NOTES AND CORRESPONDENCE

Uncertainty in Vertically Integrated Liquid Water Content due to Radar Reflectivity Observation Error

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ABSTRACT

Radar reflectivity is used to estimate meteorological quantities such as rainfall rate, liquid water content, and the related quantity, vertically integrated liquid (VIL) water content. The estimation of any of these quantities depends on several assumptions related to the characteristics of the physical processes controlling the occurrence and character of water in the atmosphere. Additionally, there are many sources of error associated with radar observations, such as those due to brightband, hail, and drop size distribution approximations. This work addresses one error of interest, the radar reflectivity observation error; other error sources are assumed to be corrected or negligible. The result is a relationship between the uncertainty in VIL water content and radar reflectivity measurement error. An example application illustrates the estimation of VIL uncertainty from typical radar reflectivity observations and indicates that the coefficient of variation in VIL is much larger than the coefficient of variation in radar reflectivity.

1. Introduction

Advances in operational remote sensing capabilities for rainfall observation [e.g., National Weather Service (NWS) WSR-88D radar network] highlight the need for objective methods relating expected or characteristic errors in radar observations to error characteristics of derived liquid water quantities describing the state of a rain cloud system. While many error types and sources exist in radar observations, this work addresses only measurement errors; other errors, such as brightband, hail, and drop size distribution approximations, are assumed to be corrected or negligible. Estimates of error magnitudes are useful in determining the confidence associated with a derived quantity, such as in-cloud liquid water content within a given radar pixel, and for updating the state or establishing the initial conditions of a rainfall forecasting model. Rainfall forecasting algorithms capable of utilizing error information are typically stochastic state–space formulations of models such as those developed by Georgakakos and Bras (1984a, 1984b), Lee and Georgakakos (1990), Seo and Smith (1992), and French and Krajewski (1994). The models address the goal of estimating the true state of a system (such as vertically integrated liquid water content), given that observations are in error and models are imperfect.

In general, estimation of the true state of a system is based on the relative confidence associated with the model and the observations (Bras and Rodriguez-Iturbe 1985). Once a "best" estimate of the model state (for example, liquid water content) is determined, stochastic equations of system dynamics can be integrated in time to obtain a forecast of the state. A discussion of the process of optimal estimation of a variable from information about the mean and variance is described by Gelb (1974). The technique provides a means for systematically employing all available external measurements, regardless of their errors, to improve the accuracy of estimate of the state (Gelb 1974).

Typically there are two sources of precipitation observations, rain gauge and radar. Rain gauge observation errors are relatively well defined in terms of the error associated with a particular observation; however, the same is not true for radar-based observations of rainfall. Additionally, since radar measures rainfall amounts indirectly, the need for a method for estimating the errors is all the more necessary. Using

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straightforward assumptions regarding radar reflectivity observation error structure, together with corresponding analytical relationships defining radar reflectivity, liquid water content, and statistical moments, a relationship between radar reflectivity observation error and uncertainty in vertically integrated liquid (VIL) water content is derived.

2. Radar reflectivity observations and VIL

Radar reflectivity is a measure of the amount of power backscattered by raindrops in a volume of the atmosphere sampled by the radar. Reflectivity magnitude depends on the number, size, and arrangement of raindrops in the sample volume. This work relies on the assumptions used in derivation of the standard weather radar equation available in references such as Battan (1973) and Doviak and Zrnić (1993).

A fundamental component in the expression relating liquid water mass to radar reflectivity is the raindrop size distribution (DSD). Doviak and Zrnić (1993) report on several observed size distributions of raindrops, and the work of Marshall and Palmer (1948) revealed a simple dependence of exponentially decreasing drop concentration on increasing diameter. Ulbrich (1983) proposed a more general form of the DSD, commonly known as a gamma-type formulation, with an additional parameter—one in which the exponential is a special case. Additionally, a well-defined function relates two of the gamma DSD parameters (Ulbrich 1983). The more tractable exponential DSD offers nearly the same flexibility as the gamma DSD and is more appealing from an analytical and computational point of view. The exponential DSD is used in this work as

