles by the α and C adjustment methods. In the examples shown, we used a constant Z–R relationship. This implies that when α is adjusted in the k–Z relationship, the corresponding k–R relationship is adjusted accordingly, and the rain profiles by the C and α adjustment methods differ even though the k profiles are the same.

In this paper we formulate the problem in terms of the reflectivity factor Z rather than the rainfall rate R because Z is more closely related to the observable quantity $P_r(r)$. This also allows us to separate the problem of attenuation correction from the issue of the ambiguities in the Z–R relation. However, if the constraint is given by the PIRR instead of PIA, then we need to introduce either a Z–R or a k–R relation into our formulation. If we rewrite the equations in terms of the rainfall rate R, two more parameters come into play. These are the coefficient a and the power b in the Z–R power-law relationship: $Z = aR^b$. If these parameters are regarded as adjustable, we have a total of five adjustable constants including α, β, and C, which makes the problem even more unwieldy. Although many different combinations of these parameters will be consistent with the measured values of $Z_m(r)$ and PIA (or PIRR), in general, the solutions will yield different rainfall rate profiles.

It may be worthwhile to mention the difference between the PIA and PIRR conditions. Since the quantity we want to estimate ultimately is the rainfall rate and not the attenuation, the PIRR condition will give a better estimate of the rain profile than the PIA condition. This is particularly true in horizontally nonuniform rain in which the PIA is, in general, less than the attenuation expected from uniform rain with the same average rainfall rate. Nevertheless, there is a serious question about how to obtain the PIRR. The surface reference method estimates the PIA and not the PIRR. For example, Fujita (1989) suggests that the PIRR can be estimated from the surface echo attenuation, which is essentially equivalent to the PIA. However, we need to assume a k–R relationship to get the PIRR from the PIA. If this is the case, the PIRR is no better than the PIA. Instead of the surface return attenuation, we can use radiometer data as the constraint. The constraint given by the radiometer data is more closely related to the PIA condition than to the PIRR condition, because the radiometer essentially measures the attenuation. In this regard, the radiometer suffers the same problem as the radar when the rain distribution is not horizontally uniform.

The N_0 adjustment method (Kozu's method) assumes a drop size distribution that relates the coefficients α, β, a, and b through the parameters in the drop size distribution function. In particular, by adjusting a drop size distribution parameter N_0 in the exponential distribution function $N(D) = N_0 \times \exp(-3.67D/D_0)$ where D is the diameter of the particle, this method, in effect, adjusts the values α, β, a, and b simultaneously to meet the PIA condition. Because the relationships among the parameters α, β, a, and b are based on a raindrop size distribution model, this approach to reduce the number of independent parameters is physically appealing.

All correction methods, in general, assume that the k–Z relationship is constant along the path of the radar beam. This may not be a valid assumption in all types of rain, particularly in convective storms. The formulation of these methods itself allows us to make the k–Z relationship range dependent. For example, α can be parameterized according to the altitude in the Hitschfeld–Bordan and in the surface reference methods. However, in practice, it is not very easy to choose a functional form of the range dependence except in a typical stratiform rain case in which the rain, snow, and ice layers can be identified.

The weakness of the surface reference methods arises from the fact that if the PIA is in error, the profiles obtained are biased. The bias is rather large if the total attenuation is small. This can be seen from the definition of the correction factor ϵ in (18). In fact, if the attenuation is weak, then $qS(r_s)$ is small and A_s^β is close to 1. Following the same argument made above for the instability of the Hitschfeld–Bordan method, we can say that the correction factor ϵ may have a large error if A_s^β is close to 1 and if the estimate is not exact. For example, the attenuation coefficient at Ka band is about 0.23 dB km^{-1} mm^{-1} h (Ulaby et al. 1981). Therefore, if the thickness (height) of the uniform rain region is 4 km and if the two-way total attenuation (two times the PIA) is 18 dB, then the rainfall rate is estimated to be 10 mm h^{-1}. If the PIA is in error by 3 dB, then this will under- or overestimate the rainfall rate by 3.3 mm h^{-1} depending on the sign of the error. If the PIA error, in dB, remains constant, the absolute error in the path-integrated rainfall rate is constant regardless of the total attenuation. This is the strength and at the same time the weakness of the surface reference methods. The relative error decreases with the increasing rainfall rate. On the other hand, if the rainfall rate is small—for example, if the rainfall rate is 3.3 mm h^{-1}—the relative error in the surface reference method may become unacceptably large (100% in our example). The actual magnitudes of biases caused by the error in the estimate of the PIA differ in the different surface reference methods. As mentioned before, the relative bias is largest in the C adjustment method and smallest in the α adjustment method. In the α adjustment method, the bias becomes 0 at the rain top or at $r = 0$, while in the C adjustment method and the final value method, the error in A_s persists to the rain top from the surface. These facts can be verified by setting $S(0) = 0$ in (14), (19), and (20) and by assuming $\epsilon \neq 1$ in (18). Figures 4 and 5 are such examples. Note that if we use a fixed k–R relationship instead of a fixed Z–R relationship, the relatively low bias in the α adjustment method will disappear in the rainfall rate R.