$$N(D) = N_0 e^{-\Lambda D}, \qquad (1)$$

with $N(D)$ ($\mathrm{m^{-3}\,m^{-1}}$) equal to the number of drops per unit volume per unit size interval having equivolume spherical diameter D (m) and where N_0 ($\mathrm{m^{-3}\,m^{-1}}$) and Λ ($\mathrm{m^{-1}}$) are parameters of the distribution. The form was originally proposed by Marshall and Palmer (1948), who also suggested that Λ varies with rainfall rate R ($\mathrm{mm\,h^{-1}}$) as Λ ($\mathrm{m^{-1}}$) $= 4100R^{-0.21}$, and N_0 has the constant value $N_0 = 8 \times 10^6\ \mathrm{m^{-3}\,m^{-1}}$. Parameter values vary according to storm type, time of measurement with respect to storm cycle, topography, season, and climate (Jameson 1991).

The liquid water content M, in a volume of the atmosphere containing raindrops, is defined as the sum of the liquid water mass of all drops in a unit volume (Doviak and Zrnić 1993), and using the DSD in (1) gives

$$M = \frac{\pi}{6}\rho \int_0^\infty D^3 N(D)dD = \pi\rho N_0 \Lambda^{-4}, \qquad (2)$$

where ρ is water density. The integral from 0 to ∞ is an approximation to reality since there is an upper limit on the known size of drops. However, the form of the DSD allows this approximation since as $D \to \infty$, $N(D) \to 0$; and the use of (2) is evident due to the appealing form of the analytical solution. Alternatively, defining the integral over the limits 0 to $D_{\max}$ (where $D_{\max}$ is an upper limit on the size of drops) is appealing from a physical point of view, but it is less appealing in an application since the solution to (2) would be mathematically more complex. Additionally, the integral from 0 to ∞ is the most commonly used form.

Using the definition of radar cross section for raindrops, it can be shown (Doviak and Zrnić 1993) that radar reflectivity, at any point in space, is defined as the sum of sixth powers of drop diameters in a unit volume, with the implicit assumption that the raindrops are small compared to the radar wavelength. Using the DSD in (1) leads to

$$Z(x, y, h) = \int_0^\infty D^6 N(x, y, h, D)dD = \Gamma(7)N_0(x, y, h)\Lambda(x, y, h)^{-7}, \qquad (3)$$

where Z is radar reflectivity, x and y are Cartesian coordinates in the horizontal, h is height, $\Gamma(\ )$ is the gamma function defined as $\Gamma(n+1) = n!$ (Gradshteyn and Ryzhik 1980) leading to $\Gamma(7) = 6! = 720$, and the expression is valid for nonattenuating radars (i.e., wavelengths of at least 10 cm where the observed raindrops are small compared to the wavelength, and for wavelengths of 5 cm at light to moderate rainfall rates).

The expression in (3) can be solved for one DSD parameter, for example N_0, to yield

$$N_0(x, y, h) = \frac{\Lambda(x, y, h)^7}{720} Z(x, y, h). \qquad (4)$$

Substitution of (4) into (2) provides a relationship for liquid water content $M(x, y, h)$ in terms of radar reflectivity

$$M(x, y, h) = \frac{\pi\rho\Lambda(x, y, h)^3}{720} Z(x, y, h). \qquad (5)$$

Using observed radar reflectivity and (5), it is possible to construct a discretized three-dimensional field of reflectivity-derived liquid water content (the vertical distribution of liquid water content in columns of the atmosphere). The vertically integrated liquid (VIL) water content in a single cloud column in such a field, above the horizontal spatial location (x, y), is defined as

$$\mathrm{VIL}(x, y) = \int_{h_b}^{h_t} M(x, y, h)dh, \qquad (6)$$

where h_b is the height of the bottom of the cloud column, and h_t is the height of the top of the cloud column.

For operational purposes, VIL(x, y) is computed as the discrete summation of M over the number of ver-

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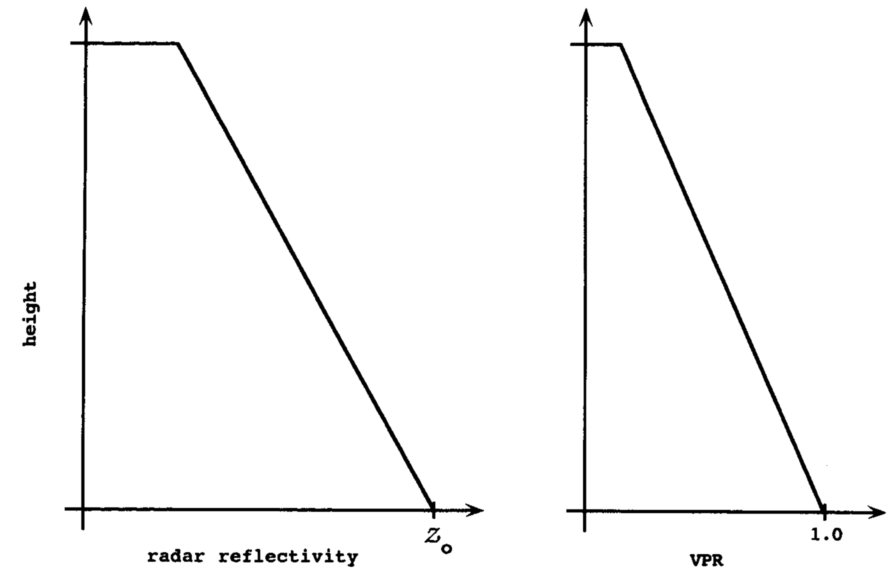

FIG. 1. Schematic illustration of vertical distribution of radar reflectivity and VPR.

tical observations n between radar echo-base and radar echo-top. The expression in (6) becomes the following:

$$\mathrm{VIL}(x, y) = \sum_{i=1}^{n} M_i(x, y)\Delta h_i(x, y), \qquad (7)$$

where $M_i(x, y)$ is liquid water content at the ith scan elevation angle, and $\Delta h_i(x, y)$ is the height of the corresponding elevation increment. Substituting a form of (5) into (7), assuming constant DSD parameters, and letting the spatial dependence of all terms be implicit leads to

$$\mathrm{VIL} = \frac{\pi\rho\Lambda^3}{720}\sum_{i=1}^{n} \Delta h_i \mathcal{Z}_i, \qquad (8)$$

where $\mathcal{Z}_i$ is reflectivity from the ith scan elevation angle over a particular (x, y) location.

In general, the scan elevation angle, range, and beamwidth will define the height Δh_i associated with a particular reflectivity observation $\mathcal{Z}_i(x, y)$. Most weather radars are configured such that the elevation angle may be either constant or variable, with the particular scan elevation sequence computer-controlled; for the purpose of this work, the elevation increment is assumed to be constant. Assuming a constant Δh for a given spatial location (x, y), over n scan elevations, (8) can be written as

$$\mathrm{VIL} = C\sum_{i=1}^{n} \mathcal{Z}_i, \qquad (9)$$

where $C = \pi\rho\Lambda^3\Delta h/720$.

3. Characteristics of radar reflectivity

Radar reflectivity ($\mathrm{mm^6\,m^{-3}}$) is typically reported as reflectivity Z in units of dBZ, with the relationship between Z and $\mathcal{Z}$ defined by

$$Z = 10\log_{10}\left(\frac{\mathcal{Z}}{1\ \mathrm{mm^6\,m^{-3}}}\right). \qquad (10)$$

To facilitate the uncertainty analysis of VIL, the expression in (10) can be rewritten in an equivalent form as

$$Z = \frac{1}{\psi}\ln\left(\frac{\mathcal{Z}}{1\ \mathrm{mm^6\,m^{-3}}}\right), \qquad (11)$$

where $\psi = 0.1\ln(10)$. The goal is to derive a formulation that is compatible with readily available statistical definitions and to express the unknown moments of VIL in terms of the more quantifiable moments of the observed variable Z. The form of (11) is appealing from a statistical point of view since the relationships defining the moments of a lognormal variable exist; use of traditional statistical definitions in the derivation of VIL uncertainty is possible.

a. Vertical profile of radar reflectivity

In general, a vertical profile of reflectivity observations at a particular horizontal location (x, y) is defined by $\mathcal{Z}(x, y, h)$. A dimensionless vertical profile of reflectivity (VPR) can be associated with the spatial domain of interest, as defined by the function $\mathrm{VPR}(h)$ (refer to Fig. 1, for example), and may be assumed constant over the entire domain. Defining a VPR for the domain is appealing since it provides a means of