When the attenuation is negligibly small, the apparent reflectivity factor $Z_m(r)$ is approximately the same as the actual reflectivity factor $Z(r)$ and no further correction is needed. As long as the attenuation is small, the Hitschfeld–Bordan solution provides a reasonable estimate. Because the Hitschfeld–Bordan estimate asymptotically converges to the Z–R solution as the attenuation goes to zero, we can use the Hitschfeld–Bordan method whenever the attenuation is small.

The problem is then how to divide the regions for the Hitschfeld–Bordan method and the surface reference method. A simple weighting method by the attenuation does not work because the Hitschfeld–Bordan solution often diverges to infinity at a certain range. We can devise a method in which the solution never diverges but asymptotically converges to the Hitschfeld–Bordan estimate when the attenuation is small and to the surface reference estimate when the attenuation is large. One such method is obtained by modifying the correction factor ϵ to

$$\epsilon = 1 + w(x)(\epsilon_0 - 1), \qquad (23)$$

where ϵ_0 is the correction factor given by (18) and the weighting function $w(x)$ converges to 0 when $x \to 0$ and to 1 when $x \to 1$. The variable x represents the normalized total attenuation; that is, $x = 0$ when there is no attenuation and $x = 1$ when the attenuation is infinite (the attenuation factor is 0). We need to chose $w(x)$ in such a way that the quantity $1 - \epsilon qS(r)$ never becomes zero for any possible value of x so that the right-hand side of (19) or (20) does not diverge. The actual form of function $w(x)$ that minimizes the relative error depends on the errors in the estimate of PIA and in the model parameters. The normalized total attenuation x can be estimated by using the PIA, $qS(r_s)$, or a combination of the two. A simple choice of x is $qS(r_s)$ itself; that is,

$$x = \begin{cases} qS(r_s), & qS(r_s) \leq 1 \\ 1.0, & qS(r_s) > 1. \end{cases} \tag{24}$$

Examples of the hybrid method are shown in Figs. 2–5, where the estimates are obtained by the α adjustment method with the modified correction factor given by (23). The weighting function used is $w(x) = x$ with x defined by (24). The hybrid estimate in Fig. 5 shows that even though no special consideration on the magnitudes of errors in PIA and the parameters is given for the selection of the weighting function $w(x)$, this formulation is already free from the large error in the surface reference method in light rain when the PIA estimate is severely biased. Preliminary trials of different weighting functions such as x^2, $1 - (1 - x)^2$, $\sin^2(\pi x/2)$ show improvement over the linear weighting function for some examples but do not show a significant overall improvement. The hybrid method is proposed here as a possible method of reducing the error and the bias in the estimate. Although a simple weighting method in Z, such as $Z = [1 - w(x)]Z_m + w(x)Z_\alpha$ appears to work well, it can be said that such a method will almost certainly give a negatively biased profile because Z_m is negatively biased. The optimum selection of the weighting method and its weighting function depends upon the variability in the model parameters and in the surface reference. We would like to discuss this question together with the possibility of other hybrid methods in a future paper.

It has been shown using airborne radar data (Meneghini et al. 1992; Kozu et al. 1991; Marzoug and Amayenc 1992) that the surface reference technique works well for stratiform rain at Ka band over the ocean. This is mainly because the variation of the scattering cross section of the sea surface is relatively small in comparison to the large attenuation by rain at this frequency. It is also important to note that the horizontal uniformity of rain within the radar beam contributes to the success. As an example of extreme nonuniformity in the rainfall rate, consider a case in which the rain area covers only half the beam. The estimate of PIA from the surface return never exceeds 3 dB no matter how heavy the rain is (provided that the ray approximation of propagation is valid). This kind of situation will cause a serious error in the surface reference estimate. Such a problem seldom occurs in airborne data because the field of view is in general much smaller than the horizontal correlation length of the storm. This is particularly true in stratiform rain.