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characterizing the three-dimensional field of reflectivity in a compact and meaningful form. The VPR(h) is related to the observed reflectivity profile by the following relationship:

$$Z(x, y, h) = Z_0(x, y)\mathrm{VPR}(h), \quad (12)$$

where Z_0 is the base-level reflectivity, $Z(x, y, h = h_b)$, VPR($h = h_b$) = 1.0, and VPR($h > h_b$) may be greater than 1.0. The mean and variance of reflectivity follow directly by taking expectations of (12) and are given by

$$E[Z(x, y, h)] = E[Z_0(x, y)]\mathrm{VPR}(h) = \bar{Z}_0(x, y)\mathrm{VPR}(h) \quad (13)$$

and

$$\mathrm{var}[Z(x, y, h)] = \mathrm{var}[Z_0(x, y)]\mathrm{VPR}(h)^2 = \sigma^2_{Z_0}(x, y)\mathrm{VPR}(h)^2, \quad (14)$$

where $E[\ ]$ is the expected value operator, $\bar{Z}_0(x, y)$ is the expected value or mean of base-level reflectivity, the variance of base-level reflectivity is $\mathrm{var}[Z] = E[(Z - E[Z])^2] = \sigma^2_{Z_0}$.

b. *Characteristics of VIL*

The definition of VIL over the depth of a cloud column at a particular horizontal spatial location (x, y) is characterized as a function of the sum of the vertically distributed reflectivity observations. In general, radar measurement errors may be correlated, especially if a radar is improperly calibrated. In this study, measurement errors associated with reflectivity are considered uncorrelated; additionally, reflectivity observations are assumed to be independent. In reality, reflectivity observations at a given horizontal location, from different scan elevation angles, are separated in time by a period equal to the radar antenna rotation time, but decorrelation is not guaranteed. A correlation function could be introduced at this point, but it would lead to more complicated expressions. Since the goal of this work is to illustrate the propagation of uncertainty in radar-based products, use of the simpler formulation is explained. Recall the definition of VIL given in (9). Taking expected values of both sides, substituting the expressions from (13) and (14), and making the spatial dependence of all terms implicit leads to the following definition of the mean and variance of VIL:

$$\overline{\mathrm{VIL}} = C\bar{Z}_0 \sum_{i=1}^{n} \mathrm{VPR}_i \quad (15)$$

and

$$\sigma^2_{\mathrm{VIL}} = C^2\sigma^2_{Z_0} \sum_{i=1}^{n} \mathrm{VPR}_i^2, \quad (16)$$

where VPR_i is the magnitude of the vertical profile of reflectivity corresponding to the ith scan elevation angle.

Introducing the coefficient of variation of a variable x, which is defined as the standard deviation divided by the mean $\delta_x = \sigma_x/\bar{x}$ (Benjamin and Cornell 1970), provides a convenient, compact, and dimensionless means of describing uncertainty in a variable. The coefficient of variation for VIL is evaluated using (15) and (16), resulting in the following expression:

$$\delta_{\mathrm{VIL}} = \frac{\sigma_{\mathrm{VIL}}}{\overline{\mathrm{VIL}}} = \frac{\sigma_{Z_0}}{\bar{Z}_0} \frac{\left(\sum_{i=1}^{n} \mathrm{VPR}_i^2\right)^{0.5}}{\sum_{i=1}^{n} \mathrm{VPR}_i} = \delta_{Z_0} \frac{\left(\sum_{i=1}^{n} \mathrm{VPR}_i^2\right)^{0.5}}{\sum_{i=1}^{n} \mathrm{VPR}_i}. \quad (17)$$

The formulation of δ_{VIL} in (17) depends on three quantities, the first and second moments of base-level reflectivity and the vertical profile of reflectivity (i.e., vertical gradient of Z).

c. *Moments of base-level radar reflectivity*

Following from the relationship in (11), the moments of base-level reflectivity are related to the mean and variance of logarithmic base-level reflectivity through the following expressions (Bras and Rodriguez-Iturbe 1985):

$$\bar{Z}_0 = \exp\left(\psi\bar{Z}_0 + \frac{\psi^2\sigma^2_{Z_0}}{2}\right) \quad (18)$$

$$\sigma^2_{Z_0} = \exp[2(\psi\bar{Z}_0 + \psi^2\sigma^2_{Z_0})] - \exp(2\psi\bar{Z}_0 + \psi^2\sigma^2_{Z_0}), \quad (19)$$

where the mean and variance of logarithmic base-level reflectivity Z_0 are $\bar{Z}_0$ and $\sigma^2_{Z_0}$.

The coefficient of variation in base-level reflectivity is defined as $\delta_{Z_0} = \sigma_{Z_0}/\bar{Z}_0$, and the expressions in (18) and (19) provide the necessary quantities in terms of the moments of Z_0. Substitution of (18) and (19) into (17) completes the definition of VIL uncertainty, and hence a means of estimating the accuracy of VIL in terms of logarithmic base-level reflectivity moments. The final expression defining VIL uncertainty is

$$\delta_{\mathrm{VIL}} = \frac{\{\exp[2(\psi\bar{Z}_0 + \psi^2\sigma^2_{Z_0})] - \exp(2\psi\bar{Z}_0 + \psi^2\sigma^2_{Z_0})\}^{0.5}}{\exp[\psi\bar{Z}_0 + (\psi^2\sigma^2_{Z_0}/2)]} \frac{\left(\sum_{i=1}^{n} \mathrm{VPR}_i^2\right)^{0.5}}{\sum_{i=1}^{n} \mathrm{VPR}_i}. \quad (20)$$

4. VIL uncertainty estimation

Use of the method of VIL uncertainty estimation is illustrated using two examples. The first example presents a scenario considering a range of base-level radar reflectivity values and three typical VPR profiles or

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gradients. The second example is an application of the proposed algorithm to an observed profile of radar reflectivity recorded by an NWS WSR-74 radar in Oklahoma City, Oklahoma, equipped with a RADAP II processor.

For the first example, values of base-level reflectivity from 15 to 55 dBZ are considered. Three VPR profiles are analyzed for each base-level reflectivity value; each profile represents a different gradient of incremental decreasing reflectivity as scan elevation angle increases. The VPR gradients correspond to incremental decreases in base-level reflectivity of 3, 5, and 10 dB per scan elevation angle. Observation error in logarithmic radar reflectivity is assumed to be 1 dB; this value is typical of the error magnitude associated with a radar such as the NWS WSR-88D. The results are summarized in Fig. 2, where the variation of δ_{VIL} with magnitude of base-level reflectivity Z_0 is shown; the solid line represents a VPR gradient of 10 dB per scan elevation angle, the center dashed line represents a VPR gradient of 5 dB per scan elevation angle, and the lower dashed line represents a VPR gradient of 3 dB per scan elevation angle. The figure shows a limited range of variation in δ_{VIL} for a given VPR gradient; with δ_{VIL} decreasing slightly with increasing base-level reflectivity. The result appears qualitatively correct since the constant observation error is lower relative to the magnitude of the observed quantity at higher reflectivity values. Additionally, δ_{VIL} decreases with a decrease in the magnitude of the incremental change or gradient in the VPR profile. One factor leading to this result is that the number of reflectivity observations for a given base-level reflectivity magnitude increases as the magnitude of the incremental change or gradient in VPR decreases.

For example, at a base-level radar reflectivity of 30 dBZ, $\bar{Z}_0$ = 30 dBZ, and with an observation error of 1 dB, σ_{Z_0} = 1 dB. The coefficient of variation in logarithmic base-level reflectivity is $\delta_{Z_0} = \sigma_{Z_0}/\bar{Z}_0 = 1/30 = 0.0333$. Considering a VPR gradient decreasing with elevation in 5-dB increments per scan elevation angle leads to the following reflectivity profile: 30, 25, 20, 15, 10, 5, 0 dBZ. Implementing the formulation in (20) leads to a coefficient of variation in VIL of δ_{VIL} = 0.1682—a value that is significantly greater than the coefficient of variation in base-level reflectivity.