For a spaceborne radar such as the Tropical Rainfall Measuring Mission precipitation radar, which has a footprint of about 5 km, the uniformity assumption may not be valid. Many small-scale convective storms have horizontal dimensions similar to the footprint size. Since the Z–R relationship is nonlinear, the measured Z factor, which is the average of Z within the beam weighted by the antenna gain pattern, will tend to overestimate the average rainfall rate of nonuniform rain if the attenuation is negligible. The attenuation effect will further complicate the problem. The PIA through a horizontally nonuniform rain is always smaller than the attenuation expected from a uniform rain with the same average rainfall rate (Nakamura 1991; Amayenc et al. 1993). Therefore, if the measured PIA is used as the constraint to determine the path-averaged rainfall rate, the estimate will result in a negative bias in the retrieved rain profile in any surface reference method, provided that nonuniform beam filling is the only error source. The amount of error depends on the nonuniformity of rain. Because the correlation length tends to decrease as the rain intensity increases, the usefulness of the surface reference technique may be severely limited. Note that the Hitschfeld–Bordan method also suffers from the nonuniform beam-filling problem (Amayenc et al. 1993).

5. Conclusions

In this paper the retrieval methods for the vertical rain profile from an airborne or spaceborne radar were reviewed and compared. The analytical solution to correct the attenuation effect does not work well when the attenuation becomes significant. To circumvent this instability problem, surface reference techniques have been proposed. With the path-integrated attenuation as a constraint, the instability problem can be solved. This has been verified experimentally by using dual-frequency airborne data (Meneghini and Nakamura 1990; Meneghini et al. 1992; Kozu et al. 1991; Marzoug and Amayenc 1992). Nevertheless, there are still many degrees of freedom in the choice of an adjustable parameter and of the values of parameters themselves. To improve the accuracy of the estimate and to accommodate different types of rain, it is necessary not only to decrease the uncertainty of each parameter but to find the interrelationships among the parameters for different rain types. This information will narrow the possible values that the parameters can take. The N_0 adjustment method follows this general principle. Aside from the rain-related parameters, other possible quantities that may be adjusted are the radar calibration constant C and the surface reference estimate of the PIA. An accurate calibration of radar will eliminate the need for C adjustment and allow the constraint of path attenuation to be used to reduce the uncertainty in the meteorological parameters.

The accuracy of all surface reference methods relies heavily on the accuracy of the surface reference estimate of the PIA. In this paper we proposed a hybrid method that avoids the large relative error associated with the surface reference method when the attenuation is small.

A related problem is how accurately can the PIA be obtained from the surface reference technique. The decrease of the surface return is not caused by rain attenuation alone. Over the ocean the surface cross section changes according to the wave height and the wind speed. It may also be modified by rain striking the ocean surface (Bliven and Giovanangeli 1993; Tsimplis and Thorpe 1989; Moore et al. 1979). The magnitude of this change and its dependence on frequency, wind speed, incidence angle, and rainfall rate has not been quantified. Over the land the uncertainty and variation of surface cross section can be more pronounced.

Even with such uncertainties, the combination of the Hitschfeld–Bordan method and the surface reference technique seems to be the most promising method currently available for profiling the rain from air or space with a single, attenuating wavelength radar.

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APPENDIX

Effect of the PIA Condition on the N_0 Adjustment Solution

The H–B solution is given by

$$Z_{\mathrm{HB}}(r) = Z_m(r)\left[1 - q\int_0^r \alpha Z^\beta ds\right]^{-1/\beta},$$

or in terms of rainfall rate,

$$\begin{aligned} R_{\mathrm{HB}}(r) &= cZ^d \\ &= cZ_m(r)^d\left[1 - q\int_0^r \alpha Z^\beta ds\right]^{-d/\beta}. \end{aligned}$$

Now suppose that this solution does not satisfy the PIA condition and that we need to modify the k–Z relation from $k = \alpha Z^\beta$ to $k = \alpha' Z^{\beta'}$. In Kozu's N_0 adjustment method (Kozu et al. 1991), this modification will change the R–Z relation through the change of N_0 from $R = cZ^d$ to $R = c'Z^{d'}$. If the k–R relation is linear, then $c \propto \alpha$ and $d = \beta$ for all N_0. Suppose that the modification of k–Z relation is done by adjusting α by a factor η: $\alpha' = \eta\alpha$ and $\beta' = \beta$. Under the above assumption, the constant c will also be adjusted to $c' = \eta c$ and the constant d remains the same. This adjustment will result in a new rainfall estimate R_{N_0}:

$$\begin{aligned} R_{N_0}(r) &= c'Z_m(r)^{d'}\left[1 - q\int_0^r \alpha' Z^{\beta'} ds\right]^{-d'/\beta'} \\ &= \eta c Z_m(r)^d\left[1 - q\int_0^r \eta\alpha Z^\beta ds\right]^{-d/\beta}. \end{aligned}$$

On the other hand, the solution given by the C adjustment method is

$$R_C(r) = \epsilon^{d/\beta} c Z_m(r)^d\left[1 - q\int_0^r \epsilon\alpha Z^\beta ds\right]^{-d/\beta}.$$

In our case, because of the linear k–R relation, $d/\beta = 1$. Therefore, R_{N_0} and R_C are the same. (Here, ϵ becomes identical to η in this case.)