The second example utilizes two observed reflectivity profiles recorded on 0624 UTC 1 July 1988 by the NWS WSR-74 radar in Oklahoma City, Oklahoma. Details of NWS WSR-74 RADAP II data characteristics, format, and quality control for attenuation, ground clutter, etc., are described in McDonald and Saffle (1989). The first reflectivity profile consists of the following magnitudes from lowest to highest scan elevation angle: 43.5, 42.0, 39.5, 39.5, 32.5, 27.0, 21.0, 21.0 dBZ (azimuth 296° and range 14 n mi). The corresponding VPR is 1.000, 0.708, 0.398, 0.398, 0.079, 0.022, 0.006, 0.006. Assuming a radar observation error of 1 dB, the coefficient of variation in logarithmic base-level reflectivity is δ_{Z_0} = 0.0230, and the coefficient of variation in VIL is δ_{VIL} = 0.1204. For an observation error of 2 dB, the results are δ_{Z_0} = 0.0460 and δ_{VIL} = 0.2509.

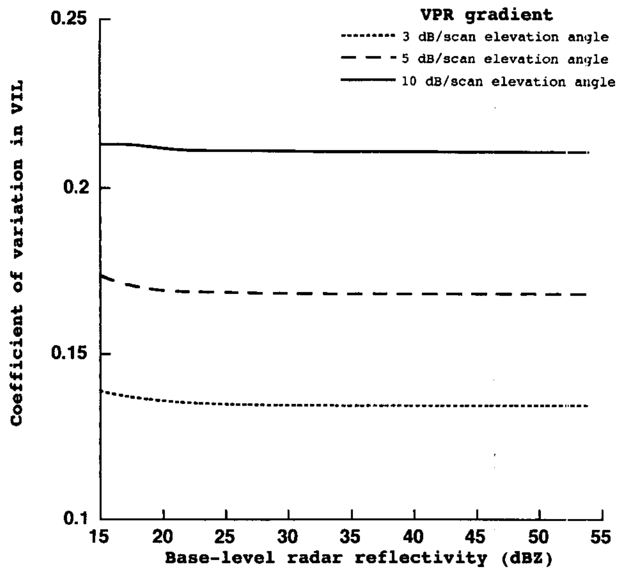

FIG. 2. Variation in VIL coefficient of variation as a function of base-level radar reflectivity for a radar reflectivity observation error of 1 dB.

A second reflectivity profile from the same radar volume scan consists of the following reflectivities, again from lowest to highest scan angle: 46.5, 46.5, 46.5, 48.0, 52.0, 39.5, 32.5, 21.0 dBZ (azimuth 60° and range 15 n mi). The corresponding VPR is 1.000, 1.000, 1.000, 1.413, 3.548, 0.200, 0.040, 0.003; and for a radar observation error of 1 dB, δ_{Z_0} = 0.0215 and δ_{VIL} = 0.1194; for an observation error of 2 dB, δ_{Z_0} = 0.0430 and δ_{VIL} = 0.2488.

5. Conclusions and closing remarks

This work presents an objective analytical approach for estimating the uncertainty in VIL from radar reflectivity observations. The approach is based on reasonable assumptions concerning the characteristics and moments of radar reflectivity observations. The formulation has potentially useful applications in the area of rainfall analysis and forecasting where an estimate of the error associated with liquid water content estimates in the atmosphere is needed. It was shown that while the uncertainty in a single radar reflectivity measurement may be low, a derived quantity, such as VIL, may have a larger uncertainty associated with it. The error may be significant enough to make it desirable to account for the error using a systematic approach. For example, observation error characteristics are desirable for applications of stochastic rainfall forecasting

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models where the uncertainty associated with VIL is utilized in an objective optimal estimation process to estimate the true state of the system.

Two example applications illustrate that VIL uncertainty, for a constant observation error, decreases slightly with increasing base-level reflectivity. The result appears qualitatively correct since the relative magnitude of the error is higher at low values of observed reflectivity. A second example presented an application of the method to observed radar reflectivity profiles and supports the conclusion that a relatively small uncertainty in radar reflectivity leads to a larger uncertainty in the associated VIL estimate.

The presented analyses are based on several assumptions. The most important are those concerning contamination of VPR by signals from non–liquid water hydrometeors. Other assumptions concerning constant parameters of DSD, spatially constant VPR, and lack of correlation in elements of VPR could be relaxed but would lead to substantially more complex expressions for VIL uncertainty. Since our purpose here was just an illustration of the problem of uncertainty propagation in radar-based products, we opted for the simple scenario described herein.

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