At X band, the k–R relation is not linear. For a fixed Z, the relative increase of the attenuation coefficient k is larger than that of R when N_0 is increased. This implies that the overall adjustment to R in this case is not as large as in the linear case for the same magnitude of adjustment to k. On the other hand, because R is still an increasing function of N_0 for a fixed Z, the adjusted R must be larger than the α adjustment estimate when $\epsilon > 1$. From these facts, we can conclude that when $\epsilon > 1$, $R_C > R_{N_0} > R_\alpha$, and that when $\epsilon < 1$, $R_C < R_{N_0} < R_\alpha$, if the parameters before the adjustment are the same for all methods. Here, of course, we assume that the parameters c and d do not change in the C and α adjustment methods.

REFERENCES

Amayenc, P., M. Marzoug, and J. Testud, 1993: Analysis of cross-beam resolution effects in rainfall rate profile retrieval from a spaceborne radar. *IEEE Trans. Geosci. Remote Sens.*, **31**, 417–425.

Bliven, L. F., and J.-P. Giovanangeli, 1993: An experimental study of microwave scattering from rain- and wind-roughened seas. *Int. J. Remote Sens.*, **14**, 855–869.

Fujita, M., 1989: An approach for rain rate profiling with a rain-attenuating-frequency radar under a constraint on path-integrated rainrate. *IGARSS'89,* 1491–1494.

Hitschfeld, W., and J. Bordan, 1954: Errors inherent in the radar measurement of rainfall at attenuating wavelengths. *J. Meteor.*, **11,** 58–67.

Iguchi, T., 1992: Radar depolarization signature of rain in cumulus clouds measured with a dual-frequency air-borne radar. *IGARSS'92,* 1728–1730.

Kozu, T., K. Nakamura, R. Meneghini, and W. C. Boncyk, 1991: Dual-parameter radar rainfall measurement from space: A test result from an aircraft experiment. *IEEE Trans. Geosci. Remote Sens.*, **29,** 690–703.

Marzoug, M., and P. Amayenc, 1991: Improved range-profiling algorithm of rainfall rate from a spaceborne radar with path-integrated attenuation constraint. *IEEE Trans. Geosci. Remote Sens.*, **29,** 584–592.

——, and ——, 1992: A new class of dual-frequency algorithms for rain rate profiling from a spaceborne radar: Principle and tests. *IGARSS'92,* 1376–1378.

——, and ——, 1994: A class of single- and dual-frequency algorithms for rain-rate profiling from a spaceborne radar. Part I: Principle and tests from numerical simulations. *J. Atmos. Oceanic Technol.*, **11,** 1480–1506.

Meneghini, R., 1978: Rain-rate estimates for an attenuating radar. *Radio Sci.*, **13,** 459–470.

——, and K. Nakamura, 1990: Range profiling of the rain rate by an airborne weather radar. *Remote Sens. Environ.*, **31,** 193–209.

——, J. Eckerman, and D. Atlas, 1983: Determination of rain rate from a space-borne radar using measurements of total attenuation. *IEEE Trans. Geosci. Remote Sens.*, **21,** 34–43.

——, T. Kozu, H. Kumagai, and W. C. Boncyk, 1992: A study of rain estimation methods from space using dual-wavelength radar measurements at near-nadir incidence over ocean. *J. Atmos. Oceanic Technol.*, **9,** 364–382.

Moore, R. K., Y. S. Yu, A. K. Fung, D. Kaneko, G. J. Dome, and R. E. Werp, 1979: Preliminary study of rain effects on radar scattering from water surfaces. *IEEE. J. Oceanic Eng.*, **OE-4,** 31–32.

Nakamura, K., 1991: Biases of rain retrieval algorithms for spaceborne radar caused by nonuniformity of rain. *J. Atmos. Oceanic Technol.*, **8,** 363–373.

Tsimplis, M., and S. A. Thorpe, 1989: Wave damping by rain. *Nature,* **342,** 893–895.

Ulaby, F. T., R. K. Moore, and A. K. Fung, 1981: *Microwave Remote Sensing.* Vol. 1. Addison–Wesley, 456 pp.